# AAE 333

Fluid Mechanics is involved in many problems aerospace engineers face.

Aerospace engineers need to predict and control

- Forces & moments (acting on bodies moving relative to a fluid)
- Rates of heat transfer (between a body and fluid)

Historical examples

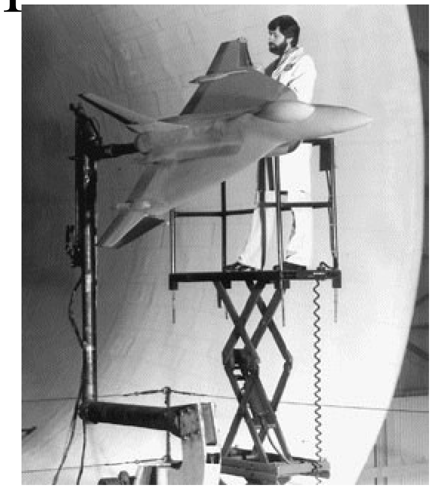
- evaluation of ship resistance
- Wright brothers's experiments to design a new wing
- blunt re-entry body

# Why Study Fluid Mechanics?

**Goal:** To build the theoretical framework which will allow to solve simple engineering problems involving fluid mechanics and give you the basic understanding needed for further study in this area or to work on interdisciplinary problems involving fluid mechanics.

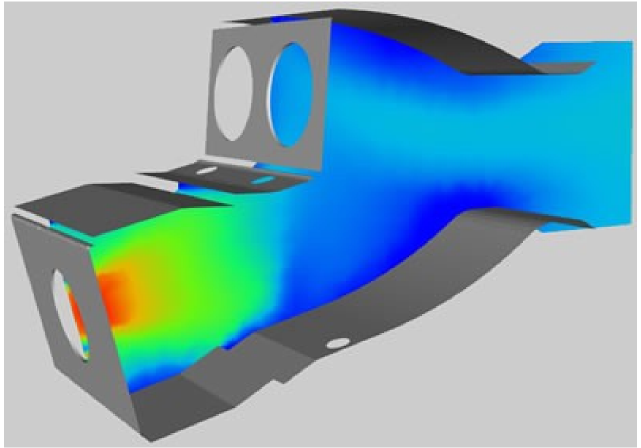
# Other examples:

## airplane lift and drag



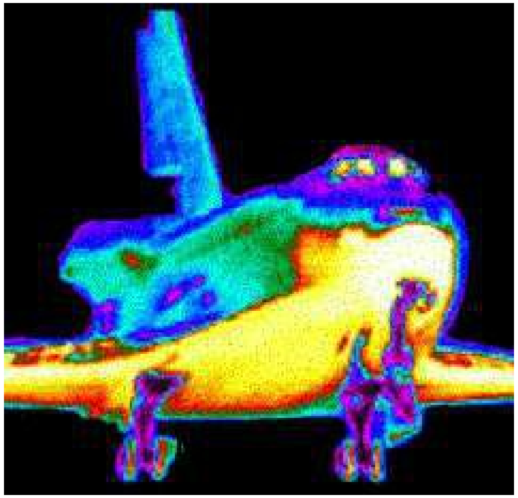
Scale model of fighter aircraft in wind tunnel. (NASA)

## combustion

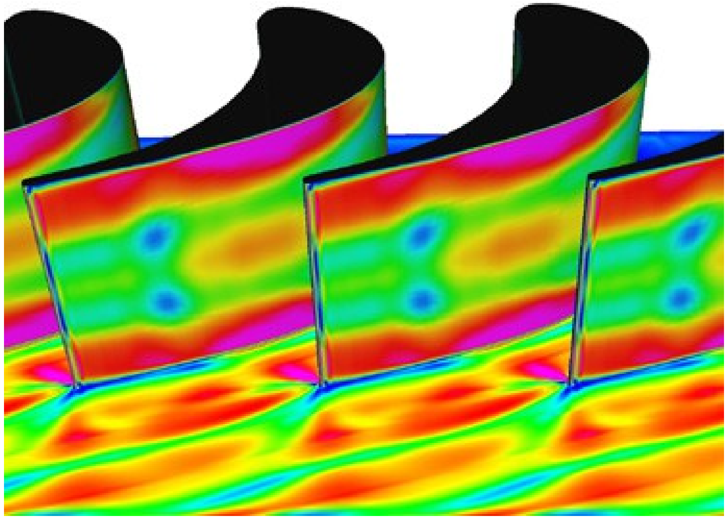


Temperature distribution within the combustion chamber. Two successive combustions followed by a quick dilution reduce significantly NOx emissions (ONERA)

## heating



IR image of a landing space shuttle (Inframetrics)

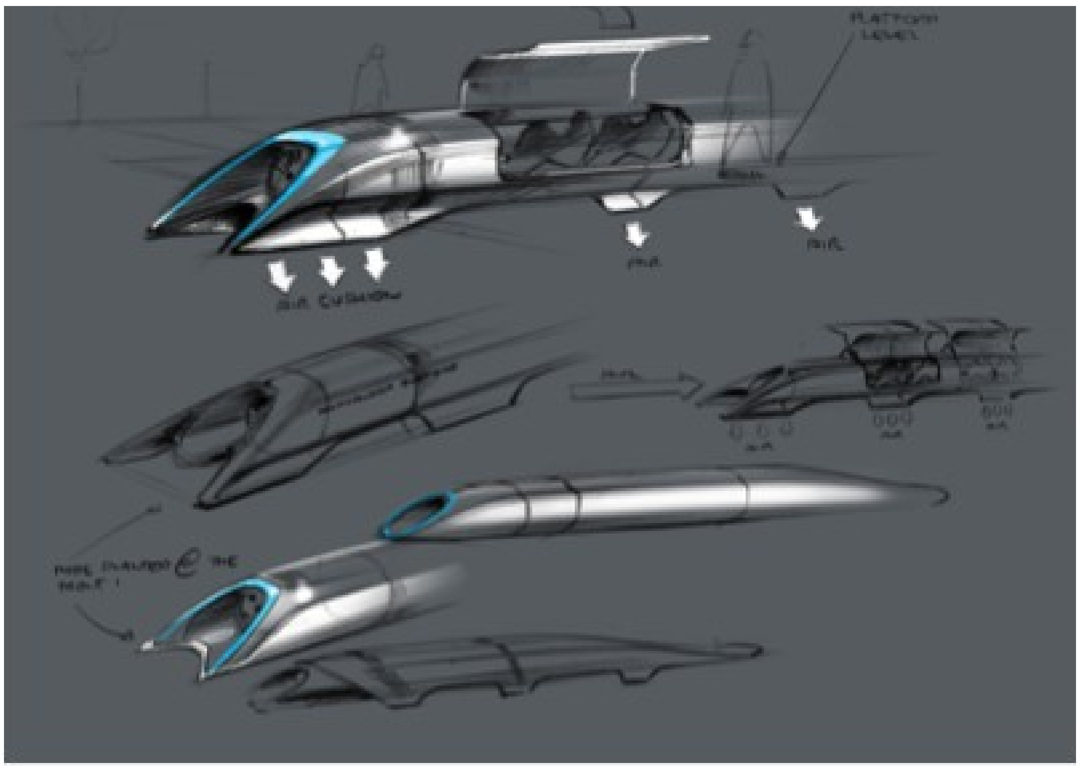


Temperature contours on a transonic turbine rotor (NASA)

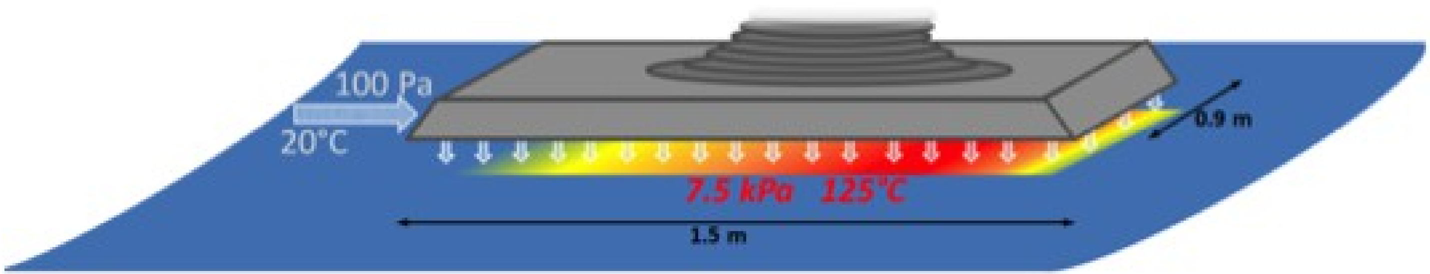
# New Applications?

"Hyperloop Preliminary Design Study"

http://www.teslamotors.com/sites/default/files/blog_images/hyperloop-alpha.pdf



- Tube at rough vacuum: 100 Pa
- Pod speed: high subsonic
- 30 min travel time between LA and SF




Air ski suspension: 0.5-1.3 mm height

Interdisciplinary problems:

- fluid & structures
  - Tail buffet
  - Flutter
  - Cavity noise (acoustic loading)
- fluid & propulsion
  - Scramjet inlet
  - Rocket nozzles
  - Powered lift systems (over-the-wing blowing/ flaps) (high lift, low noise) for V/STOL aircraft
- fluids & controls
  - Control stall of compressors
  - Vortex generators (avoid stall, tail buffet)
  - Reduce drag due to turbulence by active control

Hydrodynamics: flow of

Gas dynamics: flow of

Aerodynamics: flow of

- external
- internal

# Chapter 1: Fundamental Principles

Roadmap for 1st chapter:

introductory concepts

forces & moments

- center of pressure
- types of flow
- dimensional analysis
- flow similarity
- fluid statics

# Introductory Concepts

Fluid: A substance that continuously when a shear force is applied.

Solid: A substance that an applied shear force by static deformation.

A fluid will take the shape of a container, a solid will not.

Q: Is fluid the same as liquid?

2 types of fluids:

Liquid: A fluid in which the molecules are relatively close together and interact strongly; it will take the shape of container but maintains a constant volume.

Gas: A fluid in which the molecules are widely spaced and interact only briefly. A gas will take the shape of a container and expand to fill the entire volume.

In air at standard atmosphere conditions the spacing between molecules is 10 times larger than the molecular diameter (d~4 x$10^{-10}$ m).

# Continuum Hypothesis

Treat the fluid as a continuous medium rather than being made up of molecules separated by void.

Continuum hypothesis is valid when mean free path of molecules is negligible compared to the characteristic size of the flow problem. Continuum hypothesis is about <u>fluid flows</u> not fluids!

For air at 1 atmosphere and room temperature: mean free path is about 60 nanometers ($6 \times 10^{-8}$ m). For most aerodynamic flows of interest, characteristic size is much larger (L~1 m).

Continuum hypothesis fails for:

- Low-pressure (vacuum) systems
- High-altitude (above 100 km) flight
- Flows in micro/nanodevices

We will only consider continuum flows in AAE333.

Non-continuum flows: AAE519, AAE590D.

For continuum fluid flows, we assume that fluid properties vary continuously in space.

Q: What are the most important fluid properties?

# Fundamental Variables:

Pressure(p): normal force per unit area due to molecular motion

Let:

dA = surface element

dF= normal force on dA

p=

density(ρ): =

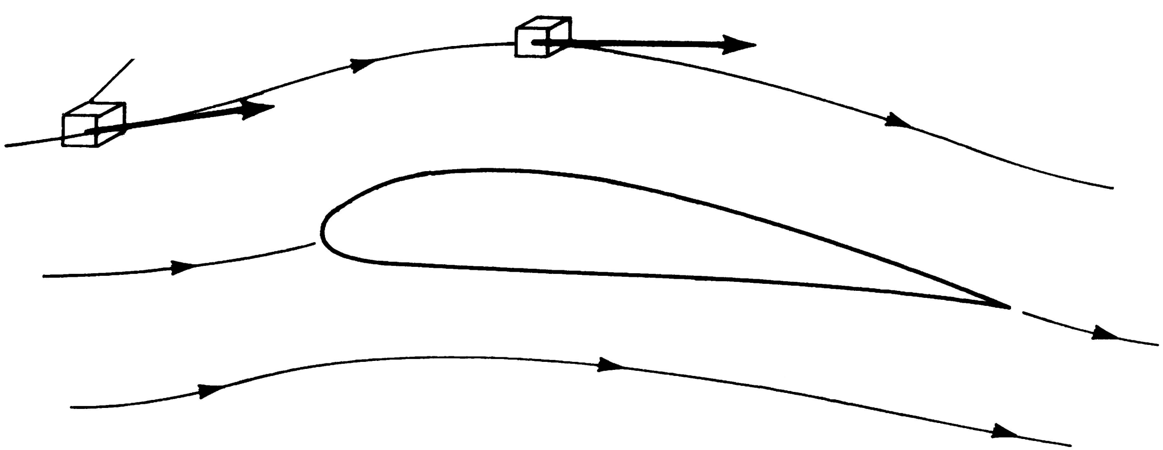
where = volume element,

= mass of

Temperature (T) = directly proportional to kinetic energy of fluid molecules

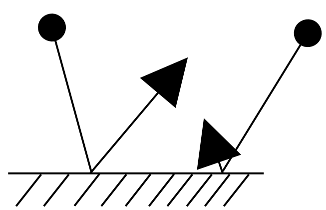
velocity ($\vec{v}$) for a fluid element can vary from point to point; velocity it is a vector

streamline: the path of a fluid element (fixed for steady flow)

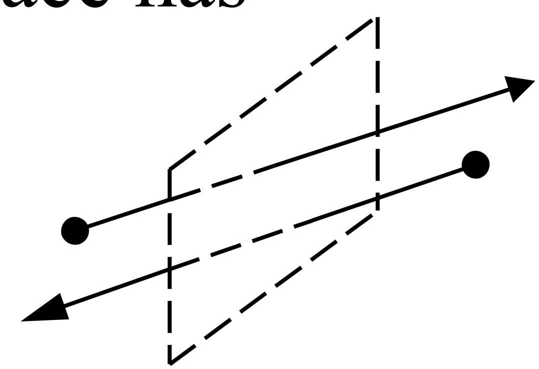


# Forces & Moments

Pressure is a scalar
Pressure force acts perpendicular to any given surface
Pressure has no direction – surface has

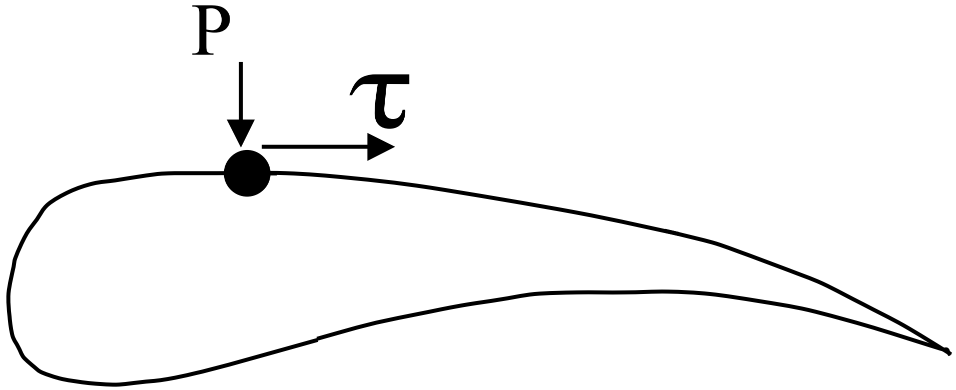


solid surface



in the fluid

Assume a body moving relative to a fluid

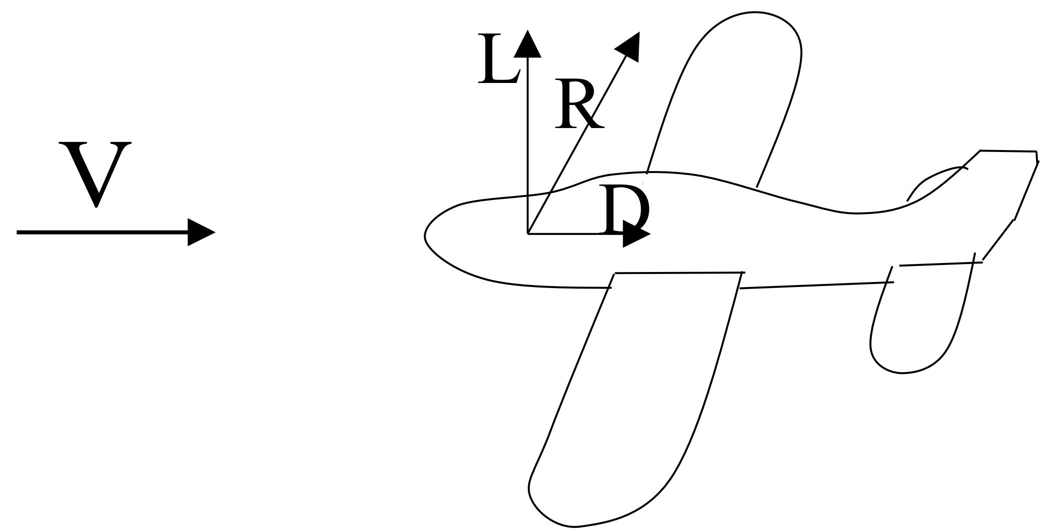



1. Pressure force (                    to surface)

2. Viscous force (                         to surface) due to transfer of momentum due to molecular motion in the presence of a velocity gradient

=viscosity coefficient

# Forces on a Body

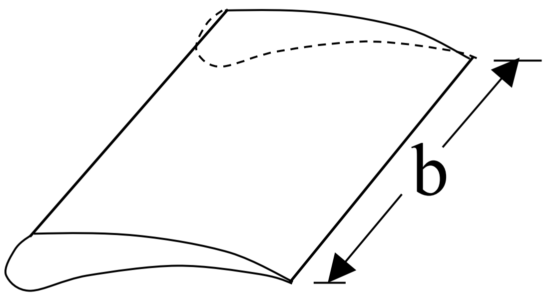


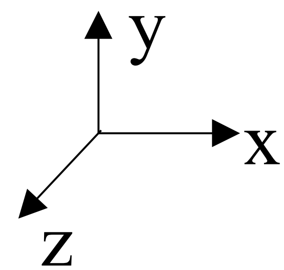

L=Lift : force

D= Drag : force

R= Resultant (total) force      R=

# 2-D Flow:




If: 1) no variation in the z direction

2) no velocity in the z direction

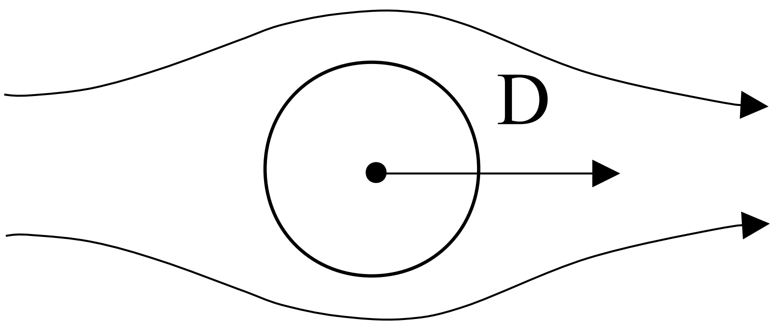
2-D flow corresponds to a wing of infinite span;

3-D effects at the tips

Denote

*L'*,*D'*,*R'* : forces/unit span (N/m)

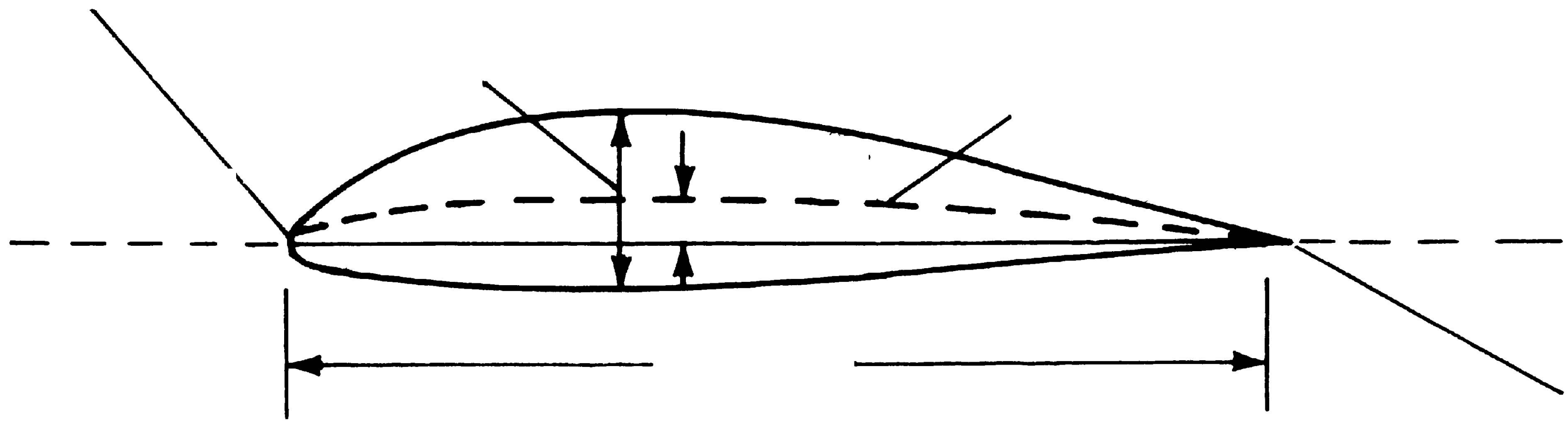
# Example: cylinder in cross flow




Let $D' = 100$ N/m

| Length of cylinder | drag force |
|---|---|
| 1m | D= |
| 10m | |
| 0.1m | |

# Airfoil nomenclature (Anderson, Ch. 4)



Leading edge (LE)

Trailing edge (TE)

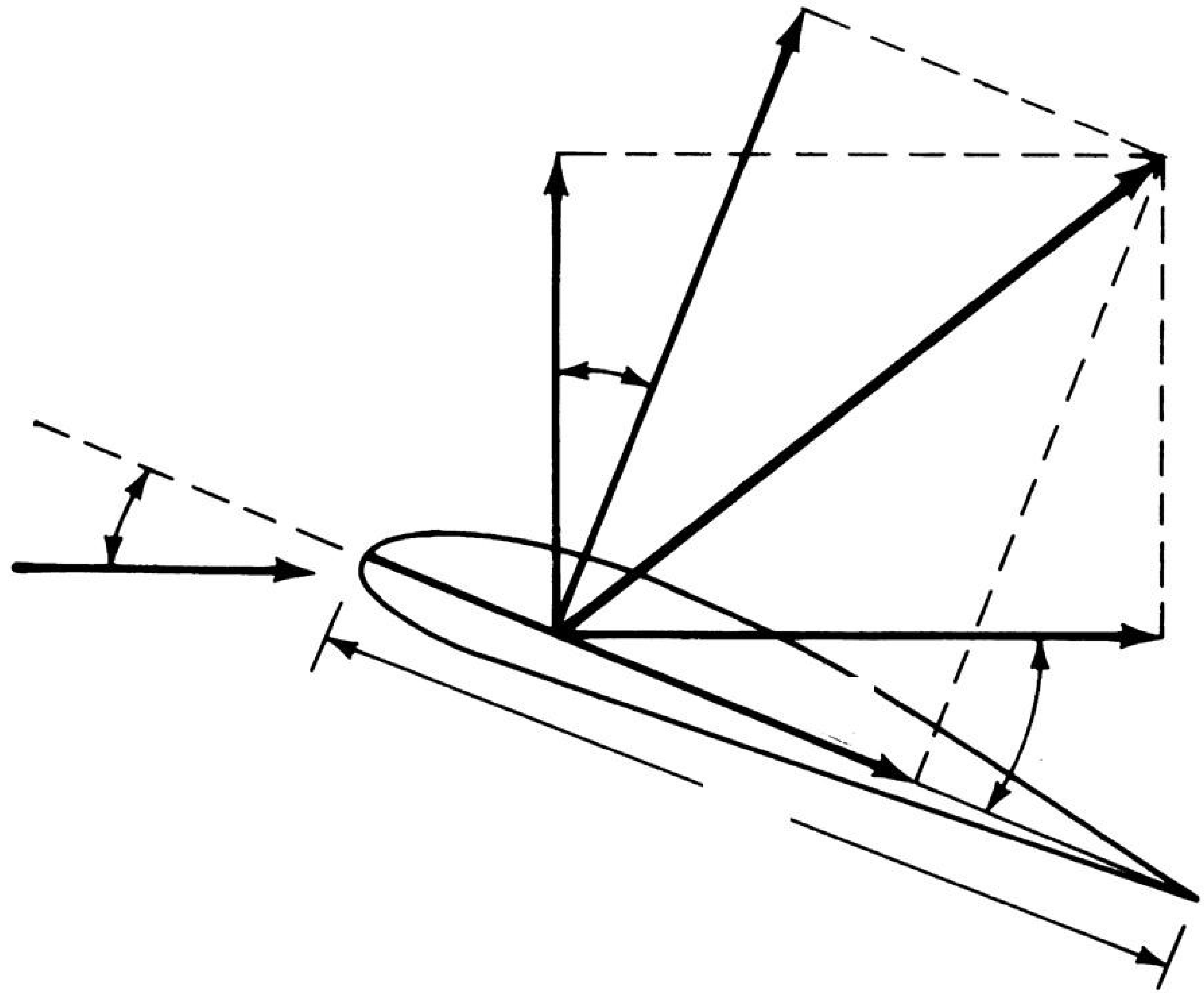
Chord (c)

Thickness (t)

Mean camber line

Camber

# Forces on airfoil



TE= trailing edge,    LE=leading edge

c= chord length,     =angle of attack

Forces: $R' =$ force/unit span

$L' =$ force/unit span

$N' =$ force/unit span

$A' =$ force/unit span

$D' =$ force/unit span

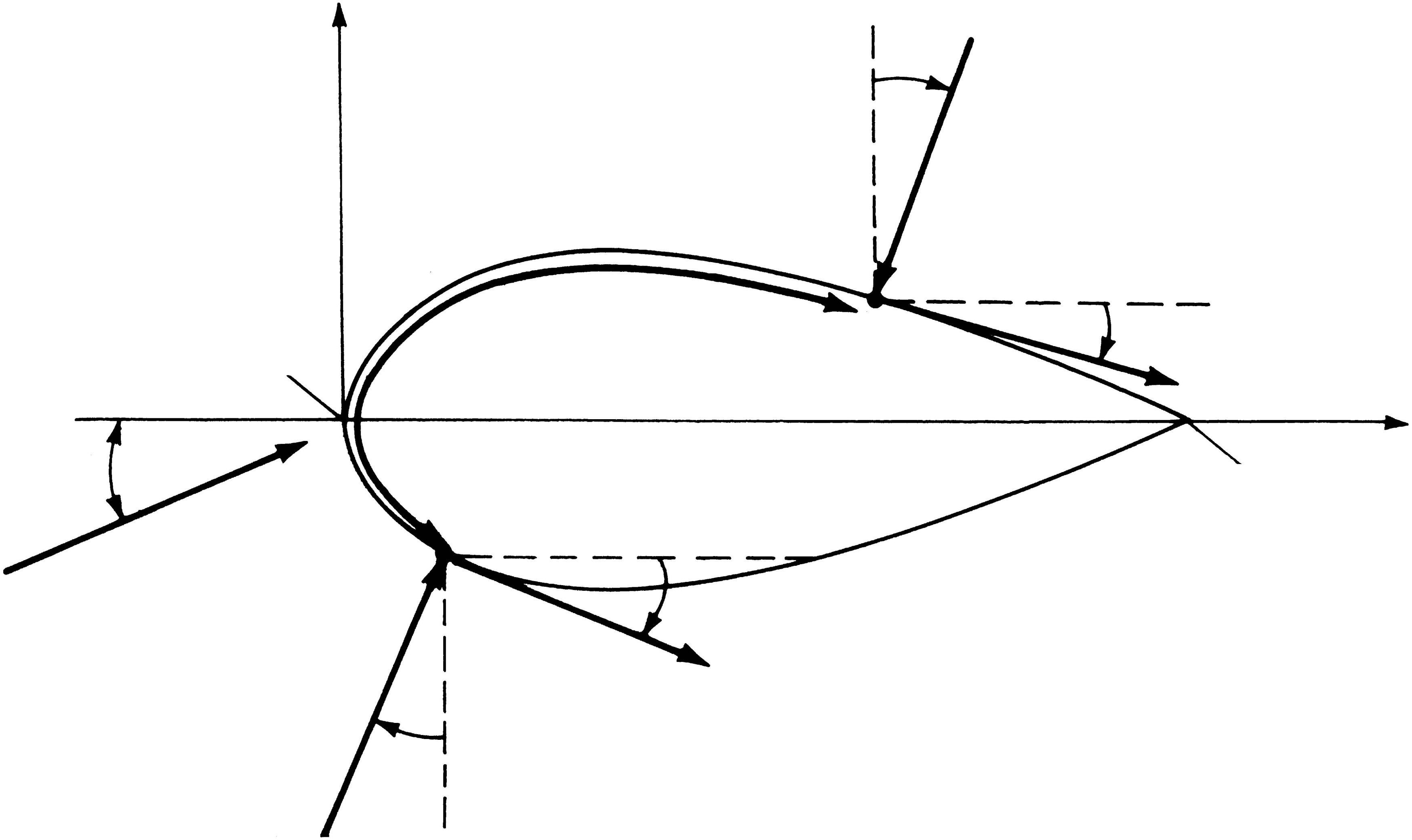
$$\begin{pmatrix} D' \\ L' \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} A' \\ N' \end{pmatrix}$$

rotation matrix

$L' = N' \cos \quad - A' \sin$

$D' = N' \sin \quad + A' \cos$

# Calculation of $N'$ and $A'$ from pressure and shear stress distributions.

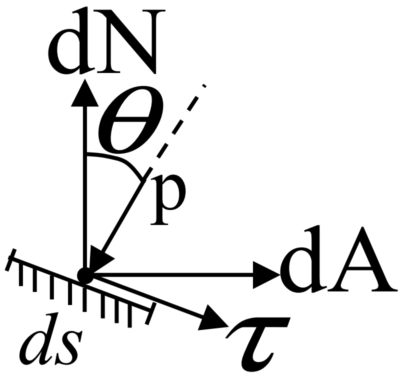


: upper surface : lower surface : outward normal

: surface inclination angle

: arclength : differential s

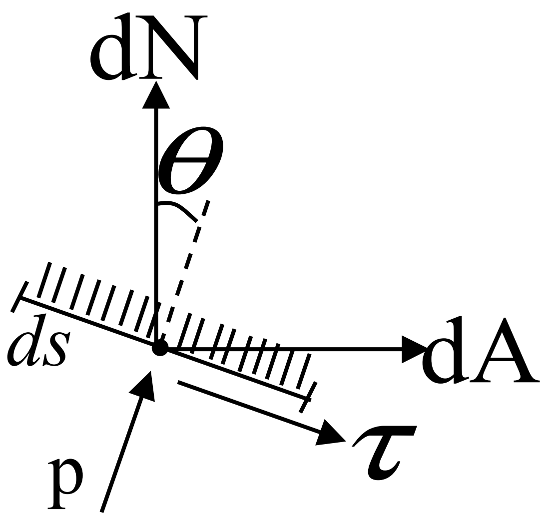
upper surface:




$$dA_u' = ( \qquad\qquad\qquad\qquad ) \, ds_u$$

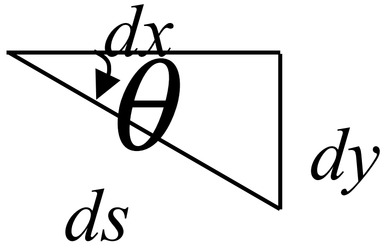
$$dN_u' = (-p_u \cos\theta - \tau_u \sin\theta) \, ds_u$$

lower surface:




$$dA_l' = (p_l \sin\theta + \tau_l \cos\theta) \, ds_l$$

$$dN_l' = ( \qquad\qquad\qquad\qquad ) \, ds_l$$




Express *ds* through *dx*, *dy* and $\theta$:

$$dx = \cos\theta\, ds$$

$$dy = -\sin\theta\, ds$$

$$\frac{dy}{dx} =$$

$$dN_u' = (-p_u \cos\theta - \tau_u \sin\theta)\, ds_u =$$

$$= (-p_u + \tau_u \frac{dy_u}{dx})\, dx$$

Similarly:

$$d\,N_l' = (p_l + \tau_l \frac{dy_l}{dx})\, dx$$

$$d\,A_u' = (p_u \frac{dy_u}{dx} + \tau_u)\, dx \qquad d\,A_l' = (-p_l \frac{dy_l}{dx} + \tau_l)\, dx$$

Integrate over the chord from LE ($x=0$) to TE ($x=c$):

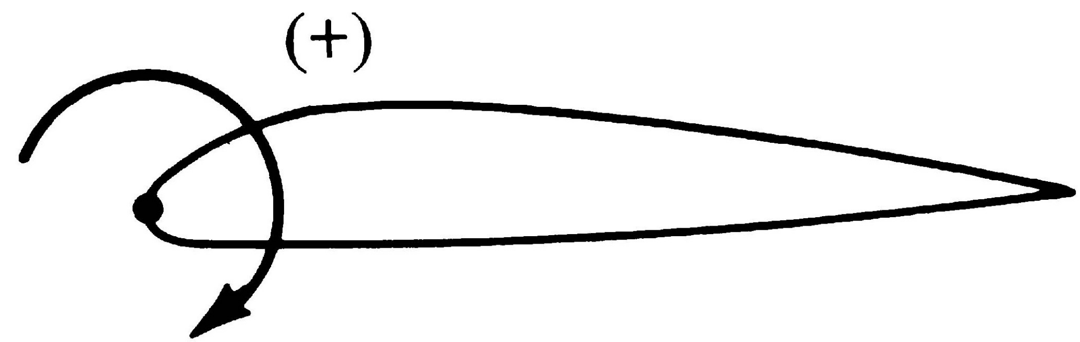
$$N' = \int_0^c dN'_u + \int_0^c dN'_l = \int_0^c (p_l - p_u)\,dx + \int_0^c \left[\tau_u \frac{dy_u}{dx} + \tau_l \frac{dy_l}{dx}\right] dx$$

$$A' = \int_0^c dA'_u + \int_0^c dA'_l = \int_0^c \left[p_u \frac{dy_u}{dx} - p_l \frac{dy_l}{dx}\right] dx + \int_0^c (\tau_u + \tau_l)\,dx$$

From $p$ and $\tau$ -----> N', A' ------> L', D'.

Aerodynamic moment – moment of the resultant force on the airfoil (or wing) around a reference axis parallel to spanwise direction

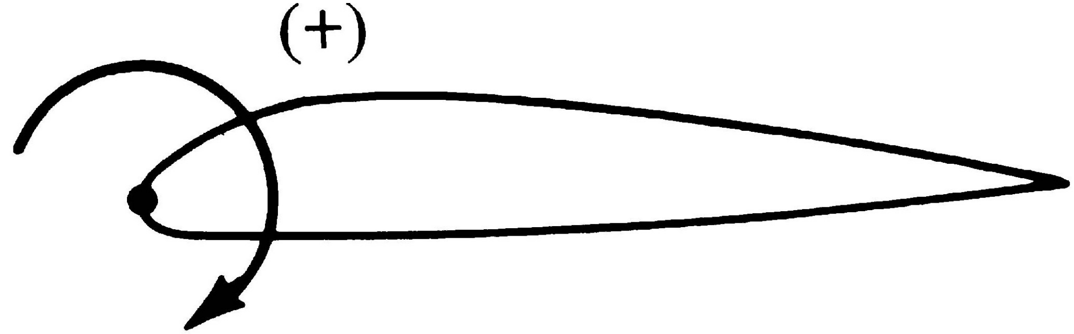
The moment tends to change the angle of attack. Also called pitching moment.




Sign convention: positive moment is pitch up (increases angle of attack)

Reference axis can be placed in different locations, e.g. LE, quarter-chord, half-chord, and so on. Notations:

Moment around LE:




upper surface: $d\,M_u^{'} = -x\,d\,N_u^{'} + y\,dA_u^{'} =$

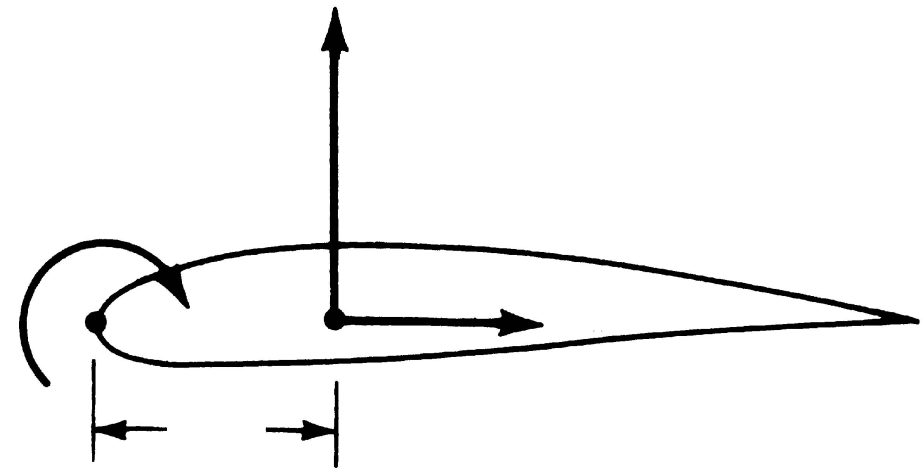
lower surface: $d\,M_l^{'} = -x\,d\,N_l^{'} + y\,dA_l^{'} =$

Integrate:

$$M_{\mathrm{LE}}^{'} = \int_0^c (p_u - p_l)\,x\,dx - \int_0^c \left[ \tau_u \frac{dy_u}{dx} + \tau_l \frac{dy_l}{dx} \right] x\,dx$$

$$+ \int_0^c \left[ p_u \frac{y_u}{dx} + \tau_u \right] y_u dx + \int_0^c \left[ -p_l \frac{y_l}{dx} + \tau_l \right] y_l dx$$

# Center of Pressure



= location along the chord where the resultant force can be applied to produce the same moment as the distributed load.

for small $\alpha$:

Moment can be taken about any point e.g, *c/4*

(for thin symmetric airfoils $x_{cp}=c/4$)

# <u>Example:</u> (Anderson, Problem 1.3)

Consider an infinitely thin flat plate of chord $c$ at an angle of attack $\alpha$ in a supersonic flow. The pressure on the upper and lower surfaces are different but constant over each surface; that is $p_u(s)=c_1$ and $p_l(s)=c_2$ where $c_1$ and $c_2$ are constants and $c_2>c_1$. Ignoring the shear stress, calculate the location of the center of pressure.

# Non-dimensionalization

for comparison of different designs

for scaling of wind tunnel tests to full scale

dynamic pressure: (force/area)

(specific kinetic energy of free stream)

Let $S$ = reference area, $l$ = reference length

Define: $C_L = \dfrac{L}{q_\infty S}$ - lift coefficient

$C_D = \dfrac{D}{q_\infty S}$ - drag coefficient

$C_M = \dfrac{M}{q_\infty S l}$ - moment coefficient

$$C_N = \frac{N}{q_\infty S}$$ - normal force coefficient

$$C_A = \frac{A}{q_\infty S l}$$ - axial force coefficient

2-D force coefficients: $(S = c \cdot 1 = c)$

$$c_l = \frac{L'}{q_\infty c} \qquad c_d = \frac{D'}{q_\infty c} \qquad c_m = \frac{M'}{q_\infty c^2}$$

Similarly we can define $c_n$ and $c_a$

$$c_p = \frac{p - p_\infty}{q}$$

$$c_f = \frac{\tau}{q}$$

Previous eqs for $N'$ and $A'$ can be non-dimensionalized to obtain $c_n$, $c_a$ and $c_m$ in terms of $C_p$, $c_f$.

$$c_n = \frac{1}{c}\left[\int_0^c \left(c_{p,l} - c_{p,u}\right) dx + \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right) dx\right]$$

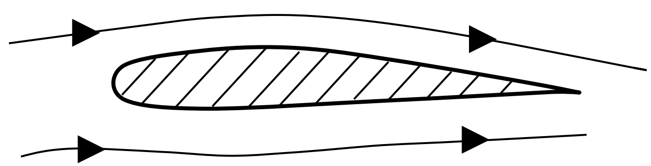
$$c_a = \frac{1}{c}\left[\int_0^c \left(c_{p,u}\frac{dy_l}{dx} - c_{p,l}\frac{dy_u}{dx}\right) dx + \int_0^c \left(c_{f,u} + c_{f,l}\right) dx\right]$$

$$c_{m_{\mathrm{LE}}} = \frac{1}{c^2}\int_0^c \left(c_{p,u} - c_{p,l}\right) x\, dx - \int_0^c \left[c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right] x\, dx$$

$$+ \int_0^c \left[c_{p,u}\frac{y_u}{dx} + c_{f,u}\right] y_u\, dx + \int_0^c \left[-c_{p,l}\frac{y_l}{dx} + c_{f,l}\right] y_l\, dx$$

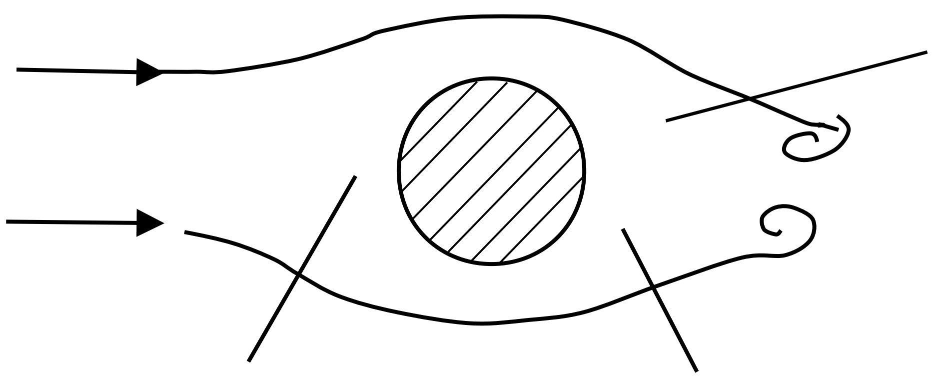
# Specifying reference areas and lengths

Streamline bodies: drag due mostly to



High/low drag

Bluff bodies= drag due mostly to



(separation)

High/low drag

low p

For streamlined bodies:

3-D S= planform area, l = mean chord

2-D S= chord · (1) , l = chord

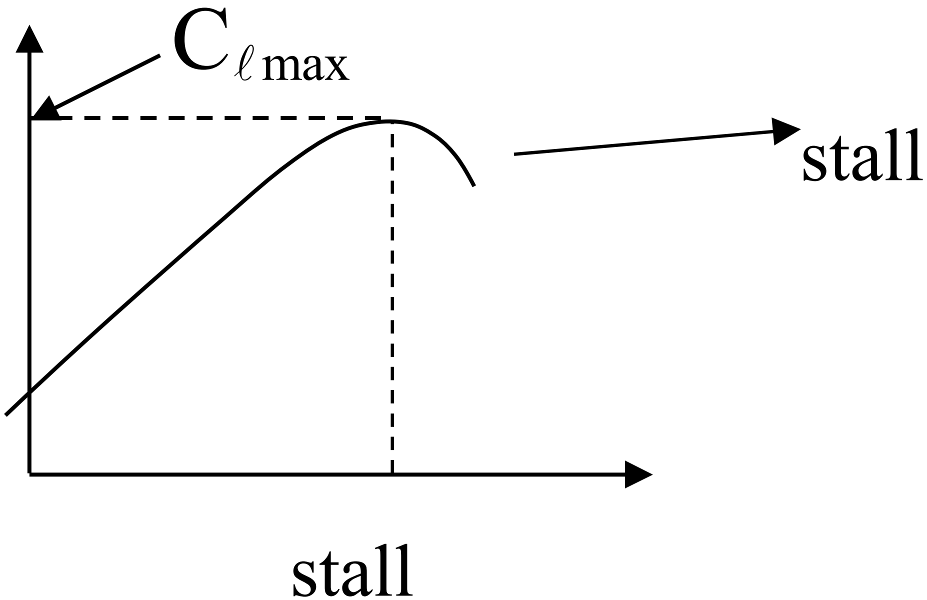
for bluff bodies

3-D S=

2-D S=

S=

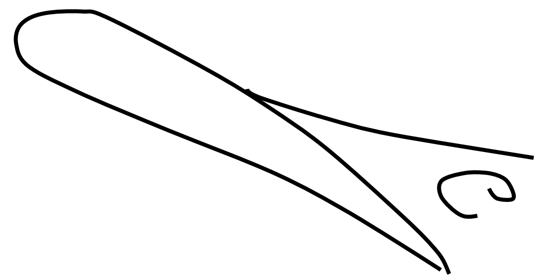
<u>Typical values</u> of $c_l$, $c_d$




Airfoils:

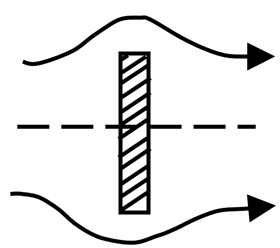
$c_{l\,max} = 1.5 - 2.0$ (single element)

$= 4$ (multi-element)



Bluff bodies:

flat plate



cylinder

Two important non-dimensional numbers:

Reynolds number: $Re=$

$\rho$=density $V$=velocity $L$=characteristic length

$\mu$= viscosity coefficient

Mach number : $M=$

$a$= speed of sound

**Types of Flow**

Continuum *vs* free molecular flow

mean free path

at sea level

at 100 km

Knudsen number: $Kn =$

[AAE 519, AAE590D]

if Kn<0.01 continuum flow

0.01 < Kn < 10 rarefied

Kn> 10 free molecule

## Incompressible vs compressible flow

if $M<0.3$ then is nearly constant

incompressible flow (AAE 333)

for compressible flow, Mach # regimes:

$M<1$ everywhere $M_\infty < 0.8$

$M<1$ regions $0.8 < M_\infty < 1.2$

$M>1$

$M>1$ everywhere $M_\infty > 1.2$

$M_\infty > 5$

## Inviscid *vs* viscous flow

inviscid flow is ideal

for high Re# viscous effects can be limited to thin regions near a body (boundary layers), except when separation occurs

inviscid flow assumption predicts drag =0.

# Some Practical Issues

Remember:

Blunt body= body where most of the drag is

Streamlined body = body where most of the drag is

Blunt bodies have a much higher $C_D$ than streamlined bodies.

The skin friction coefficient $C_f$ is higher for turbulent flows than laminar flows because velocity profile is flatter:

Explain: golf ball dipples, old baseball have smaller drag

# Buckingham Pi Theorem

The name Pi comes from the mathematical notation $\Pi$, meaning a product of variables.

Principle of Dimensional Homogeneity (PDH):

*Any equation describing a physical situation will only be true if both sides have the same units.*

The Buckingham Pi Theorem states:

*If a physical process satisfies PDH and involves* ***N*** *dimensional variables, it can be reduced to* ***N-K*** *non-dimensional variables or $\Pi$s. The reduction* ***K*** *is equal to the maximum number of variables that do not form a $\Pi$ among themselves and is always less than or equal to the number of fundamental units (mass, length, time, etc) describing the variables.*

*Method of repeating variables* is based on the application of Buckingham Pi theorem to determine the non-dimensional parameters that can be formed from a group of dimensional variables describing a physical process.

Step 1) Come up with a list of $N$ dimensional variables $P_j$ that describe the process and are related by some unknown functional relation: $f_1(P_{1,} P_{2,} \ldots, P_N) = 0$

Step 2) Let $K$ be the number of fundamental units needed to describe the $N$ dimensional variables. The fundamental units are mass, length, time, temperature and so on.

Step 3) Choose $K$ of the dimensional variables to be repeating variables such that the $K$ fundamental units can be all formed by some combination of the repeating variables and no non-dimensional parameters can be formed from the set of repeating variables.

Step 4) Form *N-K* dimensionless *Pi products* by multiplying the *K* repeating variable, as follows

$$\Pi_1 = P_1^{a_{11}} P_2^{a_{12}} \dots P_K^{a_{1K}} P_{K+1}$$

The exponents $a_{ij}$ are determined so that $\Pi_i$ are non-dimensional.

The Buckingham Pi theorem then states that the $N-K$ Pi products are related by some functional relationship:

$$f_2(\Pi_1, \Pi_2, \dots, \Pi_{N-K}) = 0$$

The advantage of using the Buckingham Pi theorem is that it can greatly reduce the number of parameters one has to deal with.

Example: Force coefficient (2-D)

Based on physical intuition we expect the resultant force vector for a given 2-D shape to depend on:

Fluid density | Freestream velocity

Body size (chord) | Viscosity | Speed of sound

Therefore the magnitude of the resultant force per unit span is

$$R' = f(\rho_\infty, V_\infty, c, \mu_\infty, a_\infty)$$

or as

The Buckingham Pi theorem will tell us how many parameters we can form from this set of variables.

Follow the steps outlined above to find the non-dimensional parameters $\Pi_i$

Step 1: The dimensional variables are

$$R', \rho_\infty, V_\infty, c, \mu_\infty, a_\infty$$

So $N=$

Step 2: What are the fundamental units needed?

Denote $m$=mass, $l$=length, $t$=time.

$[R']=$

$[\rho_\infty]=$

$[V_\infty]=$

$[\mu_\infty]=$

$[a_\infty]=$

So $K=$

Step 3: Choose $K$ *repeating variables* from among the N dimensional variables. We don't want to choose $R'$ because it is the variable of interest. We don't want to choose both $a_\infty$ and $V_\infty$ because their ratio is a non-dimensional parameter.

Choose the following as repeating variables: $\rho_\infty, V_\infty, c.$

Check that we can represent each of the fundamental units with this set of variables:

mass:

time:

length:

Step 4: Form the non-dimensional products.

Let $$\Pi_1 = \rho_\infty^{a_{11}} V_\infty^{a_{12}} c^{a_{13}} R'$$

$\Pi_1$ is dimensionless, so each exponent must be zero:

This is almost the same as the force coefficient except for a factor of 2. Since multiplying by a number doesn't change the units of $\Pi$, let

$$\Pi_1 = \frac{R'}{\frac{1}{2}\rho_\infty V_\infty^2 c}$$

Let $\Pi_2 = \rho_\infty^{a_{21}} V_\infty^{a_{22}} c^{a_{23}} \mu_\infty$

For $\Pi_2$ to be non-dimensional we need:

Thus, $\Pi_2 = \dfrac{\mu_\infty}{\rho_\infty V_\infty c}$ Similarly, we obtain $\Pi_3 = \dfrac{a_\infty}{V_\infty}$

The statement that we started with

can be rewritten as $R' = f(\rho_\infty, V_\infty, c, \mu_\infty, a_\infty)$

$$c_{R'} = f_1\left(\frac{1}{\mathrm{Re}_\infty}, \frac{1}{M_\infty}\right)$$

# Dynamic Similarity

Approaches for experimental testing:

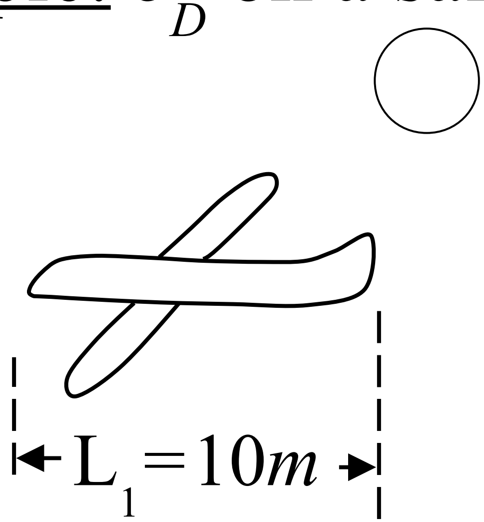
- Flight testing [expensive]
- Full scale wind tunnel testing
  (80' x 120' at NASA Ames; $V_{max}$=150 mph)
  [still expensive]
- Scaled model testing

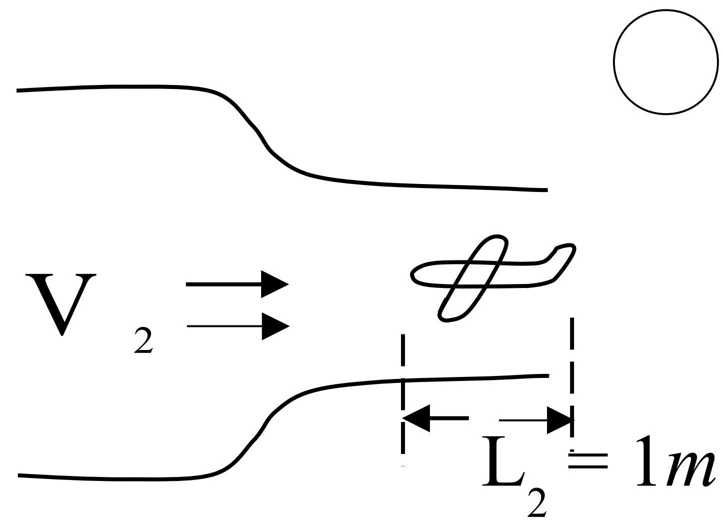

Conditions for dynamic similarity

1) Bodies are similar (scale model)
2) Governing parameters are the same

$c_D = f\,(Re, M)$

# Example: $c_D$ on a sailplane.




1 atm

1 atm

Find $V_{\infty 2}$

$$c_D = f\left(\mathrm{Re}, M\right)$$

$$M_{\infty 1} = \frac{V_{\infty 1}}{a_{\infty 1}} = \frac{10\, m/s}{340\, m/s} = 0.03$$

For incompressible flow $c_D = f(Re)$.

$$\mathrm{Re}_{\infty 1} = \mathrm{Re}_{\infty 2} \Rightarrow \left( \frac{\rho_1}{\rho_2} \right) \left( \frac{V_{\infty 1}}{V_{\infty 2}} \right) \left( \frac{L_1}{L_2} \right) \left( \frac{\mu_2}{\mu_1} \right) = 1.$$

$$\Rightarrow V_{\infty 2} = 10\, V_{\infty 1} = 100\, m/s.$$

Check Mach number for the model:

Example: $c_D$ on an airliner.

Prototype:

$L_1$=50 m

Altitude is 10 km

$M_{\infty 1}$=0.8 (transonic)

$\rho_{\infty 1}$=0.413 kg/m3

$\mu_{\infty 1}$=1.447x10$^{-5}$ kg/m s

$a_{\infty 1}$=299 m/s

Model:

$L_2$=5 m

At sea level

$\rho_{\infty 2}$=1.226 kg/m3

$\mu_{\infty 2}$=1.78x10$^{-5}$ kg/m s

$a_{\infty 2}$=340 m/s

First, find freestream velocity of the prototype:

$V_{\infty 1} = M_{\infty 1}\, a_{\infty 1}$ = 239 m/s. Need to match Mach numbers:

$V_{\infty 2}$ = = 272 m/s.

$$\mathrm{Re}_{\infty 1} = \frac{\rho_{\infty 1} V_{\infty 1} L_1}{\mu_{\infty 1}} = 3.4 \times 10^8$$

$$\mathrm{Re}_{\infty 2} = \frac{\rho_{\infty 2} V_{\infty 2} L_2}{\mu_{\infty 1}} = 9.4 \times 10^7$$

How can we increase the Re?

Increase $L$ : No, too expensive

Increase No, need to match

Increase

- increase $p$ -(see example 1.5 in Anderson)

  (pressurize the wind tunnel) expensive, dangerous

- reduce $T$ (cryogenic facility)

  expensive, difficult to instrument

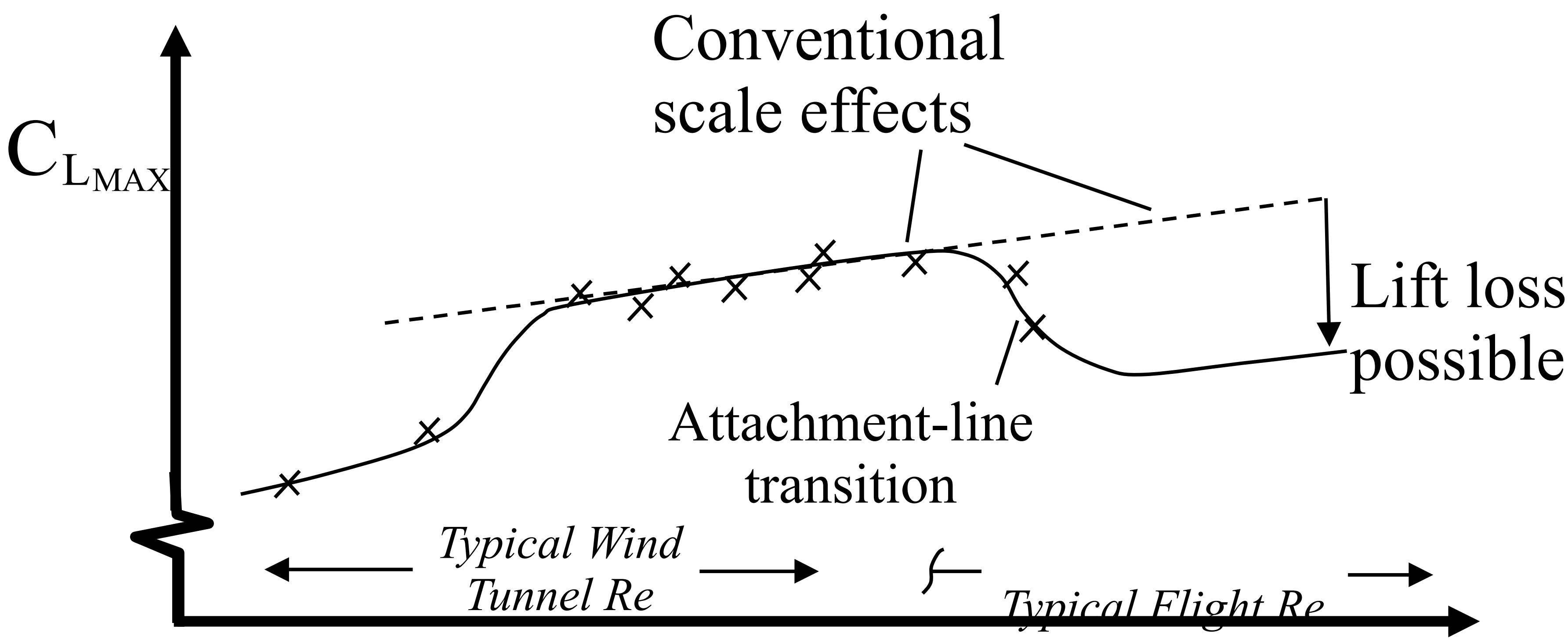
- heavy gas

Increase $\mu$ :

No clean answer what to do. Preferred approach:

M# simulation in one tunnel & Re# simulation in other.

Source: National Facilities Study, Volume 2; Subsonic and Transonic Wind Tunnel Complex, 1994.



# <u>Fluid Statics</u> describes fluids at rest.

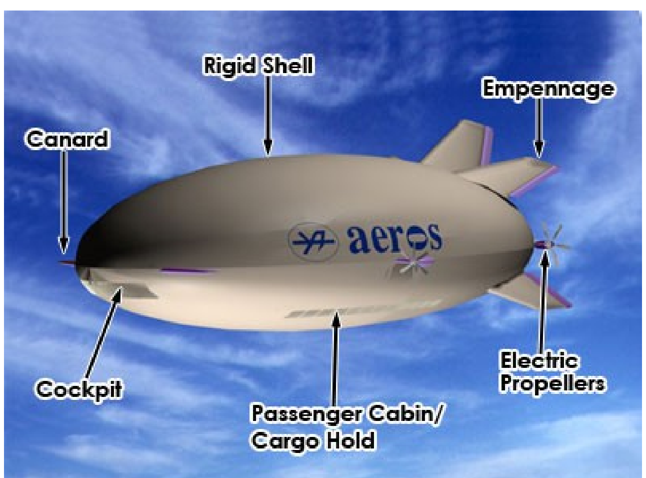
Some aerostatics applications:

- blimps (non-rigid airships)
- science balloons


Purdue blimp (Prof. John Sullivan and team)

Constellation of stratospheric balloons (NASA/Global Aerospace Corp.)

- hybrid airships (aerostatic/aerodynamic lift)

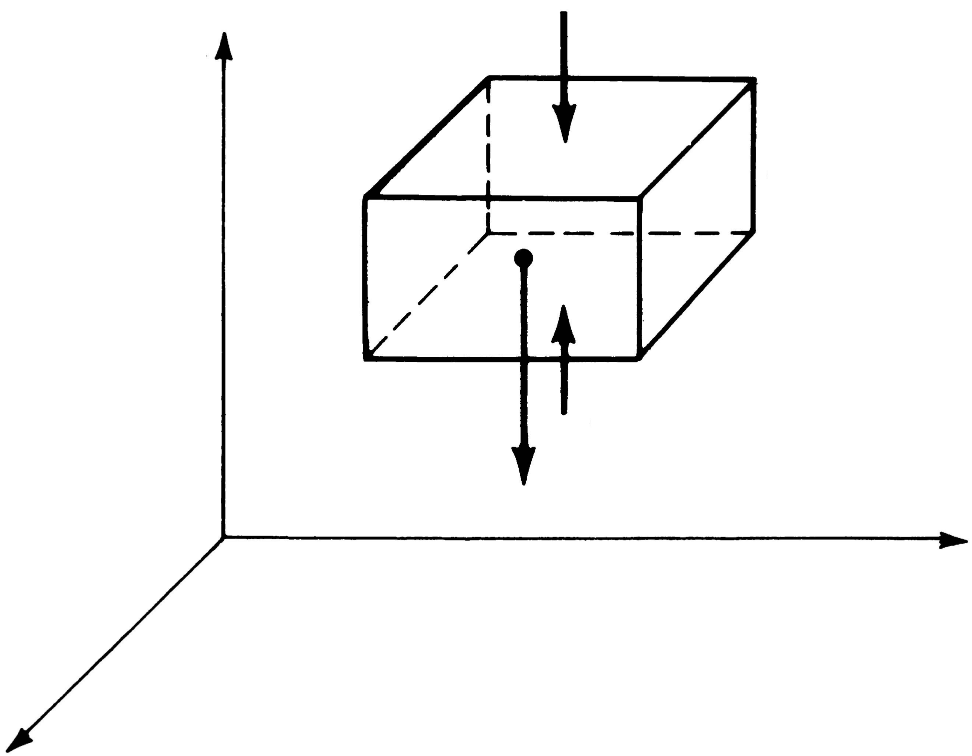




Aeroscraft is an aircraft that utilizes a combination of buoyant and dynamic lift and is designed to fly further (~10,000 miles) and lift more(~ 500 tons) than any other craft today (Worldwide Aeros Corp)

# Hydrostatic equation.

Fluid is static: no relative motion between adjacent fluid layers.



Force balance:

$$p\,dxdz-(p+dp)\,dx\,dz-(\rho\cdot dxdydz)\,g=0$$

$$\frac{dp}{dy}=-\rho\,g$$ Hydrostatic equation

For liquids $\rho=const$. Let's integrate with respect to $y$:

$$dp=-\rho\,g\,dy \quad\Rightarrow\quad \int_{p_0}^{p} dp=-\int_{y_0}^{y}\rho\,g\,dy$$

$$p=p_0-\rho\,g\,(y-y_0)$$ Hydrostatic pressure

Example 1: At what depth does p=2 atm?

$$p = p_a - \rho g y \qquad (p = p_a \text{ at } y = 0)$$

$$y = -\frac{p - p_a}{\rho g} = \frac{-1.01 \times 10^5\, N/m^2}{(1 \times 10^3\, kg/m^3)(9.8\, m/s^2)} = -10.3\, m$$

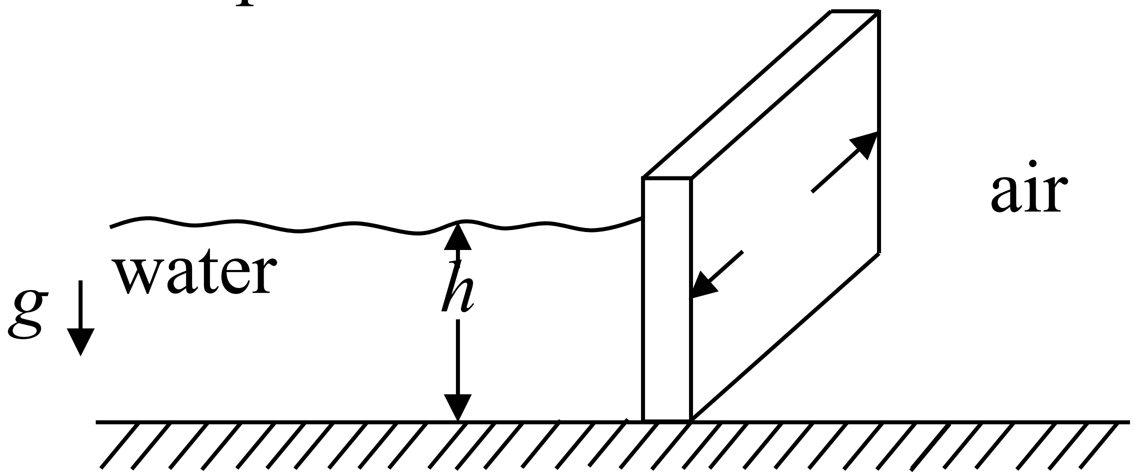
Pressure on a diver at a depth of 100 ft is 3.95 atm.

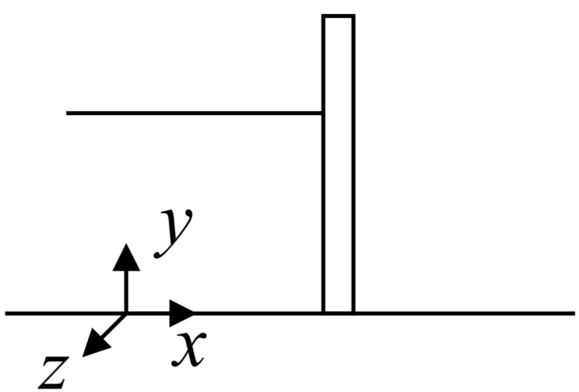
$$p_1 V_1 = p_2 V_2$$

$$\frac{V_1}{V_2} = \frac{p_2}{p_1} = 3.95$$

If the diver holds breath on ascent, the lung volume would increase by a factor of 4.

Example 2: Forces and moments on a dam.




$$p = p_0 - \rho_w g (y - y_0) \qquad y_0 = h,\ p_0 = p_a \qquad \Rightarrow$$

$$p = p_a + \rho_w g (h - y)$$

Static equilibrium: $\sum F_x = 0, \quad \sum M_z = 0.$

$$\sum F_x = R_x + \int_0^h (p(y) - p_a) W \, dy = 0 \Rightarrow$$

$$R_x = -W \int_0^h \rho_w g (h - y) dy =$$

$$\sum M_z = M_{R,z} - \int_0^h (p(y) - p_a) y W \, dy = 0 \Rightarrow$$

$$M_{R,z} = \int_0^h [-\rho_w g (y - h) y] W \, dy = -\rho_w g W \left[ \frac{y^3}{3} - h \frac{y^2}{2} \right]_0^h = \frac{\rho_w g W h^3}{6}$$

What if fluid is a gas not a liquid? ( $\rho \neq const$ )

$$\frac{dp}{dy} = -\rho g$$

Ideal gas equation of state: $p = \rho R T$

Assume $T = T_0$ (isothermal).

Substitute $\rho = \dfrac{p}{R T_0}$ into hydrostatic equation:

$$\frac{dp}{dy} = -\frac{g}{R T_0} p$$

$\dfrac{dp}{p} = -\dfrac{g}{R T_0} dy$ Integrate: $\displaystyle\int_{p_1}^{p} \frac{dp}{p} = -\frac{g}{R T_0} \int_{y_1}^{y} dy \Rightarrow$

$$\ln(p/p_1) = -\frac{g}{RT_0}(y - y_1) \quad \Rightarrow$$

Lower stratosphere: $T_0$=216.7 $^0$K (-56.5 $^0$C= -69.6 $^0$F)

$$c_0 = \frac{g}{RT_0} = \frac{9.8\, m/s^2}{296\, J/(kg \cdot K)\, 216.7\, K} = 1.53 \times 10^{-4} \frac{1}{m}$$

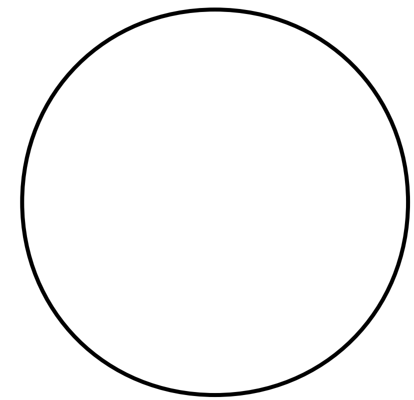
$$p = p_1 e^{-c_0 y}$$

pressure drops to a half every 4.5 km (2.8 miles)

# Force on a submerged body

*Archimedes principle*: Buoyancy force = weight of fluid displaced by the body.

Example: an immersed sphere.



Net force:

$F_y$ = Buoyancy force + gravitational force =

If $\rho_b < \rho \quad \Rightarrow$

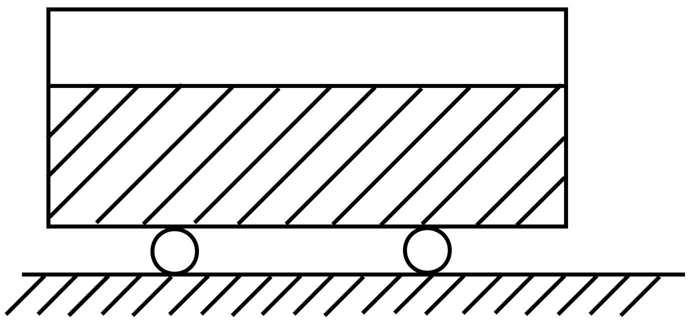
$\rho_b > \rho \quad \Rightarrow$

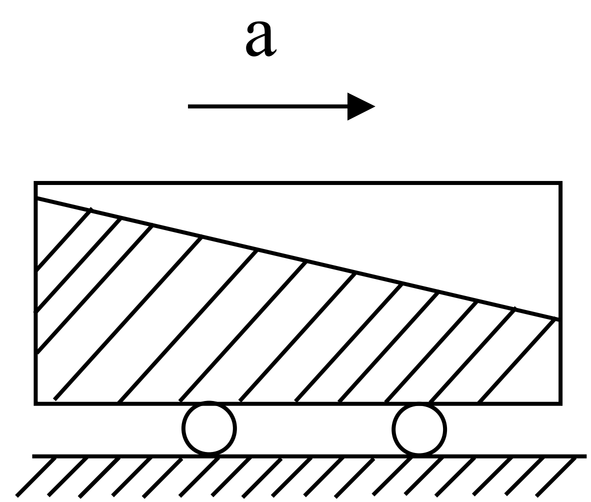


$\rho$= fluid density

# Motion with constant acceleration

accelerating box car:






$y$-direction: $\dfrac{\partial p}{\partial y} = -\rho g$ (gravity, as before)

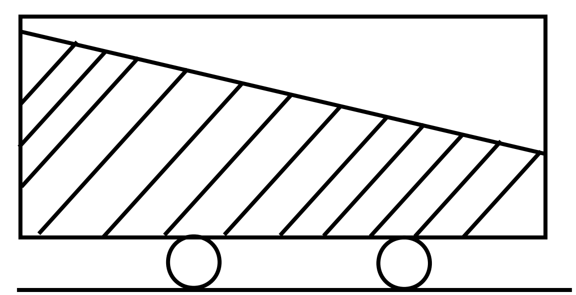
$x$-direction: $\dfrac{\partial p}{\partial x} =$ (inertial force in $-x$ direction)

$c_0$ depends on coordinate system

$$\frac{\partial p}{\partial y} = -\rho g \quad \Rightarrow \quad p = -\rho g y + c_1(x)$$

$$\frac{\partial p}{\partial x} = \quad \Rightarrow \quad \frac{dc_1}{dx} =$$

$$\Rightarrow \quad p =$$

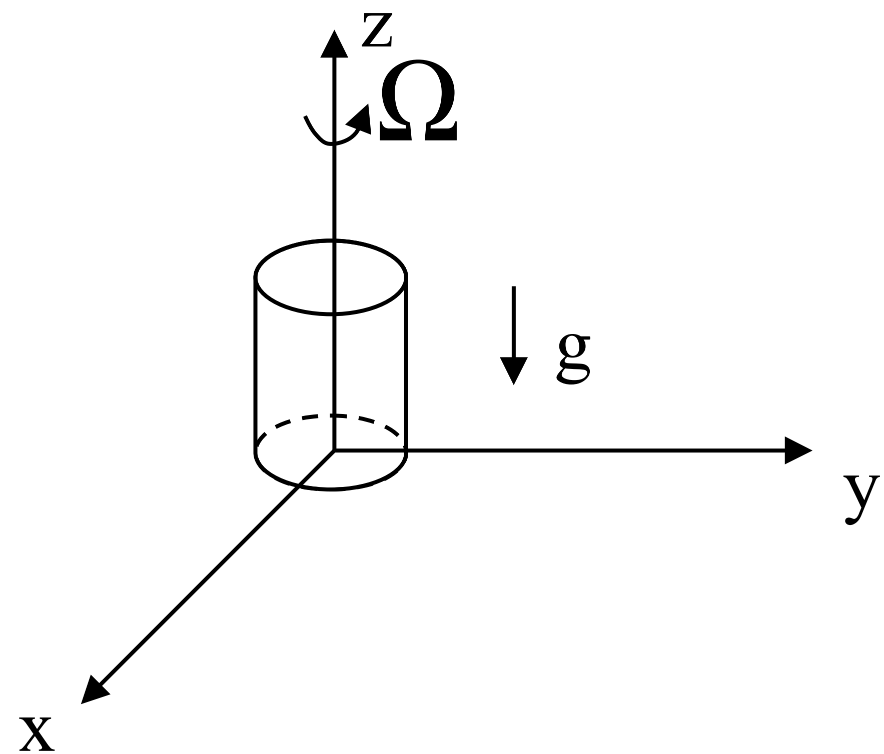


$p = p_a$ for the water surface

$$p_a = -\rho a x - \rho g y + c_0 \quad \Rightarrow \quad y =$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{a}{g}$$

# Solid Body Rotation




z-direction:

r-direction:

$$\frac{d\,p}{dz} = -\rho\, g$$

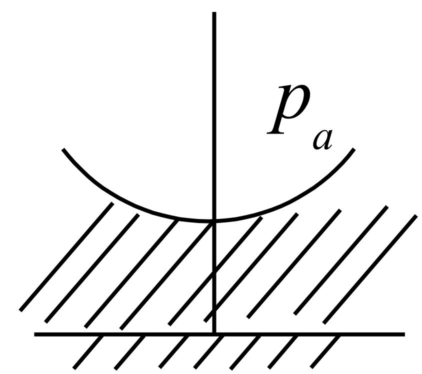
$$\frac{d\,p}{dr} =$$

Find $p(r,z)$:

$$\frac{dp}{dz} = -\rho g \quad \Rightarrow \quad p = -\rho g z + c_1(r)$$

$$\frac{dp}{dr} = \qquad\qquad\qquad\qquad \Rightarrow$$

$$p = -\rho g z + \rho \frac{r^2}{2}\Omega^2 + c_0$$




Find free surface: $p = p_a$

$$\Rightarrow \quad z =$$

Note: surface tension effects are neglected.

# Summary of Chapter 1

$$c_n = \frac{1}{c}\left[\int_0^c \left(c_{p,l} - c_{p,u}\right)dx + \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right)dx\right]$$

$$c_a = \frac{1}{c}\left[\int_0^c \left(c_{p,u}\frac{dy_l}{dx} - c_{p,l}\frac{dy_u}{dx}\right)dx + \int_0^c \left(c_{f,u} + c_{f,l}\right)dx\right]$$

$$c_d = c_a \cos\alpha + c_n \sin\alpha \qquad c_l = -c_a \sin\alpha + c_n \cos\alpha$$

$$c_{m_{\mathrm{LE}}} = \frac{1}{c^2}\int_0^c \left(c_{p,u} - c_{p,l}\right)x\,dx - \int_0^c \left[c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right]x\,dx$$
$$+ \int_0^c \left[c_{p,u}\frac{d\,y_u}{dx} + c_{f,u}\right]y_u\,dx + \int_0^c \left[-c_{p,l}\frac{d\,y_l}{dx} + c_{f,l}\right]y_l\,dx$$

Center of pressure $x_{cp} = -\frac{M_{\mathrm{LE}}}{N} \approx -\frac{M_{\mathrm{LE}}}{L}$

For dynamic similarity

- bodies must be geometrically similar
- non-dimensional similarity parameters must be equal (usually *Re, M*).

$c_l$, $c_d$ are equal

Hydrostatic equation: $\dfrac{dp}{dy} = -\rho g$

For $\rho = const$ : $p = constant - \rho g y$

- Acceleration *a* in *x*-direction: $p = constant - \rho g y - \rho a x$

surface ($p=p_a$): $y = -\dfrac{a}{g} x + \dfrac{constant - p_a}{\rho g}$ (straight line)

- Solid body rotation with angular speed $\Omega$:

$$p = constant - \rho g y + \rho \frac{r^2}{2} \Omega^2$$

surface ($p=p_a$): $y = \dfrac{1}{2} \dfrac{\Omega^2}{g} r^2 + \dfrac{constant - p_a}{\rho g}$ (parabola)