

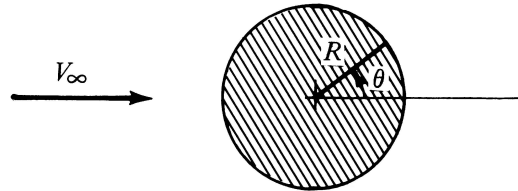
## Homework #4

Due: Friday 9/20/2013  
(3 problems; 20 points total)

1.[10 points] Velocity field for inviscid, incompressible flow around a circular cylinder of radius  $R$  has the following form in the cylindrical coordinate system:

$$\vec{V} = (V_r, V_\theta) \text{ where } V_r = V_\infty \left(1 - \frac{R^2}{r^2}\right) \cos \theta$$

$$\text{and } V_\theta = -V_\infty \left(1 + \frac{R^2}{r^2}\right) \sin \theta .$$



The pressure field for this flow is

$$p = p_\infty + \frac{1}{2} \rho_\infty V_\infty^2 - 2 \rho_\infty V_\infty^2 \sin^2 \theta .$$

We will derive these velocity and pressure fields later in the course.

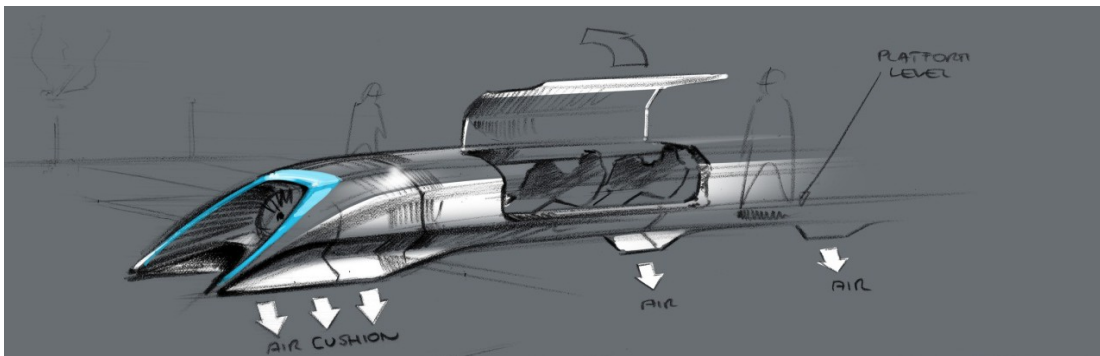
- a) Check that the velocity field satisfies the mass conservation (continuity equation).
- b) Find the magnitude of velocity at the surface of the cylinder ( $r=R$ ). Find the locations on the surface of the cylinder where the velocity magnitude is a maximum.
- c) Find the curl of the velocity field.
- d) Express the velocity and pressure fields in Cartesian coordinate system.
- e) Express the pressure in terms of the dimensionless pressure coefficient  $C_p$ . Find the locations of maximum and minimum pressure on the surface of the cylinder.
- f) Find the pressure gradient field.

2.[5 points] A rigid tank of volume  $V=1.0 \text{ m}^3$  is initially filled with air at  $20^\circ \text{ C}$  and  $p_0=100 \text{ kPa}$ . At time  $t=0$ , a vacuum pump is turned on and evacuates air at a constant flow rate  $Q=0.080 \text{ m}^3/\text{min}$  (regardless of pressure). Assume an ideal gas and an isothermal process.

- a) Set up a differential equation for this flow.
- b) Solve this equation for  $t$  as a function of  $(V, Q, p, p_0)$ .
- c) Compute the time in minutes to pump the tank down to  $p=20 \text{ kPa}$ .

4.[5 points] On August 12, 2013 Elon Musk announced the Hyperloop, a new high-speed transportation system designed to move passengers between Los Angeles and San Francisco in 35 minutes. The complete preliminary design study is at:

[http://www.teslamotors.com/sites/default/files/blog\\_images/hyperloop-alpha.pdf](http://www.teslamotors.com/sites/default/files/blog_images/hyperloop-alpha.pdf)



The preliminary design specifies that Hyperloop capsules will move inside 350-mile long tube in which the pressure would be maintained at 100 Pa, about 0.1% of sea-level atmospheric pressure.

Even if self-sealing doors are used, the air will leak out of the pressurized capsule into the low-pressure tube through microscopic gaps around the doors. In clean-room laboratory conditions very effective sealing is possible with liquid seals. But for a consumer application such seals are impractical. Microscopic gaps with a width  $\sim 10$  micron would be difficult and expensive to avoid. Using an approximate door perimeter of 2 m per passenger, leads to an effective leak area of about  $3 \text{ mm}^2$  for a 28-passenger capsule. Assumed that the volume of the capsule is  $30 \text{ m}^3$  The gas leak rate from a pressurized reservoir to near vacuum through a small gap can be approximated as

$\dot{m} = C \rho a A$  where  $\dot{m}$  is the mass flow rate (kg/s);  $\rho$  density in the reservoir;  $a$  is the speed of sound in the reservoir;  $A$  is the area of the opening (gap) in the reservoir. The discharge coefficient  $C$  depends on the geometry of the gap.

- a) Setup a differential equation describing time-variation of passenger capsule internal pressure after it is placed in vacuum. State assumptions that you have made.
- b) How long does it take for the pressure to drop below breathable level (12 kPa) with  $C=0.1$ ? How long with  $C=0.01$ ?

**Quote of the week:**

“Rocket engineering is not like ditch digging. With ditch digging you can get 100 people and dig a ditch, and you will dig it a hundred times faster if you get 100 people versus one. With rockets, you have to solve the problem of a particular level of difficulty; one person who can solve the problem is worth an infinite number of people who can't.”

Elon Musk, 1971 –