(25 pts) 3. A flat plate of length ℓ is placed inside a duct. By curving the walls of the duct, the pressure distribution on the flat plate can be set. Assume the walls of the duct are contoured in such a way that the potential flow over the plate gives the following velocity on the surface of the flat plate

$$u_e(x) = \frac{U_0}{(x/\ell + 1)}$$

where the plate starts at $x = 0$ and extends to positive values of x. The Reynolds number defined by $Re_0 = U_0\ell/\nu$ has the value $Re_0 = 10,000$.

(a) Determine what portions of the plate have a favorable pressure gradient and what portions have an adverse pressure gradient.

(b) Use Thwaites' method to find the nondimensional momentum thickness, δ_2/ℓ, as a function of x/ℓ. Determine δ_2/ℓ at $x = 0$ (the beginning of the plate). As x becomes large the momentum thickness scales as $\delta_2 \propto x^n$, determine n. (Note for a Blasius (zero pressure gradient) boundary layer, $\delta_2 \propto x^{1/2}$.)

(c) Calculate Thwaites' pressure gradient parameter, λ, as a function of x/ℓ. Determine whether boundary layer separation occurs on the plate and, if so, where it is located (in terms of x/ℓ).