DFA Minimisation Algorithm

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Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA with no unreachable states, such that $L = L(D)$.

For any two states $p, q \in Q$, we say that p *is equivalent to* q (denoted as $p \approx q$) if for all $x \in \Sigma^*$,

$$\delta(p, x) \in F \iff \delta(q, x) \in F.$$

It is easy to check that $\approx$ defines an equivalence class on Q (show it!). Now we will give an algorithm to construct a new DFA whose states are the equivalence classes of Q and transitions defined accordingly, such that this new DFA also recognises L.

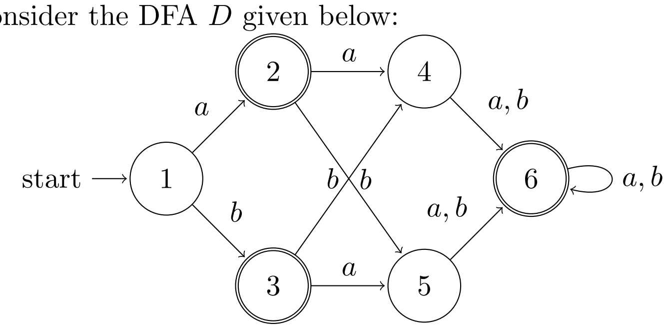
Algorithm

1. Create a table of pairs $\{p, q\}$ for all states $p, q \in Q$. Initially each pair is unmarked. (Note that we are considering unordered pairs.)
2. If for a pair $\{p, q\}$, $p \in F$ and $q \notin F$ or vice versa then mark $\{p, q\}$.
3. Repeat the following until you make an entire pass of the table without marking any pair:
   For every unmarked pair $\{p, q\}$ and for every symbol $a \in \Sigma$, if $\{\delta(p, a), \delta(q, a)\}$ is marked, then mark $\{p, q\}$.
4. Finally, if a pair $\{p, q\}$ is unmarked then $p \approx q$.

Observe that we mark a pair $\{p, q\}$ if p and q are not equivalent.

Example 1

Consider the DFA D given below:



The table of state pairs is initially as follows:

1					
-	2				
-	-	3			
-	-	-	4		
-	-	-	-	5	
-	-	-	-	-	6

Table is initially completely unmarked.

1					
$\times$	2				
$\times$	-	3			
-	$\times$	$\times$	4		
-	$\times$	$\times$	-	5	
$\times$	-	-	$\times$	$\times$	6

After first pass (marked all pairs consisting of one accept state and one non-accept state).

1					
$\times$	2				
$\times$	-	3			
-	$\times$	$\times$	4		
-	$\times$	$\times$	-	5	
$\times$	$\times$	$\times$	$\times$	$\times$	6

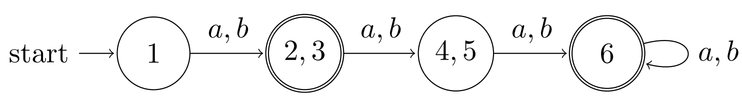
After second pass (marked pairs $\{2, 6\}$ and $\{3, 6\}$ as $\{\delta(2, a), \delta(6, a)\} = \{4, 6\}$ and $\{\delta(3, a), \delta(6, a)\} = \{5, 6\}$ and both are marked pairs).

1					
×	2				
×	-	3			
×	×	×	4		
×	×	×	-	5	
×	×	×	×	×	6

After third pass (marked pairs $\{1,4\}$ and $\{1,5\}$ as $\{\delta(1,a),\delta(4,a)\} = \{2,6\}$ and $\{\delta(1,a),\delta(5,a)\} = \{2,6\}$ and $\{2,6\}$ is now a marked pair).

At this point, we are done since we cannot make any further progress. Since the pairs $\{2,3\}$ and $\{4,5\}$ remain unmarked, hence $2 \approx 3$ and $4 \approx 5$.

The minimised DFA is as follows:



Example 2

Consider the DFA D given below in a table format:

	a	b
$\rightarrow$ 1F	6	4
2F	7	5
3	2	8
4	1	8
5	2	6
6	3	1
7	5	2
8	4	2

Here 1 is the start state and 1 and 2 are accept states.

1							
-	2						
-	-	3					
-	-	-	4				
-	-	-	-	5			
-	-	-	-	-	6		
-	-	-	-	-	-	7	
-	-	-	-	-	-	-	8

Table is initially completely unmarked.

1							
-	2						
$\times$	$\times$	3					
$\times$	$\times$	-	4				
$\times$	$\times$	-	-	5			
$\times$	$\times$	-	-	-	6		
$\times$	$\times$	-	-	-	-	7	
$\times$	$\times$	-	-	-	-	-	8

After first pass (marked all pairs consisting of one accept state and one non-accept state).

1							
-	2						
×	×	3					
×	×	-	4				
×	×	-	-	5			
×	×	×	×	×	6		
×	×	×	×	×	-	7	
×	×	×	×	×	-	-	8

After second pass (marked pairs $\{3,6\}$, $\{3,7\}$, $\{3,8\}$, $\{4,6\}$, $\{4,7\}$, $\{4,8\}$, $\{5,6\}$, $\{5,7\}$ and $\{5,8\}$).

Observe that at this point, we are done since we cannot make any further progress. From the unmarked pairs we conclude that $1 \approx 2$, $3 \approx 4 \approx 5$ and $6 \approx 7 \approx 8$.

The minimised DFA is as follows: