Practice Problem, Set 1

CS340, Semester I, 2013-14

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1. Construct a DFA and a RE for the language

$$L = \{w \in \{0,1\}^* \mid \text{every 1 in } w \text{ is immediately preceded and followed by a 0}\}.$$

   Example: The strings 00 and 0010100010 are in L whereas, 0110 and 1010010 are not in L.

2. Construct an NFA that recognises the language

$$L = \{w \in \{0,1\}^* \mid w \text{ does not contain the substring 1001}\}.$$

3. Let $N_x(w)$ = number of occurrences of the symbol x in the string w.

   Construct a CFG for the language

$$L = \{a^i b^j c^k \mid i \leq j + k \leq 2i\}.$$

4. Hamming distance between two strings, $w_1, w_2 \in \{0,1\}^n$ is said to be k if w_1 and w_2 differ in exactly k positions. This is denoted as $H(w_1, w_2)$. For example, the strings 1010010 and 1100011 have Hamming distance 3. If two strings have unequal length, we say their Hamming distance is infinite.

   For a language $L \subseteq \{0,1\}^*$, define

$$H_k(L) = \{w \in \{0,1\}^* \mid \exists x \in L,\ H(w,x) \leq k\}.$$

   (a) Show that if L is regular, then $H_2(L)$ is regular.

   (b) For any $k > 2$, show that if L is regular, then $H_k(L)$ is regular.

5. Convert the CFG below to a CFG in CNF.

$$\begin{aligned} S &\longrightarrow aSd \mid T \\ T &\longrightarrow bTc \mid \epsilon \end{aligned}$$

6. Show that the following language is not a CFL.

$$L = \{a^j b^k c^l \mid j \leq k \leq l\}$$

   (Hint: There can exist different i's for different ways of partitioning w. This is because the i is chosen *after* the partition, and hence can depend on the partition.)

7. For a language $A \subseteq \{0,1\}^*$ define $\min(L)$ as

$$\min(L) = \{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}.$$

Prove that if L is regular, then $\min(L)$ is regular.

8. Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be a function such that for some fixed $n_0 \in \mathbb{N}$,

$$f(n+1) - f(n) \geq n+1, \qquad \text{for all } n \geq n_0.$$

Consider the unary language

$$L = \{a^{f(n)} \mid n \geq 1\}.$$

Is L regular? Is it context-free?

9. Let $L \subseteq \{a\}^*$. Show that L^* is regular.

10. Give a language $L \subseteq \{0,1\}^*$ such that neither L nor $\{0,1\}^* \setminus L$ contains an infinite regular subset. Prove your answer.