cs140 – algorithms
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Bounding algorithms/problems

- algorithms: behavior over different inputs
  - upper, lower bounds on best case
  - upper, lower bounds on worst case
- problems: behavior of different algorithms
  - upper, lower bounds on worst case behavior of best algorithm

Matrix multiplication

- three nested loops
- naive divide-and-conquer
- Strassen (1969)

$$
\begin{aligned}
\mathrm{I} &= (A_{11}+A_{22})(B_{11}+B_{22}),\\
\mathrm{II} &= (A_{21}+A_{22})B_{11},\\
\mathrm{III} &= A_{11}(B_{12}-B_{22}),\\
\mathrm{IV} &= A_{22}(-B_{11}+B_{21}),\\
\mathrm{V} &= (A_{11}+A_{12})B_{22},\\
\mathrm{VI} &= (-A_{11}+A_{21})(B_{11}+B_{12}),\\
\mathrm{VII} &= (A_{12}-A_{22})(B_{21}+B_{22}),
\end{aligned}
\qquad
\begin{aligned}
C_{11} &= \mathrm{I}+\mathrm{IV}-\mathrm{V}+\mathrm{VII},\\
C_{21} &= \mathrm{II}+\mathrm{IV},\\
C_{12} &= \mathrm{III}+\mathrm{V},\\
C_{22} &= \mathrm{I}+\mathrm{III}-\mathrm{II}+\mathrm{VI},
\end{aligned}
$$

- Pan (1978)
- …
- Coppersmith and Winograd (1987)
- Vassilevska Williams (2012)

Lower bound on sorting

- comparison based sorts
- decision tree analysis for insertion sort

Decision tree analysis

- look at number of possible outputs
- derive minimum height of tree
- conclude that is a lower bound

- next topic . . . linear time sorts (?!)

Assumptions about the range

- (counting sort) the numbers are in the range [0, N-1]

- (radix sort) numbers are each a sequence of d elements, where each element in the range [0, N-1]

Assumptions about distribution

- (bucket sort) the numbers are uniformly distributed

Order statistics

- input: array of numbers A[n], index k
- output: value of k^{th} smallest number in A[n]

- specific cases?

- variations?

- bounds on the general case?

Order statistics