Name _____ Period _____ Date _____

AP Calculus AB Review Session #0C: *Limits*

1. Use Figure 1.84 to give approximate values for the following limits (if they exist).

(a) $\lim_{x \to -2} f(x)$ **(b)** $\lim_{x \to 0} f(x)$

(c) $\lim_{x \to 2} f(x)$ **(d)** $\lim_{x \to 4} f(x)$

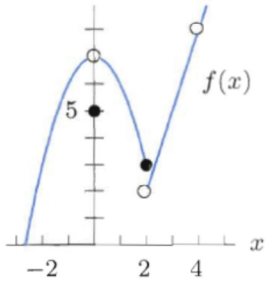

Figure 1.84

2. Using Figures 1.85 and 1.86, estimate

(a) $\lim_{x \to 1^-} (f(x) + g(x))$ **(b)** $\lim_{x \to 1^+} (f(x) + 2g(x))$

(c) $\lim_{x \to 1^-} f(x)g(x)$ **(d)** $\lim_{x \to 1^+} \frac{f(x)}{g(x)}$

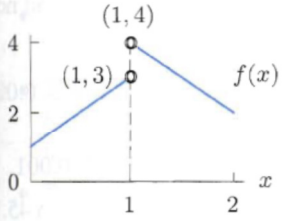

Figure 1.85

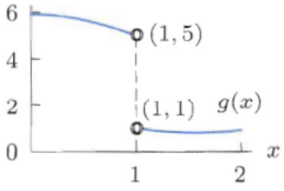

Figure 1.86

3. Carefully sketch the graph of a function $f(x)$ with domain $[-2, 4]$ that has all the following properties:
   - $f(-2) = 3$
   - $f(0) = 2$
   - $f(4) = 3$
   - $\lim_{x \to -2^+} = 3$
   - $\lim_{x \to 4^-} = 0$
   - $\lim_{x \to 0} = 1$

4. A car is driven at a constant speed. Sketch a graph of the distance the car has traveled as a function of time.

5. A car is driven at an increasing speed. Sketch a graph of the distance the car has traveled as a function of time.

6. A car starts at a high speed, and its speed then decreases slowly. Sketch a graph of the distance the car has traveled as a function of time.

Name _____ Period _____ Date _____

7. Suppose a rational function, $f(x)$, has degree 2 polynomials in the numerator and denominator. The vertical asymptotes of $f(x)$ are $x = \pm 5$. The horizontal asymptote of $f(x)$ is $y = -\frac{3}{2}$. $f(x)$ also has zeros at $x = 0$ and $x = 2$. Write one possible expression for $f(x)$.

   $f(x) =$

8. Suppose a rational function, $f(x)$, has degree 2 polynomials in the numerator and denominator. The vertical asymptote of $f(x)$ is at $x = -5$. The horizontal asymptote of $f(x)$ is $y = -\frac{3}{2}$. $f(x)$ also has a zero at $x = 2$ and a hole at $x = 5$. Write one possible expression for $f(x)$.

   $f(x) =$

   What are the coordinates of the hole? (_____ , _____)

Compute the following limits.

9. $\lim_{x\to 4} \sqrt{x+2}$

10. $\lim_{x\to -2} \frac{x^2-4}{x^2+4}$

11. $\lim_{x\to -2} \frac{x^2+4}{x^2-4}$

12. $\lim_{x\to -2} \frac{x^2-4}{x+2}$

13. A spherical balloon is being inflated. If the radius of the balloon is increasing at a rate of 1 cm per second, express the volume of the balloon as a function of time t (in seconds).

14. Express the surface area of a cube as a function of its volume.

15. If possible, choose k so that the following function is continuous on any interval:

$$f(x) = \begin{cases} \frac{5x^3 - 10x^2}{x-2} & x \neq 2 \\ k & x = 2. \end{cases}$$

16. Find k so that the following function is continuous on any interval:

$$f(x) = \begin{cases} kx & 0 \leq x < 2 \\ 3x^2 & 2 \leq x. \end{cases}$$

17. Is the following function continuous on $[-1, 1]$?

$$f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Explain.

18. **(a)** Sketch the graph of a continuous function f with *all* of the following properties:
   (i) $f(0) = 2$
   (ii) $f(x)$ is decreasing for $0 \leq x \leq 3$
   (iii) $f(x)$ is increasing for $3 < x \leq 5$
   (iv) $f(x)$ is decreasing for $x > 5$
   (v) $f(x) \to 9$ as $x \to \infty$

   (b) Is it possible that the graph of f is concave down for all $x > 6$? Explain.