

Assignment #4D: The Derivative at a Point

Last week, we saw that the **derivative's** y -values represent the slope values of the original function. We spent time sketching what the derivative looks like. This week, we'll look at using the numeric value of the derivative to give us clues about the behavior of the original function. We start by getting a sense for characterizing slope numerically.

1. Match the points labeled on the curve in Figure 2.6 with the given slopes. Then sketch the derivative.

Slope	Point
-3	
-1	
0	
$1/2$	
1	
2	

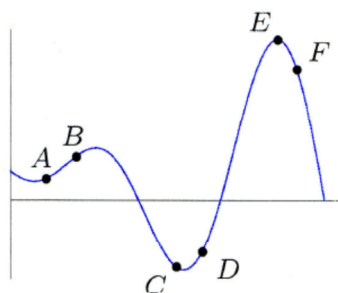


Figure 2.6

2. Label points A, B, C, D, E, and F on the graph of $y = f(x)$ in Figure 2.19.

[Note that there may be several legal placements for each point]

- Point A is a point on the curve where the derivative is negative.
- Point B is a point on the curve where the value of the function is negative.
- Point C is a point on the curve where the derivative is largest.
- Point D is a point on the curve where the derivative is zero.
- Points E and F are different points on the curve where the derivative is about the same.

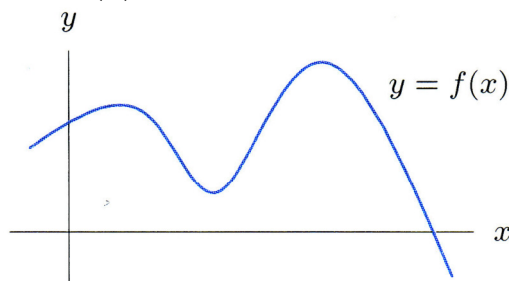


Figure 2.19

3. Use Figure 2.22 to fill in the blanks in the following statements about the function g at point B.

- (a) $g(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$ (b) $g'(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$

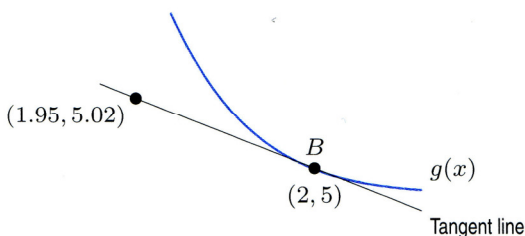


Figure 2.22

4. Suppose $h(x) = \ln(x)$. Use your calculator to find $h(2.1)$ and $h'(2.1)$. Find an equation of the line tangent to $h(x)$ at $x = 2.1$.
5. Find an equation of the line tangent to $y = x^2$ at $x = 2.1$.

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6. Find an equation of the line tangent to $y = \sin(x)$ at $x = -1$.
7. The temperature, H , in degrees Celsius, of a cup of coffee placed on the kitchen counter is given by $H = f(t)$, where t is in minutes since the coffee was put on the counter.
- (a) Is $f'(t)$ positive or negative? Why?
- (b) What are the units of $f'(20)$? What is its practical meaning in terms of the temperatures of the coffee?
8. The cost, C (in dollars) to produce g gallons of ice cream can be expressed as $C = f(g)$. Using units, explain the meaning of the following statements in terms of ice cream.
- (a) $f(200) = 350$
- (b) $f'(200) = 1.4$
9. First, ***without computing any integrals***, explain why the average value of $f(x) = \sin(x)$ on $[0, \pi]$ must be between 0.5 and 1.
10. Find the average value of $f(x) = \sin(x)$ on $[0, \pi]$.
11. Find the area under $y = 5 \ln(2x)$ and above $y = 3$ for $3 \leq x \leq 5$.