Name ____ Period _____ Date ______________________

Assignment #5C: *Methods for Approximating Area*

We have been using definite integrals to find the area under a curve. However, to this point, we have relied on the calculator to supply the answers to the problems. To gain an appreciation for how the calculator does this, we begin to develop some algorithms that can be used to approximate the area under a curve.

The area under a curve can be approximated through the use of Riemann sums: $A \approx \sum_{k=1}^{n} f\left(x_k\right)\Delta x_k$.

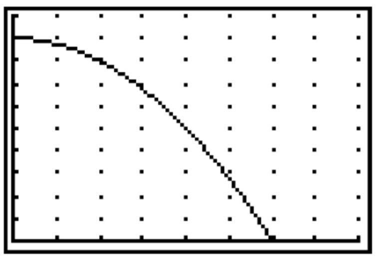
Let $L_n =$ Sum of n rectangles using the *left-hand x-coordinate* of each interval to find the height of the rectangle.

Let $R_n =$ Sum of n rectangles using the *right-hand x-coordinate* of each interval to find the height of the rectangle.

Let $M_n =$ Sum of n rectangles using the *midpoint x-value* of each interval to find the height of the rectangle.

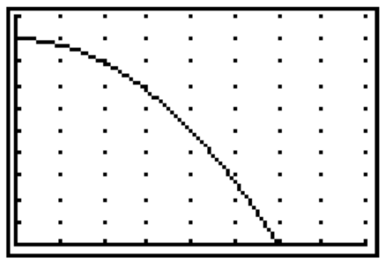
1. Let $f\left(x\right) = 9 - x^2$ on the interval $\left[0,3\right]$. Use six rectangles that have the same width. Draw a sketch of the function, along with the indicated rectangles. Then calculate the approximate area for each method.

$L_6 =$



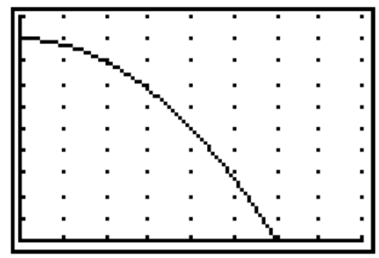
Left-hand Rule

$R_6 =$



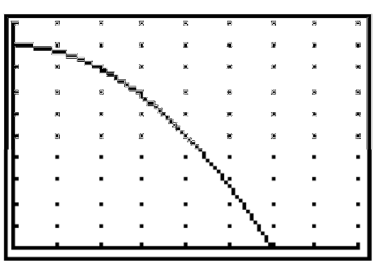
Right-hand Rule

$M_6 =$



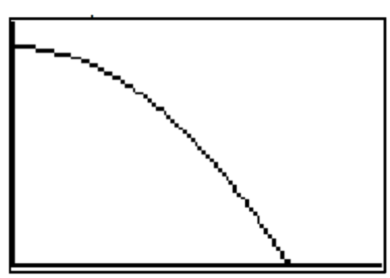
Midpoint Rule

In our interval, we don't have to use the left, right, or midpoint. Use six rectangles that have the same width, but instead of using the left, right, or midpoint x-coordinate to find the height of the rectangle, choose a random x-coordinate in each interval to find the height of the rectangle. Compute the area of each rectangle and find the total area.



Random x-coordinate

Rectangles don't have the have the same width. Choose 6 randomly-sized intervals and choose a point at random in each interval. Compute the area of each rectangle and find the total area.



Random rectangles

Based on material originally developed by Larry Petersen

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2. Let $f(x) = 3^x$ on the interval $[-1, 3]$. Draw a sketch and calculate the approximate area for each method below.

   (a) Find L_8, the left-hand Riemann sum approximation using 8 subintervals of equal length.

   (b) Find R_8, the right-hand Riemann sum approximation using 8 subintervals of equal length.

   (c) Find M_8, the midpoint Riemann sum approximation using 8 subintervals of equal length.

3. Find the left-hand and right-hand Riemann sums for $\int_2^3 \frac{1}{\ln x}\,dx$ using five subintervals of equal length and determine whether these sums under- or overestimate the exact value of the integral. Draw diagrams and use words to support your answer.