AP Calc
Final Exam Practice

You will be given two hours to work on this exam. It is divided into two sections, multiple choice (28 questions) and free response (4 questions). You will have 1 hour to work on the multiple choice and 1 hour to work on the free response. In addition, there will be no calculator allowed for the first 20 multiple choice and the first 2 free response.

Section 1: Multiple Choice

No Calculators allowed

1. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

   (A) The function has no relative extremum.

   (B) The graph of the function has one point of inflection and the function has two relative extrema.

   (C) The graph of the function has two points of inflection and the function has one relative extremum.

   (D) The graph of the function has two points of inflection and the function has two relative extrema.

   (E) The graph of the function has two points of inflection and the function has three relative extrema.

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2. What is $\lim_{x\to\infty} \dfrac{x^2 - 4}{2 + x - 4x^2}$?

(A) -2 (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{2}$ (D) 1 (E)The limit does not exist.

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3. What is $\lim_{x\to\infty} \dfrac{x^3 + 8}{1 + 3x - 8x^2}$

(A) -1 (B) 1 (C) $\dfrac{1}{2}$ (D) 8 (E) The limit does not exist.

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4. If the graph of $y = \dfrac{ax + b}{x + c}$ has a horizontal asymptote y = 2 and a vertical asymptote $x = -3$, then $a + c$ =

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

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5. For what x value does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\dfrac{7}{3}$ (C) $-\dfrac{5}{2}$ (D) $\dfrac{7}{3}$ (E) $\dfrac{5}{2}$

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6. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

(A) -6 (B) -3 (C) 3 (D) 6 (E) 8

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7. If $x^3 + 3xy + 2y^3 = 17$, then in terms of *x* and *y*, $\frac{dy}{dx} =$

(A) $-\frac{x^2 + y}{x + 2y^2}$ (B) $-\frac{x^2 + y}{x + y^2}$ (C) $-\frac{x^2 + y}{x + 2y}$ (D) $\frac{-x^2 + y}{2y^2}$ (E) $-\frac{x^2}{1 + 2y^2}$

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8. Let *f* be the function defined by $f(x) = \begin{cases} x^3 \text{ for } x \le 0 \\ x \text{ for } x > 0 \end{cases}$. Which of the following statements is true?

(A) *f* is an odd function (B) *f* is discontinuous at x = 0. (C) *f* has a relative maximum (D) *f'(0)* = 0 (E) *f'*(*x*)>0 for $x \neq 0$

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9. At what value of *x* does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of *x*

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10. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

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11. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, the *f'*(0) =

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

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12. The function *f* given by $f(x) = x^3 + 12x - 24$ is

(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) decreasing for *x* < 0, increasing for *x* > 0
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

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13. If $\frac{dy}{dx} = 2x$ and if y = 1 when x = -1, then when *x* = 2, *y* =

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) 4 (E) $\frac{1}{4}$

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14. The top of a 25-ft. ladder is sliding down a vertical wall at a constant rate of 3 ft. per minute. When the top of the ladder is 7 ft. from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}\ ft/\text{min}$ (B) $-\frac{7}{24}\ ft/\text{min}$ (C) $\frac{7}{24}\ ft/\text{min}$ (D) $\frac{7}{8}\ ft/\text{min}$ (E) $\frac{21}{25}\ ft/\text{min}$

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15. An equation of the line tangent to the graph of $y=\frac{2x+3}{3x-2}$ at the point (1, 5) is

(A) $13x-y=8$ (B) $13x+y=18$ (C) $x-13y=64$ (D) $x+13y=66$
(E) $-2x+3y=13$

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16. If f and g are continuous functions, and if $f(x)\geq 0$ for all real numbers x, which of the following must be true?

I. $\int_a^b f(x)g(x)dx=\left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

II. $\int_a^b \left(f(x)+g(x)\right)dx=\int_a^b f(x)dx+\int_a^b g(x)dx$

III. $\int_a^b \sqrt{f(x)}dx=\sqrt{\int_a^b f(x)dx}$

(A) I only (B) II only (C) III only (D) II and III only
(E) I, II, and III

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17. If the units of $R'(t)$ are $\frac{mm}{\text{sec}}$, where t is measured in seconds, what are the units of $\int R'(t)dt$?

(A) mm (B) mm^2 (C) $\frac{mm}{\text{sec}}$ (D) $\frac{mm}{\text{sec}^2}$ (E) $\frac{mm^2}{\text{sec}}$

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18. $\int (x-1)\sqrt{x}\ dx=$

(A) $\frac{3}{2}\sqrt{x}-\frac{1}{\sqrt{x}}+C$ (B) $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+C$ (C) $\frac{1}{2}x^2-x+C$

(D) $\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{3}x^{\frac{3}{2}}+C$ (E) $\frac{1}{2}x^2+2x^{\frac{3}{2}}-x+C$

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19. $\int_0^1 \left(x^2+\sqrt{x}\right)dx\ =$

(A) 1 (B) -1 (C) 0 (D) $\frac{1}{2}$ (E) $-\frac{1}{2}$

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20. Which of the following statements are true?

I. $\int_a^b -f(x)\ dx = \int_b^a f(x)\ dx$

II. $\int_a^b f(x)+g(x)\ dx = \int_b^a f(x)\ dx + \int_b^a g(x)\ dx$

III. $\int_a^b -f(x)\ dx = \int_a^b f(x)\ dx$

(A) I only (B) II only (C) III only (D) none (E) I, II, and III

For questions 21-25 you may use a calculator

21. A particle moves in a straight line, and its velocity at any time t is given by $v(t) = 5 - e^t$.
What is the total distance the particle travels from $t = 0$ to $t = 3$?

(A) 4.086 (B) 5.086 (C) 11.086 (D) 12.180 (E) 19.086

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22. Oil is leaking from a tanker at the rate of $R(t) = -.5t^2 + 63.5$ gallons per hour, where t is measured in hours. How much oil has leaked out of the tanker after 8 hours?

(A) 53 gallons (B) 423 gallons (C) 43 gallons (D) 530 gallons (E) 400 gallons

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23.

Time (sec)	0	10	25	37	46	60
Rate (gal/sec)	500	400	350	280	200	180

The table above gives the values for the rate (in gal/sec) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \le t \le 60$. What is this estimate?

(A) 1,910 gal (B) 14,100 gal (C) 16,930 gal (D) 18,725 gal (E) 20,520 gal

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24. The radius of a circle is increasing at a non-zero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\frac{1}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) 1 (E) 2

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25. A square bottomed box with no top has a fixed volume, V. What dimensions minimize the surface area?

(A) $x = 2v, y = \frac{V}{4}$ (B) $x = \frac{v}{2}, y = \frac{V}{4}$ (C) $x = \sqrt[3]{2v}, y = \sqrt[3]{\frac{V}{4}}$

(D) $x = (2v)^3, y = \left(\frac{V}{4}\right)^3$ (E) $x = \sqrt[3]{v}, y = \sqrt[3]{\frac{V}{2}}$

AP Calculus Part II

No Calculators

1. Let f be the function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

   (a) On what intervals is *f* increasing?
   (b) On what intervals is the graph of *f* concave downward?
   (c) Find the value of *k* for which *f* has 11 as its relative minimum.

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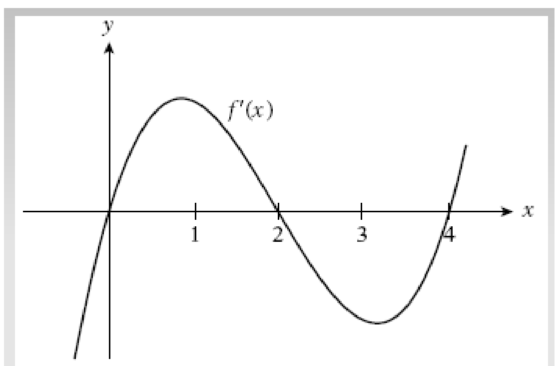
2. The figure below shows the graph of *f'*, the derivative of a function *f*.



   (a) Using the graph, find the critical values of *f*. Identify each as a relative maximum, a relative minimum, or neither. Justify your answer.
   (b) Use the graph of f' to state the intervals where *f* is increasing.
   (c) Find the approximate *x*-coordinates of the points of inflection of *f*, given the graph of f'.
   (d) State the interval(s) where *f* is concave down, given the graph of f'.
   (e) Sketch a graph of $y = f(x)$.

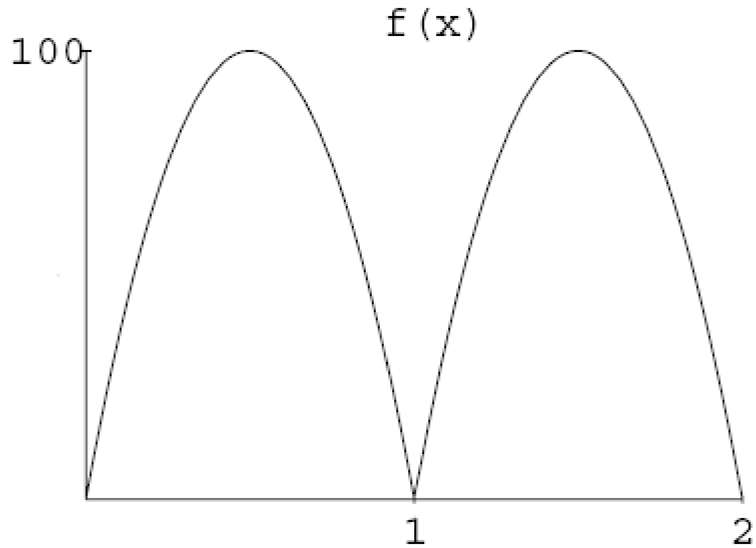
Calculators Allowed

1. Consider the curve given by $xy^2 - x^3y = 6$

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.



2. A graph of the continuous function f is given above. Rank the following integral in ascending numerical order. ***Explain your reasons.***

(a) $\int_0^2 f(x)dx$ (b) $\int_0^1 f(x)dx$ (c) $\int_0^2 \left(f(x)\right)^{\frac{1}{2}} dx$ (d) $\int_0^2 \left(f(x)\right)^2 dx$