# PH325: Advanced Statistical Mechanics

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https://piazza.com/iisc.ernet.in/fall2013/ph325/home

# Overview

- What is this course about?
- Course outline
- Logistics

# What is this course about?

# The World Around (Outside?) Us

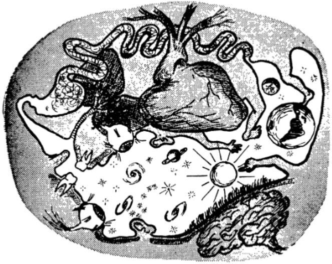

FIGURE 20

Inside-out universe. This surrealistic drawing represents a man walking on the surface of the Earth and looking up at the stars. The picture is transformed topologically according to the method indicated in Figure 19. Thus the Earth, sun, and stars are crowded in a comparatively narrow channel running through the body of the man, and surrounded by his internal organs.

(Gamow from One, Two, Three...Infinity)

- We know that our world is made of "constituents"!
- Our experiences about "agglomerate objects" are encapsulated in the laws of thermodynamics
- *Phenomenological* description of the world...all physics *is* this!

# Statistical Mechanics

- Desire: Describe a system in terms of its constituents (can help us come up with fantastic new things!)
- System: Large collection of constituents, described by
  - Classical: $\Omega = (r, p)$– phase space, Hamiltonain $H(\Omega)$
  - Quantum: $|\psi\rangle$ – Hilbert space, Hamiltonian operator $H$
- Difficulty: Too many "microscopic" degrees of freedom! Cannot "solve" the problem!
- Gibbs: Give up "deterministic" solution to the problem...motivated by the fact that only some *key* aspects of microscopic physics determines outcome of experiments...*not* all the microscopic details
- Describe the system by "probability distribution"
  - Classical: Phase space probability distribution function $\rho(\Omega)$
  - Quantum: Density matrix $\rho = \sum_\alpha p_\alpha |\alpha\rangle\langle\alpha|$
- For a given set of "equilibrium macroscopic conditions" statistical mechanics provides a solution for $\rho$ (both in classical and quantum)– these are the well known equilibrium ensembles

# Statistical Mechanics

- For a system with fixed number of particles, and at a temperature $T$,
  - Classical: $\rho(\Omega) \sim \frac{e^{-\beta H(\Omega)}}{Z}$, $Z = \# \int \mathrm{d}\Omega e^{-\beta\Omega}$
  - Quantum: $\rho = \frac{e^{-\beta H}}{Z}$, $Z = \mathrm{tr}(e^{-\beta H})$

  where $\beta = 1/T$ and $Z$ is the *partition function*...this is the canonical ensemble
- The partition function is related to the free energy $F = -T \ln Z$ (Helmholtz free energy for canonical ensemble)
- Any observable $O$ can now be obtained as
  - Classical: $\langle O \rangle = \int \mathrm{d}\Omega \, \rho(\Omega) O(\Omega)$
  - Quantum: $\langle O \rangle = \mathrm{tr}(\rho O)$
- In principle we have solved all problems...which is a quite mindless thing to say!
- Lets see whats out there...

# The World Around Us...Again!

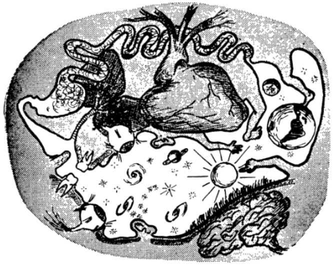

Figure 20

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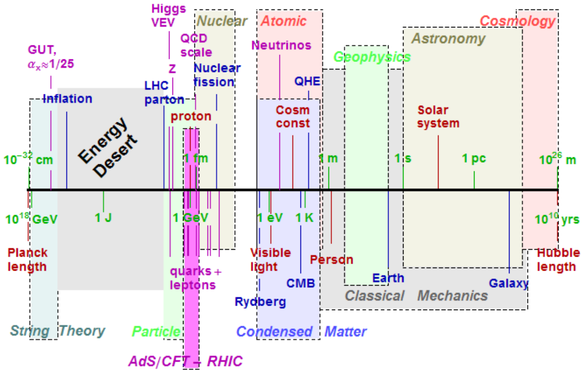
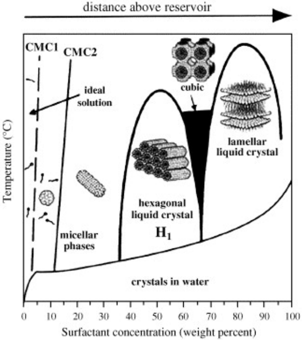
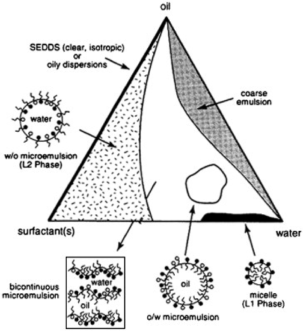



(Internet)

- Many phenomena over a large range of energy scales
- "Fundamental objects" at one scale become constituents of systems at larger scales..
- Things happen when we put lots of stuff together..

# Phases of Micelles etc.

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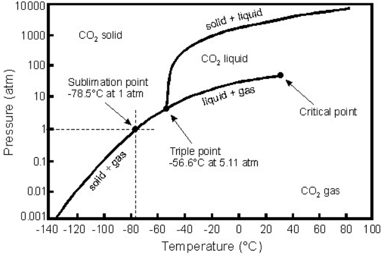
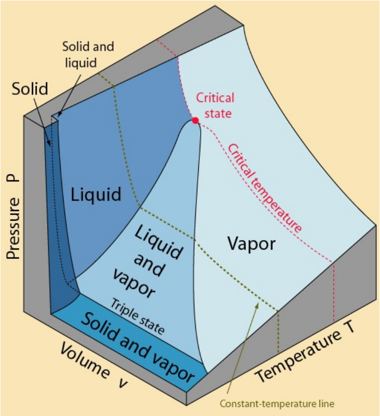
## Key Point

- There are many “phases” and “phase transitions” in systems with many constituents!
- One can go from one phase to another by changing “macroscopic” parameters

# ...of molecules like $CO_2$ and $H_2O$...




Pressure-Temperature phase diagram for $CO_2$.

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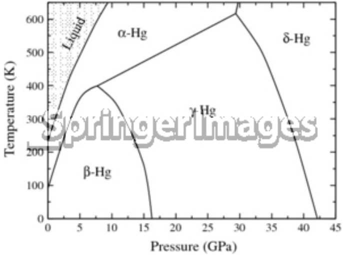
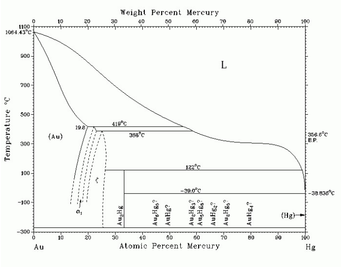
# ...and of elements!



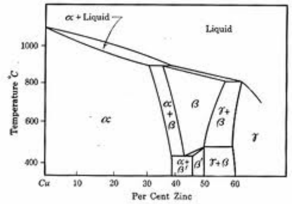




Fig. 57.—Copper-zinc diagram. (*Metals Handbook*, American Society for Metals, 1939 ed.)

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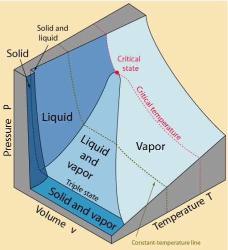
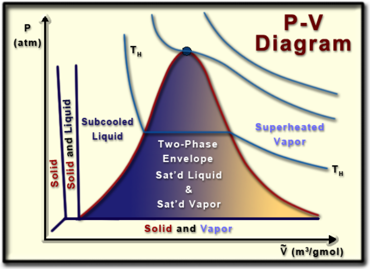
..and their “mixtures”

# First Attempt to Obtain The Phase Diagram

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- Take a shot at obtaining this phase diagram...based on knowledge of PH202
- ...
- and, fail miserably!

## Key Point

- Interactions are *CRUCIAL*!

# Key Degrees of Freedom

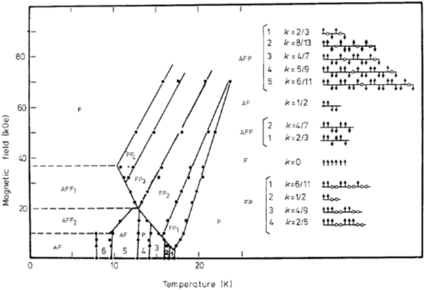



Figure 2. Phase diagram ($H$, $T$) of CeSb deduced from the specific heat peaks and determined in increasing temperature. Notation of the phases is the same as in Rossat-Mignod *et al* (1979a, b).

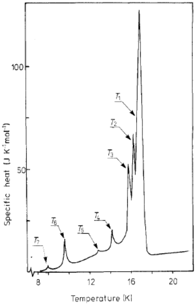
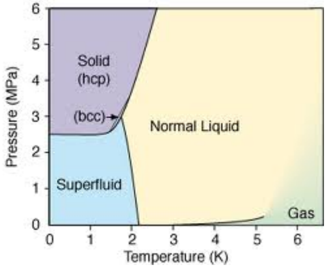



Figure 4. Specific heat of CeSb in zero applied magnetic field.

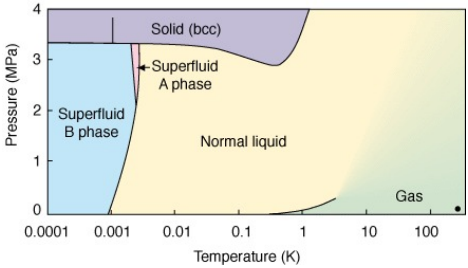
## Key Point

- Only some degrees of freedom may be involved in a phase transition. Not all degrees of freedom may participate in a phase transition!

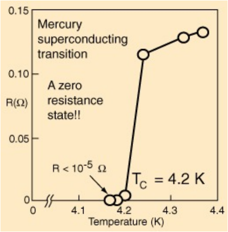
# Further Examples




$^4$He




$^3$He



# Zero Temperature!

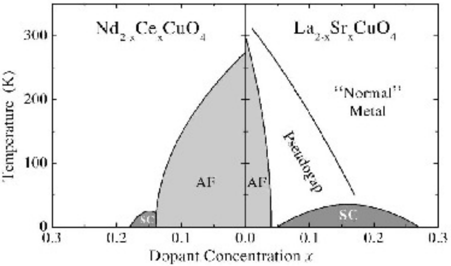
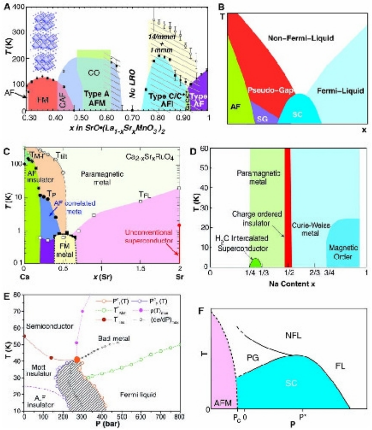



Fig. 1. Phase diagrams of representative materials of the strongly correlated electron family (notations are standard and details can be found in the original references). (A) Temperature versus hole density phase diagram of bilayer manganites (74), including several types of antiferromagnetic (AF) phases, a ferromagnetic (FM) phase, and even a globally disordered region at $x = 0.75$. (B) Generic phase diagram for HTSC. SG stands for spin glass. (C) Phase diagram of single layered ruthenates (75, 76), evolving from a superconducting (SC) state at $x = 2$ to an AF insulator at $x = 0$ ($x$ controls the bandwidth rather than the carrier density). Ruthenates are believed to be clean metals at least at large $x$, thus providing a family of oxides where competition and complexity can be studied with less quenched disorder than in Mn oxides. (D) Phase diagram of Co oxides (77), with SC, charge-ordered (CO), and magnetic regimes. (E) Phase diagram of the organic κ-(BEDT-TTF)$_2$Cu[N(CN)$_2$]Cl salt (57). The hatched region denotes the coexistence of metal and insulator phases. (F) Schematic phase diagram of the Ce-based heavy fermion materials (51).

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## Key Point

- Phase transitions can also occur at zero temperature – Quantum phase transitions

# Phase Transitions

- Can be first order or continuous!
- Continuous transitions have “singularities”

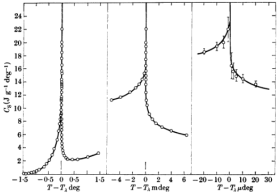



Fig. 1.16. Specific heat of $^4$He as a function of $T - T_\lambda$ in K. Notice that the shape of the specific heat curve is rather like the Greek letter $\lambda$, whence the origin of the term ‘$\lambda$-transition’. The fact that the specific heat is only about ten times its ‘normal’ value even at temperatures only a few microdegrees from $T_\lambda$ is correlated with the fact that the critical-point exponent is extremely small (in fact, $\alpha$ is probably zero, corresponding to a logarithmic divergence). The width of the small vertical line just above the origin indicates the portion of the diagram that is expanded in width in the curve directly to the right. After Buckingham and Fairbank (1965).

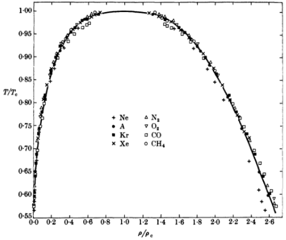



Fig. 1.8. Measurements on eight fluids of the coexistence curve (a reflection of the $P\rho T$ surface in the $\rho T$ plane analogous to Fig. 1.3). The solid curve corresponds to a fit to a cubic equation, i.e. to the choice $\beta = \frac{1}{3}$, where $\rho - \rho_0 \sim (-\epsilon)^\beta$. From Guggenheim (1945).

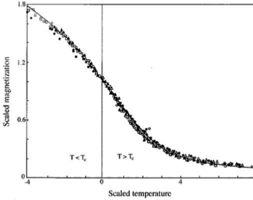



FIG. 1. Experimental $MHT$ data on five different magn
materials plotted in scaled form. The five materials are CrB
EuO, Ni, YIG, and $Pd_3Fe$. None of these materials is an
alized ferromagnet: $CrBr_3$ has considerable lattice anisotro
EuO has significant second-neighbor interactions. Ni is
itinerant-electron ferromagnet, YIG is a ferrimagnet,
$Pd_3Fe$ is a ferromagnetic alloy. Nonetheless, the data for
materials collapse onto a single scaling function, which is
calculated for the $d = 3$ Heisenberg model [after Milošević
Stanley (1976)].

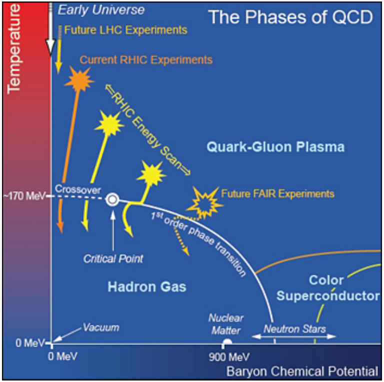
(Stanley, 1971)

- Continuous transitions have “universal features”

## Key Point

- Phase transitions can be first order or continuous
- Continuous transitions have singularities, and universal features

# ...Not "Just" Condensed Matter

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## Key Point

- Phases and phase transitions pervade all of physics!

# Summary

## Key Points

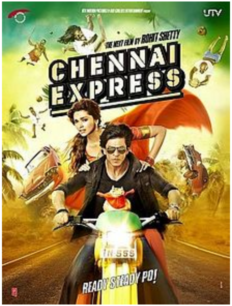
- There are many “phases” and “phase transitions” in systems with many constituents!
- Can go from one phase to another by changing “macroscopic” parameters
- Interactions are *CRUCIAL*!
- Only some degrees of freedom may be involved in a phase transition.
- Phase transitions can also occur at zero temperature – Quantum phase transitions
- Phase transitions can be first order or continuous
- Continuous transitions have singularities, and universal features
- Phases and phase transitions pervade all of physics!

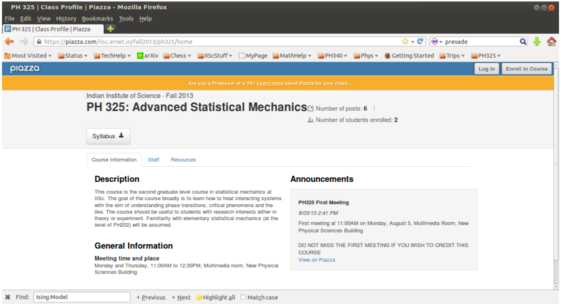
## Questions

- What is a “phase”?
- How do we describe (“understand”) a phase transition?

# What's Coming...Course Outline

# What You Should Expect...

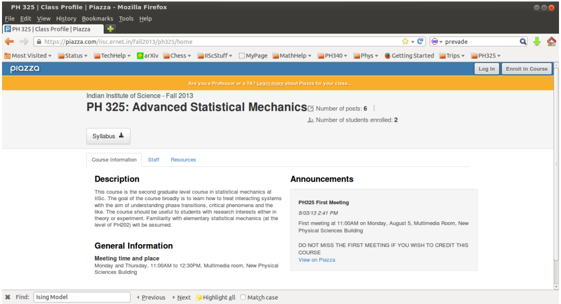


# Why Should You Do This Course

- The ideas to be introduced in this course provide the paradigmatic foundations for modern physics
- This paves way for realizing the deep unity between seemingly different branches of physics

# Logistics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Indian Institute of Science - Fall 2013
PH 325: Advanced Statistical Mechanics
Number of posts: 6
Number of students enrolled: 2
Syllabus
Course Information
Staff
Resources
Description
This course is the second graduate level course in statistical mechanics at IISc. The goal of the course broadly is to learn how to treat interacting systems with the aim of understanding phase transitions, critical phenomena and the like. The course should be useful to students with research interests either in theory or experiment. Familiarity with elementary statistical mechanics (at the level of PH202) will be assumed.
General Information
Meeting time and place
Monday and Thursday, 11:00AM to 12:30PM, Multimedia room, New Physical Sciences Building
Announcements
PH325 First Meeting
8/03/13 2:41 PM
First meeting at 11:00AM on Monday, August 5, Multimedia Room, New Physical Sciences Building
DO NOT MISS THE FIRST MEETING IF YOU WISH TO CREDIT THIS COURSE
View on Piazza
Find: Ising Model
Previous
Next
Highlight all
Match case