

CS 5363, Fall 2013

Dataflow Analysis

1 Dataflow Analysis

Reading / Source Material

- Cooper, Chapter 9

Objectives

1. explain how data flow analyses reach a fixed point
2. simulate an iterative dataflow algorithm described by its dataflow equations

Vocabulary

live (variable/value/register), gen set, kill set, liveout, dataflow problem, iterative data-flow analysis, forward/backward flow problem, fixed-point, monotone function.

Outline

1. Example: single block-liveness
2. dataflow analysis / global live out
 - (a) control flow graph
 - (b) gather local information (usually as sets)
 - (c) set of equations relating sets in different basic blocks
 - (d) searching for the least fixed-point that satisfies all equations
 - (e) forward vs. backwards flow analysis
 - (f) requirements
 - i. termination
 - ii. correctness: meets over all paths solution
 - iii. efficiency: ordering matters
 - A. forward analysis: reverse post order traversal on control flow graph
 - B. backwards analysis: reverse post order traversal on reverse control flow graph

2 Example

2.1 Local Liveness

live before	Instruction	live after
\emptyset	loadI 1 $\Rightarrow r_0$	$\{r_0\}$
$\{r_0\}$	loadI 2 $\Rightarrow r_1$	$\{r_0, r_1\}$
$\{r_0, r_1\}$	loadI 3 $\Rightarrow r_2$	$\{r_0, r_1, r_2\}$
$\{r_0, r_1, r_2\}$	add $r_1, r_2 \Rightarrow r_1$	$\{r_0, r_1\}$
$\{r_0, r_1\}$	loadI 4 $\Rightarrow r_3$	$\{r_0, r_1, r_3\}$
$\{r_0, r_1, r_3\}$	mul $r_0, r_3 \Rightarrow r_4$	$\{r_1, r_4\}$
$\{r_1, r_4\}$	add $r_4, r_1 \Rightarrow r_5$	\emptyset
\emptyset	exit	\emptyset

2.2 LiveOut

Lo: loadI 1 $\Rightarrow r_0$
loadI 5 $\Rightarrow r_1$
jumpI $\rightarrow L_1$

L1: cmp $r_0, r_1 \Rightarrow r_2$
cbr $r_2 \rightarrow L_2, L_3$

L2: loadI 1 $\Rightarrow r_3$
add $r_0, r_3 \Rightarrow r_0$
jumpI $\rightarrow L_1$

L3: writeInt r_2
exit

2.2.1 local information: gen (UEVar) and kill (VarKill) sets

UEVar = upward exposed variables (variables used with no earlier definition in block)

UEVar(Lo) = \emptyset

UEVar(L1) = $\{r_0, r_1\}$

UEVar(L2) = $\{r_0\}$

UEVar(L3) = $\{r_2\}$

VarKill = variables defined within the block (thus marking top "end" of the life range)

VarKill(Lo) = $\{r_0, r_1\}$

VarKill(L1) = $\{r_2\}$

VarKill(L2) = $\{r_0, r_3\}$

VarKill(L3) = \emptyset

2.3 Equations

$$\begin{aligned}
\text{LIVEIN}(b) &= \text{UEVAR}(b) \cup (\text{LIVEOUT}(b) \setminus \text{VARKILL}(b)) \\
&= \text{UEVAR}(b) \cup (\text{LIVEOUT}(b) \cap \overline{\text{VARKILL}(b)}) \\
\text{LIVEOUT}(b_1) &\supseteq \text{LIVEIN}(b_2) \text{ if } b_2 \in \text{succ}(b_1) \\
\text{LIVEOUT}(b_1) &\supseteq \bigcup_{b_2 \in \text{succ}(b_1)} \text{LIVEIN}(b_2) \\
\text{LIVEOUT}(b_1) &\supseteq \bigcup_{b_2 \in \text{succ}(b_1)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)}) \\
\text{LIVEOUT}(b_1) &\supseteq \bigcup_{b_2 \in \text{succ}(b_1)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)}) \\
\text{LIVEOUT}(L3) &\supseteq \emptyset
\end{aligned}$$

2.4 Computation

Treat LiveOut as an array. Repeat for each block until values do not change:

$$\begin{aligned}
\text{LIVEOUT}(b_1) &\leftarrow \bigcup_{b_2 \in \text{succ}(b_1)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)}) \\
\text{LIVEOUT}(L3) &\leftarrow \bigcup_{b_2 \in \text{succ}(L3)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)}) \cup \emptyset
\end{aligned}$$

	L0	L1	L2	L3
—	\emptyset	\emptyset	\emptyset	\emptyset
1	$\{r0, r1\}$	$\{r0, r2\}$	$\{r0, r1\}$	\emptyset
2	$\{r0, r1\}$	$\{r0, r1, r2\}$	$\{r0, r1\}$	\emptyset
3	$\{r0, r1\}$	$\{r0, r1, r2\}$	$\{r0, r1\}$	\emptyset

Iterations 2 and 3 are the same, so this is a fixed point of a function F:

$$\begin{aligned}
&F(\{(L0 \mapsto \{r0, r1\}), (L1 \mapsto \{r0, r1, r2\}), (L2 \mapsto \{r0, r1\}), (L3 \mapsto \emptyset)\}) \\
&= \{(L0 \mapsto \{r0, r1\}), (L1 \mapsto \{r0, r1, r2\}), (L2 \mapsto \{r0, r1\}), (L3 \mapsto \emptyset)\}
\end{aligned}$$

Where we define the function $F : (\text{Block} \rightarrow \mathcal{P}(\text{Var})) \rightarrow (\text{Block} \rightarrow \mathcal{P}(\text{Var}))$:

$$F(\text{LIVEOUT}) = \lambda b_1. \begin{cases} \bigcup_{b_2 \in \text{succ}(b_1)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)}) & \text{if } b_1 \neq L3 \\ (\bigcup_{b_2 \in \text{succ}(L3)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)})) \cup \emptyset & \text{if } b_1 = L3 \end{cases}$$

or equivalently without the lambda:

$$F(\text{LIVEOUT})(b_1) = \begin{cases} \bigcup_{b_2 \in \text{succ}(b_1)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)}) & \text{if } b_1 \neq L3 \\ (\bigcup_{b_2 \in \text{succ}(L3)} \text{UEVAR}(b_2) \cup (\text{LIVEOUT}(b_2) \cap \overline{\text{VARKILL}(b_2)})) \cup \emptyset & \text{if } b_1 = L3 \end{cases}$$