Newton's work on gravitational and central forces to the attention of the world. After observing the comet personally in 1682, Halley became interested. Partly as a result of a bet between Christopher Wren and Robert Hooke, Halley asked Newton in 1684 what paths the planets must follow if the Sun pulled them with a force inversely proportional to the square of their distances. To the astonishment of Halley, Newton replied, "Why, in ellipses, of course." Newton had worked it out 20 years previously but had not published the result. With painstaking effort, Halley was able in 1705 to predict the next occurrence of the comet, now bearing his name, to be in 1758.

8.8 Orbital Dynamics

The use of central-force motion is nowhere more useful, important, and interesting than in space dynamics. Although space dynamics is actually quite complex because of the gravitational attraction of a spacecraft to various bodies and the orbital motion involved, we examine two rather simple aspects: a proposed trip to Mars and flybys past comets and planets.

Orbits are changed by single or multiple thrusts of the rocket engines. The simplest maneuver is a single thrust applied in the orbital plane that does not change the direction of the angular momentum but does change the eccentricity and energy simultaneously. The most economical method of interplanetary transfer consists of moving from one circular heliocentric (Sun-oriented motion) orbit to another in the same plane. Earth and Mars represent such a system reasonably well, and a Hohmann transfer (Figure 8-10) represents the path of minimum total energy expenditure.* Two engine burns are required: (1) the first burn injects the spacecraft from the circular Earth orbit to an elliptical transfer orbit that intersects Mars' orbit; (2) the second burn transfers the spacecraft from the elliptical orbit into Mars' orbit.

We can calculate the velocity changes needed for a Hohmann transfer by calculating the velocity of a spacecraft moving in the orbit of Earth around the Sun (r_1 in Figure 8-10) and the velocity needed to "kick" it into an elliptical transfer orbit that can reach Mars' orbit. We are considering only the gravitational attraction of the Sun and not that of Earth and Mars.

For circles and ellipses we have, from Equation 8.42,

$$E = -\frac{k}{2a}$$

For a circular path around the Sun, this becomes

$$E = -\frac{k}{2r_1} = \frac{1}{2}mv_1^2 - \frac{k}{r_1} \tag{8.50}$$

*See Kaplan (Ka76, Chapter 3) for the proof. Walter Hohmann, a German pioneer in space travel research, proposed in 1925 the most energy-efficient method of transferring between elliptical (planetary) orbits in the same plane using only two velocity changes.

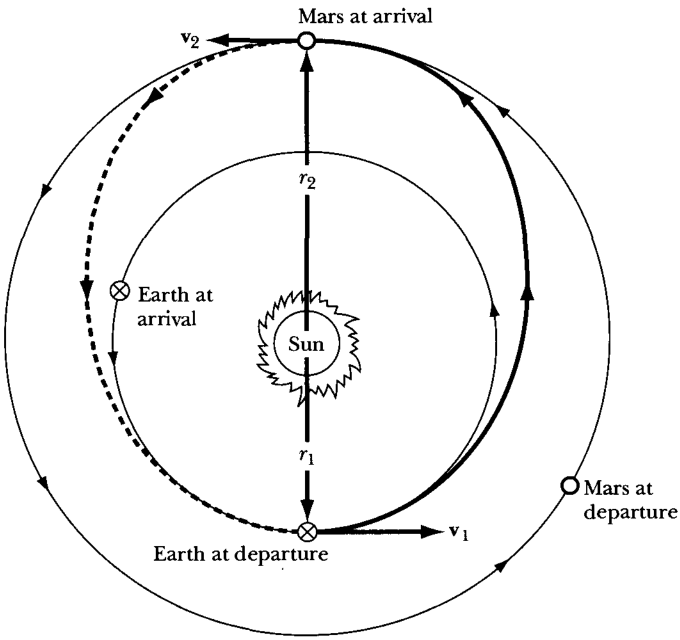

FIGURE 8-10 The Hohmann transfer for a round trip between Earth and Mars. It represents the minimum energy expenditure.

where we have $E = T + U$. We solve Equation 8.50 for v_1:

$$v_1 = \sqrt{\frac{k}{mr_1}} \tag{8.51}$$

We denote the semimajor axis of the transfer ellipse by a_t:

$$2a_t = r_1 + r_2$$

If we calculate the energy at the perihelion for the transfer ellipse, we have

$$E_t = \frac{-k}{r_1 + r_2} = \frac{1}{2} m v_{t1}^2 - \frac{k}{r_1} \tag{8.52}$$

where v_{t1} is the perihelion transfer speed. The direction of v_{t1} is along $\mathbf{v}_1$ in Figure 8-10. Solving Equation 8.52 for v_{t1} gives

$$v_{t1} = \sqrt{\frac{2k}{mr_1}\left(\frac{r_2}{r_1 + r_2}\right)} \tag{8.53}$$

The speed transfer Δv_1 needed is just

$$\Delta v_1 = v_{t1} - v_1 \tag{8.54}$$

Similarly, for the transfer from the ellipse to the circular orbit of radius r_2, we have

$$\Delta v_2 = v_2 - v_{t2} \tag{8.55}$$

where

$$v_2 = \sqrt{\frac{k}{mr_2}} \tag{8.56}$$

and

$$\left.\begin{aligned} v_{t2} &= \sqrt{\frac{2}{m}\left(E_t + \frac{k}{r_2}\right)} \\ v_{t2} &= \sqrt{\frac{2k}{mr_2}\left(\frac{r_1}{r_1 + r_2}\right)} \end{aligned}\right\} \tag{8.57}$$

The direction of v_{t2} is along $\mathbf{v}_2$ in Figure 8-10. The total speed increment can be determined by adding the speed changes, $\Delta v = \Delta v_1 + \Delta v_2$.

The total time required to make the transfer T_t is a half-period of the transfer orbit. From Equation 8.48, we have

$$T_t = \frac{\tau_t}{2}$$

$$T_t = \pi\sqrt{\frac{m}{k}}\, a_t^{3/2} \tag{8.58}$$

EXAMPLE 8.5

Calculate the time needed for a spacecraft to make a Hohmann transfer from Earth to Mars and the heliocentric transfer speed required assuming both planets are in coplanar orbits.

Solution. We need to insert the appropriate constants in Equation 8.58.

$$\begin{aligned} \frac{m}{k} &= \frac{m}{GmM_{\text{Sun}}} = \frac{1}{GM_{\text{Sun}}} \\ &= \frac{1}{(6.67 \times 10^{-11}\ \text{m}^3/\text{s}^2 \cdot \text{kg})(1.99 \times 10^{30}\ \text{kg})} \\ &= 7.53 \times 10^{-21}\ \text{s}^2/\text{m}^3 \end{aligned} \tag{8.59}$$

Because k/m occurs so often in solar system calculations, we write it as well.

$$\begin{aligned} \frac{k}{m} &= 1.33 \times 10^{20}\ \text{m}^3/\text{s}^2 \\ a_t &= \frac{1}{2}(r_{\text{Earth-Sun}} + r_{\text{Mars-Sun}}) \\ &= \frac{1}{2}(1.50 \times 10^{11}\ \text{m} + 2.28 \times 10^{11}\ \text{m}) \\ &= 1.89 \times 10^{11}\ \text{m} \\ T_t &= \pi(7.53 \times 10^{-21}\ \text{s}^2/\text{m}^3)^{1/2}(1.89 \times 10^{11}\ \text{m})^{3/2} \\ &= 2.24 \times 10^7\ \text{s} \\ &= 259\ \text{days} \end{aligned} \tag{8.60}$$

The heliocentric speed needed for the transfer is given in Equation 8.53.

$$v_{t1} = \left[\frac{2(1.33 \times 10^{20}\,\text{m}^3/\text{s}^2)(2.28 \times 10^{11}\,\text{m})}{(1.50 \times 10^{11}\,\text{m})(3.78 \times 10^{11}\,\text{m})}\right]^{1/2}$$

$$= 3.27 \times 10^4\ \text{m/s} = 32.7\ \text{km/s}$$

We can compare v_{t1} with the orbital speed of Earth (Equation 8.51).

$$v_1 = \left[\frac{1.33 \times 10^{20}\,\text{m}^3/\text{s}^2}{1.50 \times 10^{11}\,\text{m}}\right]^{1/2} = 29.8\ \text{km/s}$$

For transfers to the outer planets, the spacecraft should be launched in the direction of Earth's orbit in order to gain Earth's orbital velocity. To transfer to the inner planets (e.g., to Venus), the spacecraft should be launched opposite Earth's motion. In each case, it is the relative velocity Δv_1 that is important to the spacecraft (i.e., relative to Earth).

Although the Hohmann transfer path represents the least energy expenditure, it does not represent the shortest time. For a round trip from Earth to Mars, the spacecraft would have to remain on Mars for 460 days until Earth and Mars were positioned correctly for the return trip (see Figure 8-11a). The total trip (259 + 460 + 259 = 978 days = 2.7 yr) would probably be too long. Other schemes either use more fuel to gain speed (Figure 8-11b) or use the slingshot effect of flybys. Such a flyby mission past Venus (see Figure 8-11c) could be done in less than 2 years with only a few weeks near (or on) Mars.

Several spacecraft in recent years have escaped Earth's gravitational attraction to explore our solar system. Such **interplanetary transfer** can be divided into three segments: (1) the escape from Earth, (2) a heliocentric transfer to the area of interest, and (3) an encounter with another body—so far, either a planet or a comet. The spacecraft fuel required for such missions can be enormous, but a clever trick has been designed to "steal" energy from other solar system bodies. Because the mass of a spacecraft is so much smaller than the planets (or their moons), the energy loss of the heavenly body is negligible.

We examine a simple version of this flyby or slingshot effect that utilizes gravity assist. A spacecraft coming from infinity approaches a body (labeled B), interacts with B, and recedes. The path is a hyperbola (Figure 8-12). The initial and final velocities, *with respect to B*, are denoted by v_i' and v_f', respectively. The net effect on the spacecraft is a deflection angle of δ with respect to B.

If we examine the system in some inertial frame in which the motion of B occurs, the velocities of the spacecraft can be quite different *because of the motion of B*. The initial velocity v_i is shown in Figure 8-13a, and both v_i and v_f are shown in Figure 8-13b. Notice that the spacecraft has increased its speed as well as changed its direction. An increase in velocity occurs when the spacecraft passes *behind* B's direction of motion. Similarly, a decrease in velocity occurs when the spacecraft passes in *front* of B's motion.

During the 1970s, scientists at the Jet Propulsion Laboratory of the National Aeronautics and Space Administration (NASA) realized that the four largest planets of our solar system would be in a fortuitous position to allow a spacecraft

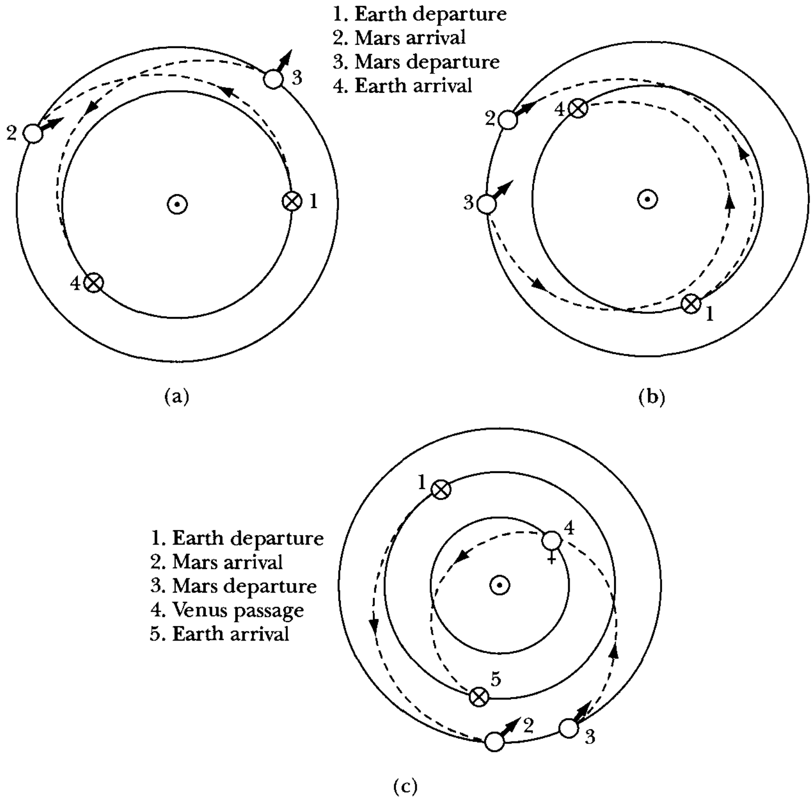

FIGURE 8-11 Round trips from Earth to Mars. (a) The minimum energy mission (Hohmann transfer) requires a long stopover on Mars before returning to Earth. (b) A shorter mission to Mars requires more fuel and a closer orbit to the Sun. (c) The fuel required for the shorter mission of (b) can be further improved if Venus is positioned for a gravity assist during flyby.

to fly past them and many of their 32 known moons in a single, relatively short "Grand Tour" mission using the gravity-assist method just discussed. This opportunity of the planets' alignment would not occur again for 175 years. Because of budget constraints, there was not time to develop the new technology needed, and a mission to last only 4 years to visit just Jupiter and Saturn was approved and planned. No special equipment was put on board the twin *Voyager* spacecrafts for an encounter with Uranus and Neptune. *Voyagers 1* and *2* were launched in 1977 for visits to Jupiter in 1979 and Saturn in 1980 (*Voyager 1*) and 1981 (*Voyager 2*). Because of the success of these visits to Jupiter and Saturn, funding was later approved to extend *Voyager 2*'s mission to include Uranus and Neptune. The *Voyagers* are now on their way out of our solar system.

The path of *Voyager 2* is shown in Figure 8-14. The slingshot effect of gravity allowed the path of *Voyager 2* to be redirected, for example, toward Uranus as it passed Saturn by the method shown in Figure 8-12. The gravitational attraction from Saturn was used to pull the spacecraft off its straight path and redirect it at a different angle. The effect of the orbital motion of Saturn allows an increase in the

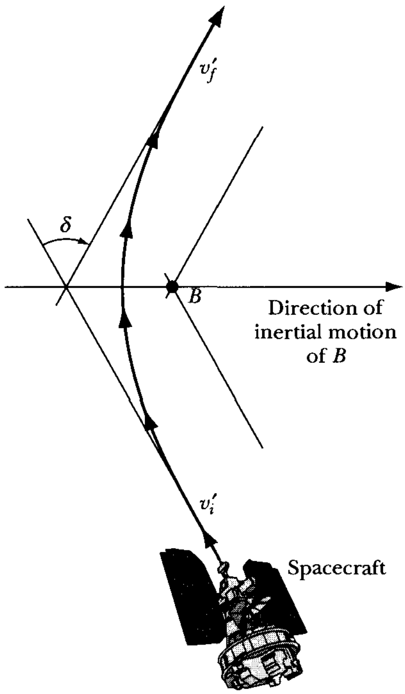

FIGURE 8-12 A spacecraft flies by a large body B (like a planet) and gains speed when it flies behind B's direction of motion. Similarly, the spacecraft loses speed when it passes in front of B's direction of motion. The direction of the spacecraft also changes.

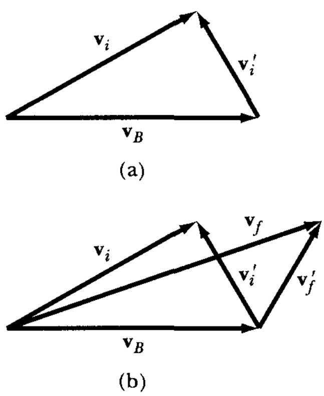

FIGURE 8-13 The vectors $\mathbf{v}_i'$ and $\mathbf{v}_f'$ are the initial and final velocities of the spacecraft with respect to B. The vectors $\mathbf{v}_i$ and $\mathbf{v}_f$ are the velocities in an inertial frame. (a) $\mathbf{v}_i = \mathbf{v}_B + \mathbf{v}_i'$. (b) $\mathbf{v}_f = \mathbf{v}_B + \mathbf{v}_f'$.

spacecraft's speed. It was only by using this gravity-assist technique that the spectacular mission of *Voyager 2* was made possible in only a brief 12-year period. *Voyager 2* passed Uranus in 1986 and Neptune in 1989 before proceeding into interstellar space in one of the most successful space missions ever undertaken. Most planetary missions now take advantage of gravitational assists; for example, the *Galileo* satel-

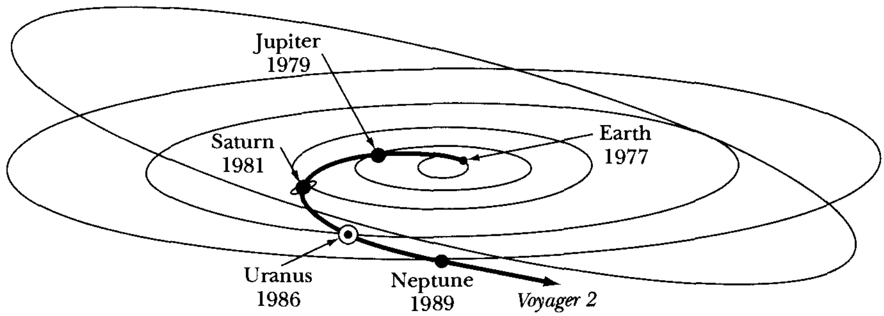

FIGURE 8-14 *Voyager 2* was launched in 1977 and passed by Jupiter, Saturn, Uranus, and Neptune. Gravitational assists were used in the mission.

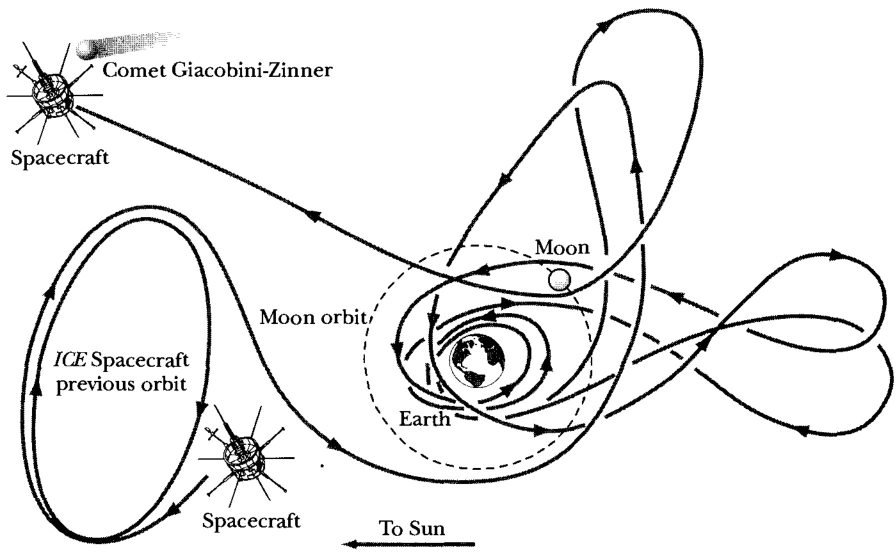

FIGURE 8-15 The NASA spacecraft initially called *ISEE-3* was reprogrammed to be the International Cometary Explorer and was sent on a spectacular three-year journey utilizing gravity assists on its way by the Comet Giacobini-Zinner.

lite, which photographed the spectacular collisions of the Shoemaker-Levy comet with Jupiter in 1994 and reached Jupiter in 1995, was launched in 1989 but went by Earth twice (1990 and 1992) as well as Venus (1990) to gain speed and redirection.

A spectacular display of flybys occurred in the years 1982–1985 by a spacecraft initially called the International Sun-Earth Explorer 3 (*ISEE-3*). Launched in 1978, its mission was to monitor the solar wind between the Sun and Earth. For 4 years, the spacecraft circled in the ecliptical plane about 2 million miles from Earth. In 1982—because the United States had decided not to participate in a joint European, Japanese, and Soviet spacecraft investigation of Halley's comet in 1986—NASA decided to reprogram the *ISEE-3*, renamed it the *International Cometary Explorer* (*ICE*), and sent it through the Giacobini-Zinner comet in September 1985, some 6 months before the flybys of other spacecraft with Halley's comet. The subsequent three-year journey of *ICE* was spectacular (Figure 8-15). The path of *ICE* included two close trips to Earth and five flybys of the moon along its billion-mile trip to the comet. During one flyby, the satellite came within 75 miles of the lunar surface. The entire path could be planned precisely because the force law is very well known. The eventual interaction with the comet, some 44 million miles from Earth, included a 20-minute trip through the comet—about 5,000 miles behind the comet's nucleus.

8.9 Apsidal Angles and Precession (Optional)

If a particle executes bounded, noncircular motion in a central-force field, then the radial distance from the force center to the particle must always be in the range $r_{max} \geq r \geq r_{min}$; that is, r must be bounded by the apsidal distances. Figure 8-6 indicates that only *two* apsidal distances exist for bounded, noncircular motion. But in executing one complete revolution in θ, the particle may not return to its original position (see Figure 8-4). The angular separation between two successive values of $r = r_{max}$ depends on the exact nature of the force. The angle between any two consecutive apsides is called the **apsidal angle,** and because a closed orbit must be symmetric about any apsis, it follows that all apsidal angles for such motion must be equal. The apsidal angle for elliptical motion, for example, is just π. If the orbit is not closed, the particle reaches the apsidal distances at different points in each revolution; the apsidal angle is not then a rational fraction of 2π, as is required for a closed orbit. If the orbit is *almost* closed, the apsides *precess*, or rotate slowly in the plane of the motion. This effect is exactly analogous to the slow rotation of the elliptical motion of a two-dimensional harmonic oscillator whose natural frequencies for the x and y motions are almost equal (see Section 3.3).

Because an inverse-square-law force requires that all elliptical orbits be exactly closed, the apsides must stay fixed in space for all time. If the apsides move with time, however slowly, this indicates that the force law under which the body moves does not vary exactly as the inverse square of the distance. This important fact was realized by Newton, who pointed out that any advance or regression of a planet's perihelion would require the radial dependence of the force law to be slightly different from $1/r^2$. Thus, Newton argued, the observation of the time dependence of the perihelia of the planets would be a sensitive test of the validity of the form of the universal gravitation law.

In point of fact, for planetary motion within the solar system, one expects that, because of the perturbations introduced by the existence of all the other planets, the force experienced by any planet does not vary exactly as $1/r^2$, if r is measured from the Sun. This effect is small, however, and only slight variations of planetary perihelia have been observed. The perihelion of Mercury, for example, which shows the largest effect, advances only about 574″ of arc per century.* Detailed calculations of the influence of the other planets on the motion of

*This precession is in addition to the general precession of the equinox with respect to the "fixed" stars, which amounts to 5025.645″ ± 0.050″ per century.