We integrate the last term using the definite integral, $\int \ln x \, dx = x \ln x - x$, to obtain after collecting terms,

$$H_{bo} = -\frac{g(m_0 - m_f)^2}{2\alpha^2} + \frac{u}{\alpha}\left[m_f \ln\left(\frac{m_f}{m_0}\right) + m_0 - m_f \right] \tag{9.167}$$

If we insert the numbers from the last example, we find the same answer for the burnout height.

The speed at burnout can be determined directly from Equation 9.165.

$$v_{bo} = -gT + u \ln\left(\frac{m_0}{m_f}\right)$$

$$= -\frac{g(m_0 - m_f)}{\alpha} + u \ln\left(\frac{m_0}{m_f}\right) \tag{9.168}$$

PROBLEMS

9-1. Find the center of mass of a hemispherical shell of constant density and inner radius r_1 and outer radius r_2.

9-2. Find the center of mass of a uniformly solid cone of base diameter $2a$ and height h.

9-3. Find the center of mass of a uniformly solid cone of base diameter $2a$ and height h and a solid hemisphere of radius a where the two bases are touching.

9-4. Find the center of mass of a uniform wire that subtends an arc θ if the radius of the circular arc is a, as shown in Figure 9-A.

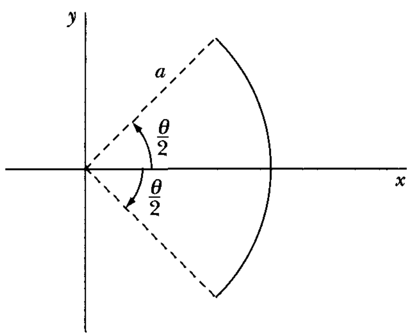

FIGURE 9-A Problem 9-4.

9-5. The center of gravity of a system of particles is the point about which external gravitational forces exert no net torque. For a uniform gravitational force, show that the center of gravity is identical to the center of mass for the system of particles.

9-6. Consider two particles of equal mass m. The forces on the particles are $\mathbf{F}_1 = 0$ and $\mathbf{F}_2 = F_0\mathbf{i}$. If the particles are initially at rest at the origin, what is the position, velocity, and acceleration of the center of mass?

9-7. A model of the water molecule H_2O is shown in Figure 9-B. Where is the center of mass?

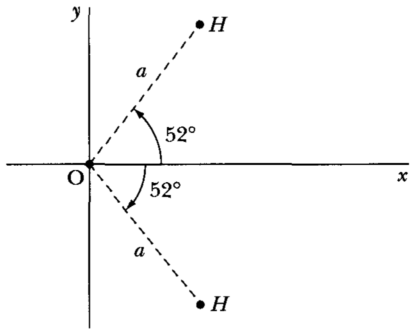

FIGURE 9-B Problem 9-7.

9-8. Where is the center of mass of the isosceles right triangle of uniform areal density shown in Figure 9-C?

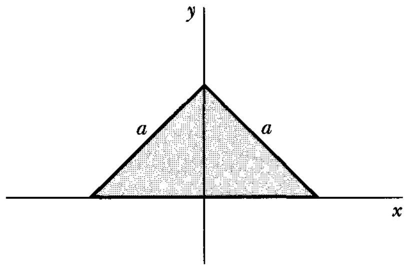

FIGURE 9-C Problem 9-8.

9-9. A projectile is fired at an angle of 45° with initial kinetic energy E_0. At the top of its trajectory, the projectile explodes with additional energy E_0 into two fragments. One fragment of mass m_1 travels straight down. What is the velocity (magnitude and direction) of the second fragment of mass m_2 and the velocity of the first? What is the ratio of m_1/m_2 when m_1 is a maximum?

9-10. A cannon in a fort overlooking the ocean fires a shell of mass M at an elevation angle θ and muzzle velocity v_0. At the highest point, the shell explodes into two fragments (masses $m_1 + m_2 = M$), with an additional energy E, traveling in the original horizontal direction. Find the distance separating the two fragments when they land in the ocean. For simplicity, assume the cannon is at sea level.

9-11. Verify that the second term on the right-hand side of Equation 9.9 indeed vanishes for the case $n = 3$.

9-12. Astronaut Stumblebum wanders too far away from the space shuttle orbiter while repairing a broken communications satellite. Stumblebum realizes that the orbiter is moving away from him at 3 m/s. Stumblebum and his maneuvering unit have a mass of 100 kg, including a pressurized tank of mass 10 kg. The tank includes only 2 kg of gas that is used to propel him in space. The gas escapes with a constant velocity of 100 m/s.

(a) Will Stumblebum run out of gas before he reaches the orbiter?
(b) With what velocity will Stumblebum have to throw the empty tank away to reach the orbiter?

9-13. Even though the total force on a system of particles (Equation 9.9) is zero, the net torque may not be zero. Show that the net torque has the same value in any coordinate system.

9-14. Consider a system of particles interacting by magnetic forces. Are Equations 9.11 and 9.31 valid? Explain.

9-15. A smooth rope is placed above a hole in a table (Figure 9-D). One end of the rope falls through the hole at $t = 0$, pulling steadily on the remainder of the rope. Find the velocity and acceleration of the rope as a function of the distance to the end of the rope x. Ignore all friction. The total length of the rope is L.

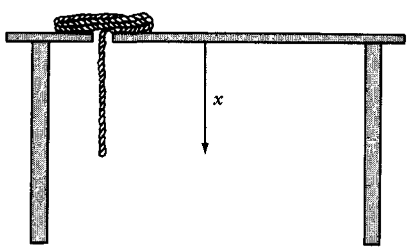

FIGURE 9-D Problem 9-15.

9-16. For the energy-conserving case of the falling chain in Example 9.2, show that the tension on either side of the bottom bend is equal and has the value $\rho\dot{x}^2/4$.

9-17. Integrate Equation 9.17 in Example 9.2 numerically and make a plot of the speed versus the time using dimensionless parameters, $\dot{x}/\sqrt{2gb}$ vs. $t/\sqrt{2b/g}$ where $\sqrt{2b/g}$ is the free fall time, $t_{\text{free fall}}$. Find the time it takes for the free end to reach the bottom. Define natural units by $\tau \equiv t\sqrt{g/2b}$, $\alpha \equiv x/2b$ and integrate $d\tau/d\alpha$ from $\alpha = \varepsilon$ (some small number greater than 0) to $\alpha = 1/2$. One can't integrate numerically from $\alpha = 0$ because of a singularity in $d\tau/d\alpha$. The expression $d\tau/d\alpha$ is

$$\frac{d\tau}{d\alpha} = \sqrt{\frac{1 - 2\alpha}{2\alpha(1 - \alpha)}}$$

9-18. Use a computer to make a plot of the tension versus time for the falling chain in Example 9.2. Use dimensionless parameters (T/Mg) versus $t/t_{\text{free fall}}$, where $t_{\text{free fall}} = \sqrt{2b/g}$. Stop the plot before T/Mg becomes greater than 50.

9-19. A chain such as the one in Example 9.2 (with the same parameters) of length b and mass ρb is suspended from one end at a point that is a height b above a table so that the free end barely touches the tabletop. At time $t = 0$, the fixed end of the chain is released. Find the force that the tabletop exerts on the chain after the original fixed end has fallen a distance x.

9-20. A uniform rope of total length $2a$ hangs in equilibrium over a smooth nail. A very small impulse causes the rope to slowly roll off the nail. Find the velocity of the rope as it just clears the nail. Assume the rope is prevented from lifting off the nail and is in free fall.

9-21. A flexible rope of length 1.0 m slides from a frictionless table top as shown in Figure 9-E. The rope is initially released from rest with 30 cm hanging over the edge of the table. Find the time at which the left end of the rope reaches the edge of the table.

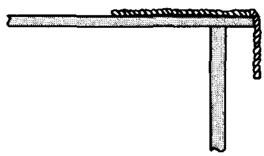

FIGURE 9-E Problem 9-21.

9-22. A deuteron (nucleus of deuterium atom consisting of a proton and a neutron) with speed 14.9 km/s collides elastically with a neutron at rest. Use the approximation that the deuteron is twice the mass of the neutron. **(a)** If the deuteron is scattered through a LAB angle $\psi = 10°$, what are the final speeds of the deuteron and neutron? **(b)** What is the LAB scattering angle of the neutron? **(c)** What is the maximum possible scattering angle of the deuteron?

9-23. A particle of mass m_1 and velocity u_1 collides with a particle of mass m_2 at rest. The two particles stick together. What fraction of the original kinetic energy is lost in the collision?

9-24. A particle of mass m at the end of a light string wraps itself about a fixed vertical cylinder of radius a (Figure 9-F). All the motion is in the horizontal plane (disregard gravity). The angular velocity of the cord is ω_0 when the distance from the particle to the point of contact of the string and cylinder is b. Find the angular velocity and tension in the string after the cord has turned through an additional angle θ.

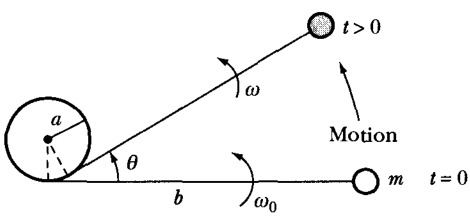

FIGURE 9-F Problem 9-24.

9-25. Slow-moving neutrons have a much larger absorption rate in ^{235}U than fast neutrons produced by $^{235}U^*$ fission in a nuclear reactor. For that reason, reactors consist of moderators to slow down neutrons by elastic collisions. What elements are best to be used as moderators? Explain.

9-26. The force of attraction between two particles is given by

$$\mathbf{f}_{12} = k\left[(\mathbf{r}_2 - \mathbf{r}_1) - \frac{r}{v_0}(\dot{\mathbf{r}}_2 - \dot{\mathbf{r}}_1)\right]$$

where k is a constant, v_0 is a constant velocity, and $r \equiv |\mathbf{r}_2 - \mathbf{r}_1|$. Calculate the internal torque for the system; why does this quantity not vanish? Is the system conservative?

9-27. Derive Equation 9.90.

9-28. A particle of mass m_1 elastically collides with a particle of mass m_2 at rest. What is the maximum fraction of kinetic energy loss for m_1? Describe the reaction.

9-29. Derive Equation 9.91.

9-30. A tennis player strikes an incoming tennis ball of mass 60 g as shown in Figure 9-G. The incoming tennis ball velocity is $v_i = 8$ m/s, and the outgoing velocity is $v_f = 16$ m/s.

(a) What impulse was given to the tennis ball?

(b) If the collision time was 0.01 s, what was the average force exerted by the tennis racket?

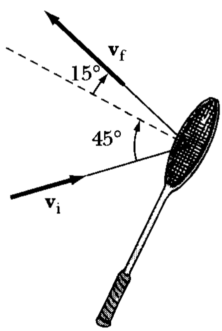

FIGURE 9-G Problem 9-30.

9-31. Derive Equation 9.92.

9-32. A particle of mass m and velocity u_1 makes a head-on collision with another particle of mass $2m$ at rest. If the coefficient of restitution is such to make the loss of total kinetic energy a maximum, what are the velocities v_1 and v_2 after the collision?

9-33. Show that T_1/T_0 can be expressed in terms of $m_2/m_1 \equiv \alpha$ and $\cos\psi \equiv y$ as

$$\frac{T_1}{T_0} = (1 + \alpha)^{-2}\left(2y^2 + \alpha^2 - 1 + 2y\sqrt{\alpha^2 + y^2 - 1}\right)$$

Plot T_1/T_0 as a function of ψ for $\alpha = 1$, 2, 4, and 12. These plots correspond to the energies of protons or neutrons after scattering from hydrogen ($\alpha = 1$), deuterium ($\alpha = 2$), helium ($\alpha = 4$), and carbon ($\alpha = 12$), or of alpha particles scattered from helium ($\alpha = 1$), oxygen ($\alpha = 4$), and so forth.

9-34. A billiard ball of initial velocity u_1 collides with another billiard ball (same mass) initially at rest. The first ball moves off at $\psi = 45°$. For an elastic collision, what are the velocities of both balls after the collision? At what LAB angle does the second ball emerge?

9-35. A particle of mass m_1 with initial laboratory velocity u_1 collides with a particle of mass m_2 at rest in the LAB system. The particle m_1 is scattered through a LAB angle ψ and has a final velocity v_1, where $v_1 = v_1(\psi)$. Find the surface such that the time of travel of the scattered particle from the point of collision to the surface is independent of the scattering angle. Consider the cases (**a**) $m_2 = m_1$, (**b**) $m_2 = 2m_1$, and (**c**) $m_2 = \infty$. Suggest an application of this result in terms of a detector for nuclear particles.

9-36. In an elastic collision of two particles with masses m_1 and m_2, the initial velocities are $\mathbf{u}_1$ and $\mathbf{u}_2 = \alpha\mathbf{u}_1$. If the initial kinetic energies of the two particles are equal, find the conditions on u_1/u_2 and m_1/m_2 such that m_1 is at rest after the collision. Examine both cases for the sign of α.

9-37. When a bullet fires in a gun, the explosion subsides quickly. Suppose the force on the bullet is $F = (360 - 10^7 t^2 \text{s}^{-2})$ N until the force becomes zero (and remains zero). The mass of the bullet is 3 g.
(**a**) What impulse acts on the bullet?
(**b**) What is the muzzle velocity of the gun?

9-38. Show that

$$\frac{T_1}{T_0} = \frac{m_1^2}{(m_1 + m_2)^2} \cdot S^2$$

where

$$S \equiv \cos\psi + \frac{\cos(\theta - \psi)}{\left(\dfrac{m_1}{m_2}\right)}$$

9-39. A particle of mass m strikes a smooth wall at an angle θ from the normal. The coefficient of restitution is ε. Find the velocity and the rebound angle of the particle after leaving the wall.

9-40. A particle of mass m_1 and velocity u_1 strikes head-on a particle of mass m_2 at rest. The coefficient of restitution is ε. Particle m_2 is tied to a point a distance a away as shown in Figure 9-H. Find the velocity (magnitude and direction) of m_1 and m_2 after the collision.

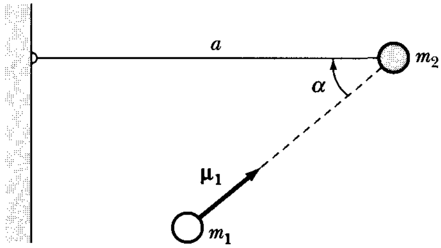

FIGURE 9-H Problem 9-40.

9-41. A rubber ball is dropped from rest onto a linoleum floor a distance h_1 away. The rubber ball bounces up to a height h_2. What is the coefficient of restitution? What fraction of the original kinetic energy is lost in terms of ε?

9-42. A steel ball of velocity 5 m/s strikes a smooth, heavy steel plate at an angle of 30° from the normal. If the coefficient of restitution is 0.8, at what angle and velocity does the steel ball bounce off the plate?

9-43. A proton (mass m) of kinetic energy T_0 collides with a helium nucleus (mass $4m$) at rest. Find the recoil angle of the helium if $\psi = 45°$ and the inelastic collision has $Q = -T_0/6$.

9-44. A uniformly dense rope of length b and mass density μ is coiled on a smooth table. One end is lifted by hand with a constant velocity v_0. Find the force of the rope held by the hand when the rope is a distance a above the table ($b > a$).

9-45. Show that the equivalent of Equation 9.129 expressed in terms of θ rather than ψ is

$$\sigma(\theta) = \sigma(\psi) \cdot \frac{1 + x \cos \theta}{(1 + 2x \cos \theta + x^2)^{3/2}}$$

9-46. Calculate the differential cross section $\sigma(\theta)$ and the total cross section σ_t for the elastic scattering of a particle from an impenetrable sphere; the potential is given by

$$U(r) = \begin{cases} 0, & r > a \\ \infty, & r < a \end{cases}$$

9-47. Show that the Rutherford scattering cross section (for the case $m_1 = m_2$) can be expressed in terms of the recoil angle as

$$\sigma_{\text{LAB}}(\zeta) = \frac{k^2}{T_0^2} \cdot \frac{1}{\cos^3 \zeta}$$

9-48. Consider the case of Rutherford scattering in the event that $m_1 \gg m_2$. Obtain an approximate expression for the differential cross section in the LAB coordinate system.

9-49. Consider the case of Rutherford scattering in the event that $m_2 \gg m_1$. Obtain an expression of the differential cross section in the CM system that is correct to first order in the quantity m_1/m_2. Compare this result with Equation 9.140.

9-50. A fixed force center scatters a particle of mass m according to the force law $F(r) = k/r^3$. If the initial velocity of the particle is u_0, show that the differential scattering cross section is

$$\sigma(\theta) = \frac{k\pi^2(\pi - \theta)}{mu_0^2\theta^2(2\pi - \theta)^2 \sin \theta}$$

9-51. It is found experimentally that in the elastic scattering of neutrons by protons ($m_n \cong m_p$) at relatively low energies, the energy distribution of the recoiling protons in the LAB system is constant up to a maximum energy, which is the energy of the incident neutrons. What is the angular distribution of the scattering in the CM system?

9-52. Show that the energy distribution of particles recoiling from an elastic collision is always directly proportional to the differential scattering cross section in the CM system.

9-53. The most energetic α-particles available to Ernest Rutherford and his colleagues for the famous Rutherford scattering experiment were 7.7 MeV. For the scattering of 7.7 MeV α-particles from ^{238}U (initially at rest) at a scattering angle in the lab of 90° (all calculations are in the LAB system unless otherwise noted), find the following:
(a) the recoil scattering angle of ^{238}U.
(b) the scattering angles of the α-particle and ^{238}U in the CM system.
(c) the kinetic energies of the scattered α-particle and ^{238}U.
(d) the impact parameter b.
(e) the distance of closest approach r_{min}.
(f) the differential cross section at 90°.
(g) the ratio of the probabilities of scattering at 90° to that of 5°.

9-54. A rocket starts from rest in free space by emitting mass. At what fraction of the initial mass is the momentum a maximum?

9-55. An extremely well-constructed rocket has a mass ratio (m_0/m) of 10. A new fuel is developed that has an exhaust velocity as high as 4500 m/s. The fuel burns at a constant rate for 300 s. Calculate the maximum velocity of this single-stage rocket, assuming constant acceleration of gravity. If the escape velocity of a particle from the earth is 11.3 km/s, can a similar single-stage rocket with the same mass ratio and exhaust velocity be constructed that can reach the moon?

9-56. A water droplet falling in the atmosphere is spherical. Assume that as the droplet passes through a cloud, it acquires mass at a rate equal to kA where k is a constant(>0) and A its cross-sectional area. Consider a droplet of initial radius r_0 that enters a cloud with a velocity v_0. Assume no resistive force and show **(a)** that the radius increases linearly with the time, and **(b)** that if r_0 is negligibly small then the speed increases linearly with the time within the cloud.

9-57. A rocket in outer space in a negligible gravitational field starts from rest and accelerates uniformly at a until its final speed is v. The initial mass of the rocket is m_0. How much work does the rocket's engine do?

9-58. Consider a single-stage rocket taking off from Earth. Show that the height of the rocket at burnout is given by Equation 9.166. How much farther in height will the rocket go after burnout?

9-59. A rocket has an initial mass of m and a fuel burn rate of α (Equation 9.161). What is the minimum exhaust velocity that will allow the rocket to lift off immediately after firing?

9-60. A rocket has an initial mass of 7×10^4 kg and on firing burns its fuel at a rate of 250 kg/s. The exhaust velocity is 2500 m/s. If the rocket has a vertical ascent from resting on the earth, how long after the rocket engines fire will the rocket lift off? What is wrong with the design of this rocket?

9-61. Consider a multistage rocket of n stages, each with exhaust speed u. Each stage of the rocket has the same mass ratio at burnout ($k = m_i/m_f$). Show that the final speed of the nth stage is $nu \ln k$.

9-62. To perform a rescue, a lunar landing craft needs to hover just above the surface of the moon, which has a gravitational acceleration of $g/6$. The exhaust velocity is 2000 m/s, but fuel amounting to only 20 percent of the total mass may be used. How long can the landing craft hover?

9-63. A new projectile launcher is developed in the year 2023 that can launch a 10^4 kg spherical probe with an initial speed of 6000 m/s. For testing purposes, objects are launched vertically.

(a) Neglect air resistance and assume that the acceleration of gravity is constant. Determine how high the launched object can reach above the surface of Earth.

(b) If the object has a radius of 20 cm and the air resistance is proportional to the square of the object's speed with $c_w = 0.2$, determine the maximum height reached. Assume the density of air is constant.

(c) Now also include the fact that the acceleration of gravity decreases as the object soars above Earth. Find the height reached.

(d) Now add the effects of the decrease in air density with altitude to the calculation. We can very roughly represent the air density by $\log_{10}(\rho) = -0.05h + 0.11$ where ρ is the air density in kg/m^3 and h is the altitude above Earth in km. Determine how high the object now goes.

9-64. A new single-stage rocket is developed in the year 2023, having a gas exhaust velocity of 4000 m/s. The total mass of the rocket is 10^5 kg, with 90% of its mass being fuel. The fuel burns quickly in 100 s at a constant rate. For testing purposes, the rocket is launched vertically at rest from Earth's surface. Answer parts **(a)** through **(d)** of the previous problem.

9-65. In a typical model rocket (Estes Alpha III) the Estes C6 solid rocket engine provides a total impulse of 8.5 N-s. Assume the total rocket mass at launch is 54 g and that it has a rocket engine of mass 20 g that burns evenly for 1.5 s. The rocket diameter is 24 mm. Assume a constant burn rate of the propellent mass (11 g), a rocket exhaust speed 800 m/s, vertical ascent, and drag coefficient $c_w = 0.75$. Determine

(a) The speed and altitude at engine burnout,

(b) Maximum height and time it occurs,

(c) Maximum acceleration,

(d) Total flight time, and

(e) Speed at ground impact.

Produce a plot of altitude and speed versus time. For simplicity, because the propellent mass is only 20% of the total mass, assume a constant mass during rocket burning.

9-66. For the previous problem, take into account the change of rocket mass with time and omit the effect of gravity. **(a)** Find the rocket's speed at burn out. **(b)** How far has the rocket traveled at that moment?

9-67. Complete the derivation for the burnout height H_{bo} in Example 9.13. Use the numbers for the *Saturn V* rocket in Example 9.12 and use Equations 9.167 and 9.168 to determine the height and speed at burnout.