Image Alignment

A lot of slides from Darrell and Szeliski

Roadmap

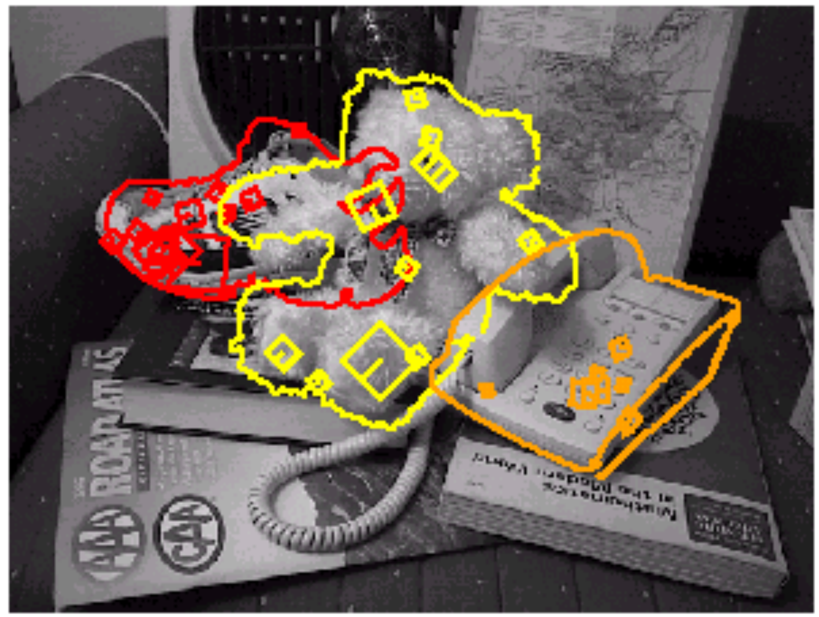
- Previous: Image formation, filtering, local features, connected components, object features, matching, …
- **Today: Feature-based Alignment**
  - **Stitching images together**
  - **Homographies, RANSAC, Warping, Blending**
  - **Global alignment of planar models**

Today: Alignment

- Homographies
- Rotational Panoramas
- RANSAC
- Global alignment
- Warping
- Blending

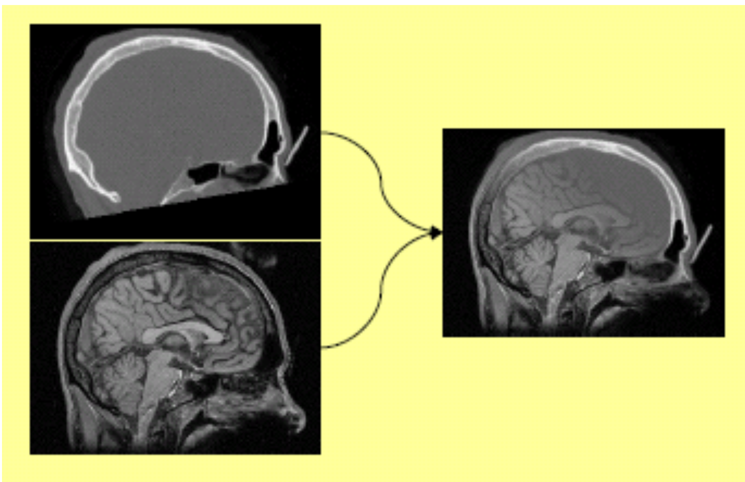
(a) (b) (c)

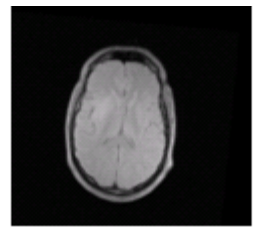
Motivation: Recognition

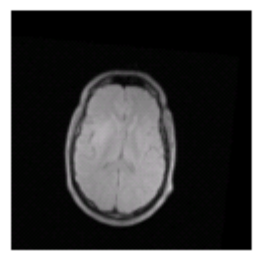


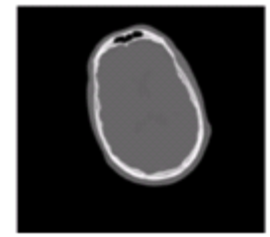
Figures from David Lowe

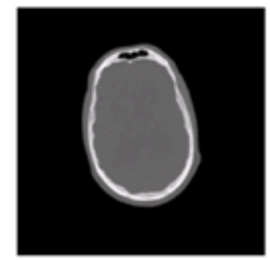
Motivation: medical image registration

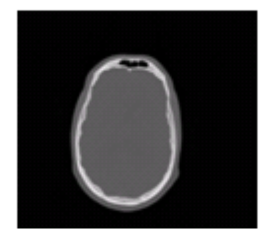












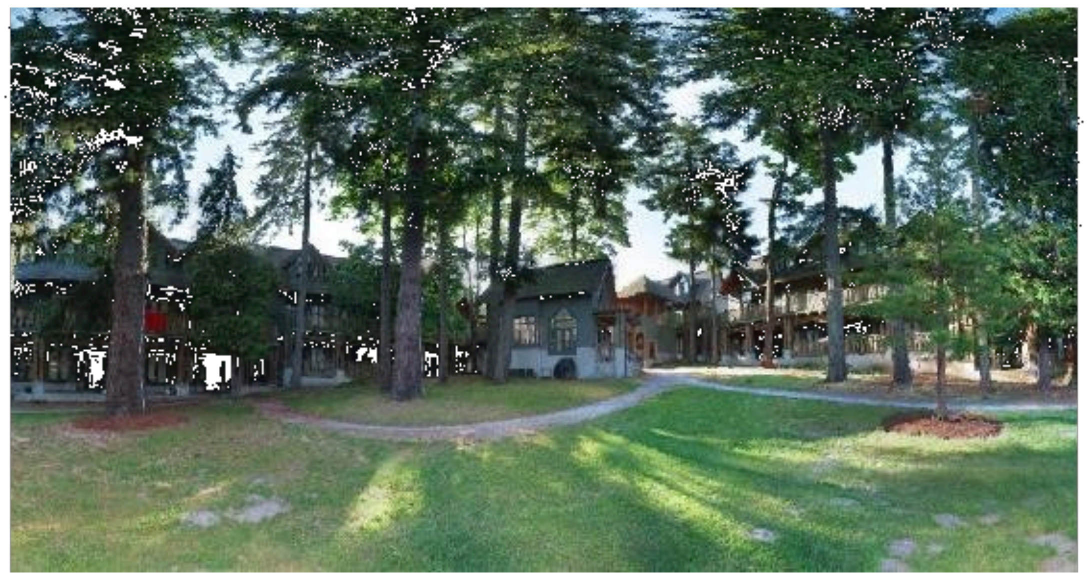
Motivation: Mosaics

- Getting the whole picture
  - Consumer camera: 50˚ x 35˚

Slide from Brown & Lowe 2003

Motivation: Mosaics

- Getting the whole picture
  - Consumer camera: 50˚ x 35˚
  - Human Vision: 176˚ x 135˚



Slide from Brown & Lowe 2003

Motivation: Mosaics

- Getting the whole picture
  - Consumer camera: 50˚ x 35˚
  - Human Vision: 176˚ x 135˚

- Panoramic Mosaic = up to 360 x 180°

Slide from Brown & Lowe 2003

Motion models

Motion models

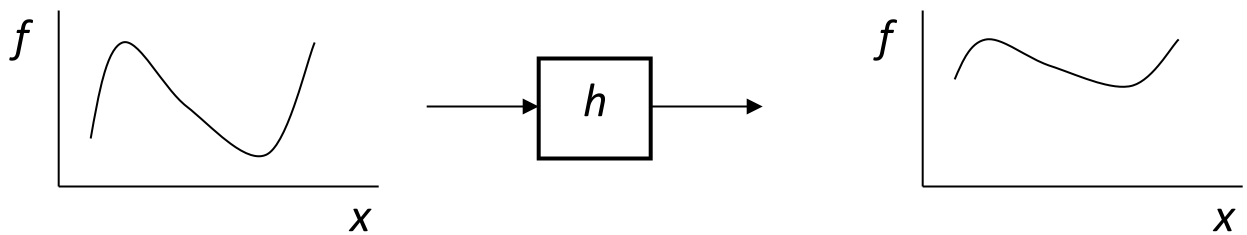
- What happens when we take two images with a camera and try to align them?
- translation?
- rotation?
- scale?
- affine?
- perspective?
- … see interactive demo (VideoMosaic)

Szeliski

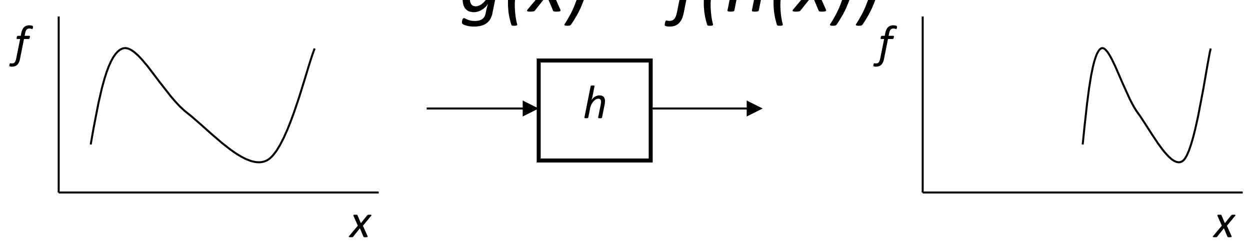
Image Warping

Image Warping

- image filtering: change *range* of image
  - $g(x) = h(f(x))$



- image warping: change *domain* of image
  - $g(x) = f(h(x))$

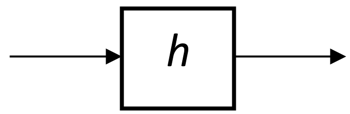


Szeliski

Image Warping

- image filtering: change *range* of image
  - $g(x) = h(f(x))$

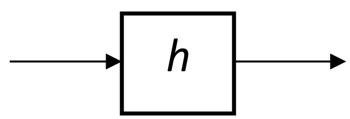
f



g

- image warping: change *domain* of image
  - $g(x) = f(h(x))$

f



g

Szeliski

Parametric (global) warping

- Examples of parametric warps:

translation

rotation

aspect

affine

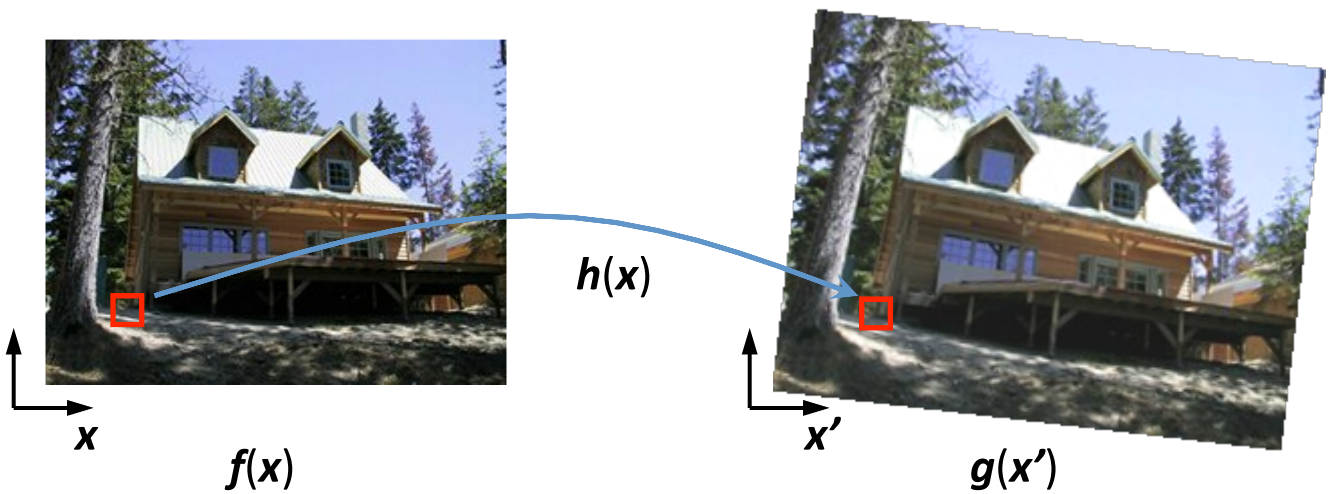
perspective

cylindrical

Szeliski

Image Warping

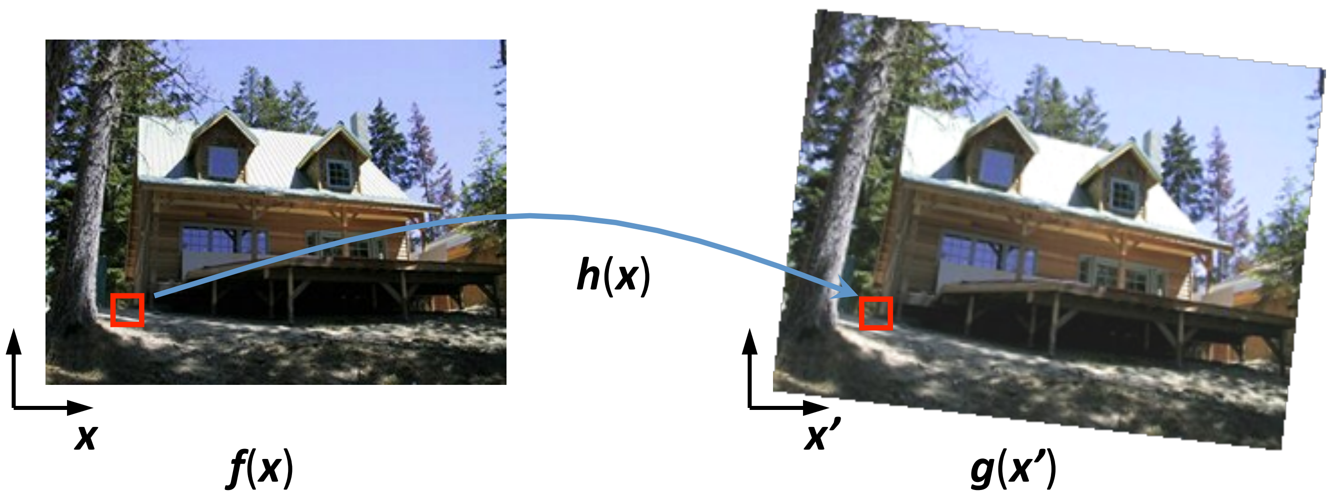
- Given a coordinate transform $\boldsymbol{x'} = \boldsymbol{h}(\boldsymbol{x})$ and a source image $\boldsymbol{f}(\boldsymbol{x})$, how do we compute a transformed image $\boldsymbol{g}(\boldsymbol{x'}) = \boldsymbol{f}(\boldsymbol{h}(\boldsymbol{x}))$?



Szeliski

Forward Warping

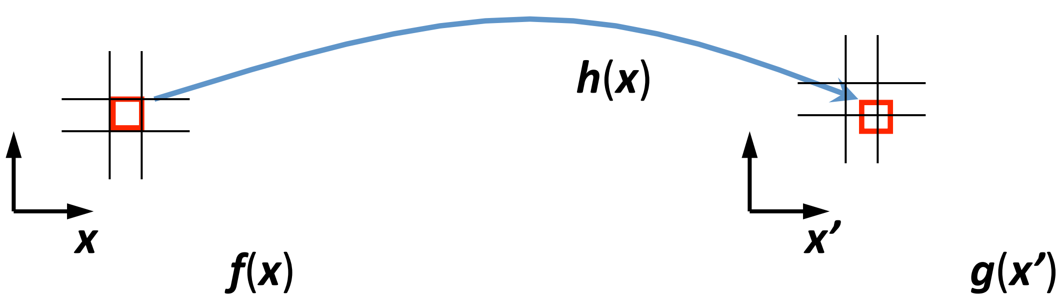
- Send each pixel $\boldsymbol{f}(\boldsymbol{x})$ to its corresponding location $\boldsymbol{x}' = \boldsymbol{h}(\boldsymbol{x})$ in $\boldsymbol{g}(\boldsymbol{x}')$
  - What if pixel lands "between" two pixels?



Szeliski

Forward Warping

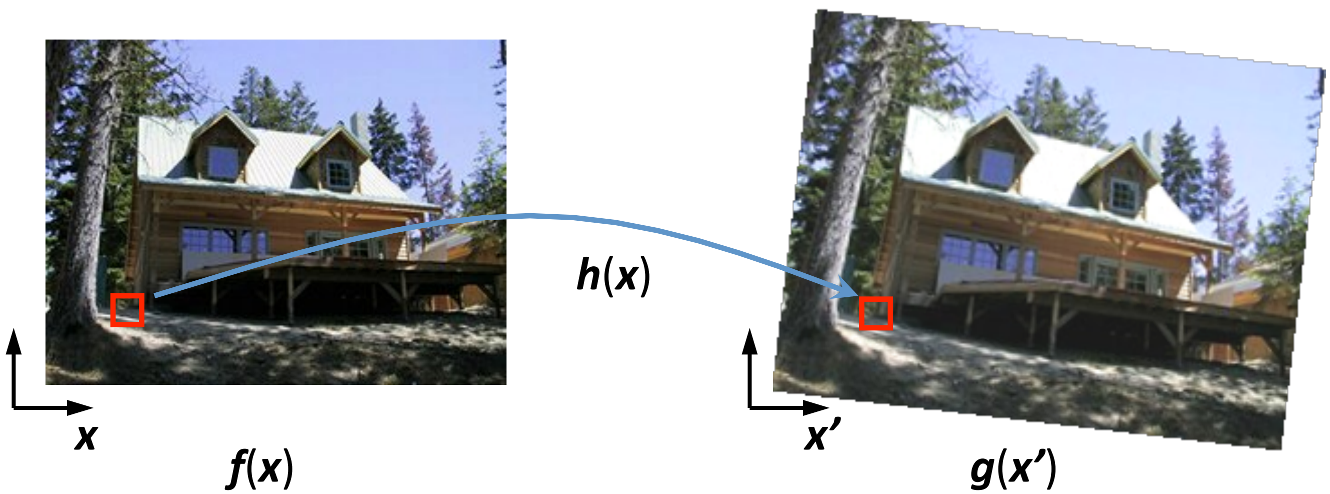
- Send each pixel ***f***(***x***) to its corresponding location ***x'*** = ***h***(***x***) in ***g***(***x'***)
  - What if pixel lands "between" two pixels?
  - Answer: add "contribution" to several pixels, normalize later (*splatting*)



Szeliski

Inverse Warping

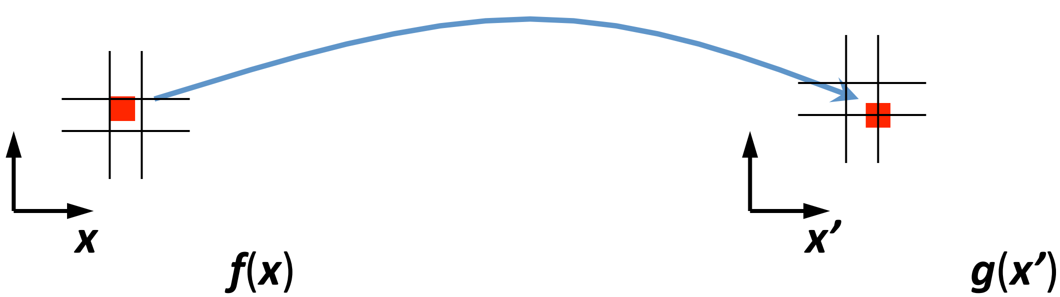
- Get each pixel ***g***(***x'***) from its corresponding location ***x'*** = ***h***(***x***) in ***f***(***x***)
  - What if pixel comes from "between" two pixels?



Szeliski

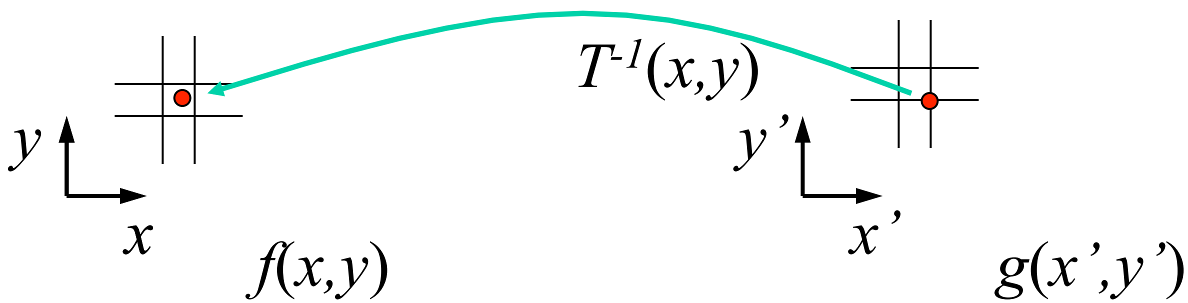
Inverse Warping

- Get each pixel ***g***(***x'***) from its corresponding location ***x'*** = ***h***(***x***) in ***f***(***x***)
  - What if pixel comes from "between" two pixels?
  - Answer: *resample* color value from *interpolated* (*prefiltered*) source image



Szeliski

Inverse warping



Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

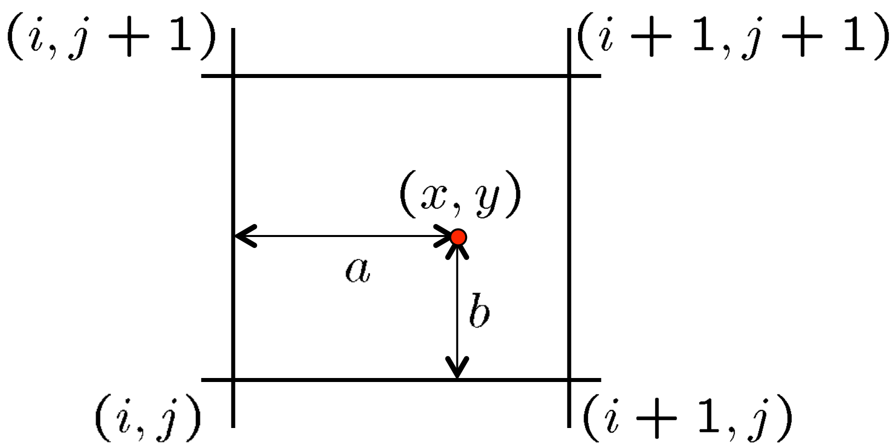
- nearest neighbor, bilinear…

`>> help interp2`

Slide from Alyosha Efros, CMU

Bilinear interpolation

Sampling at *f(x,y):*



$$
\begin{aligned}
f(x,y) = \quad & (1-a)(1-b) && f[i,j] \\
& +a(1-b) && f[i+1,j] \\
& +ab && f[i+1,j+1] \\
& +(1-a)b && f[i,j+1]
\end{aligned}
$$

Slide from Alyosha Efros, CMU

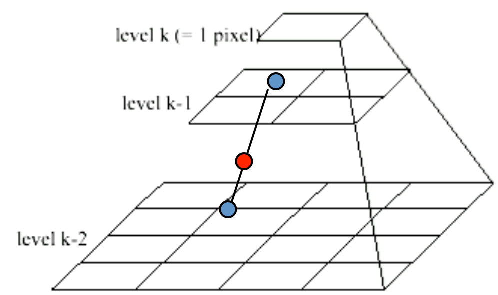
Interpolation

- Possible interpolation filters
  - nearest neighbor
  - bilinear
  - bicubic (interpolating)
  - sinc / FIR
- Needed to prevent "jaggies" and "texture crawl"

Szeliski

Prefiltering

- Essential for *downsampling* (*decimation*) to prevent *aliasing*
- MIP-mapping [Williams'83]:
  1. build pyramid (but what decimation filter?):
     - block averaging
     - Burt & Adelson (5-tap binomial)
     - 7-tap wavelet-based filter (better)
  2. *trilinear* interpolation
     - bilinear within each 2 adjacent levels
     - linear blend *between* levels (determined by pixel size)



2D coordinate transformations

- translation: $\boldsymbol{x'} = \boldsymbol{x} + \boldsymbol{t}$ $\boldsymbol{x} = (x,y)$
- rotation: $\boldsymbol{x'} = \boldsymbol{R}\,\boldsymbol{x} + \boldsymbol{t}$
- similarity: $\boldsymbol{x'} = s\,\boldsymbol{R}\,\boldsymbol{x} + \boldsymbol{t}$
- affine: $\boldsymbol{x'} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{t}$
- perspective: $\underline{\boldsymbol{x}}' \cong \boldsymbol{H}\,\underline{\boldsymbol{x}}$ $\underline{\boldsymbol{x}} = (x,y,1)$
  ($\underline{\boldsymbol{x}}$ is a *homogeneous* coordinate)
- These all form a nested *group* (closed w/ inv.)

Szeliski

Basic 2D Transformations

Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 & 0 \\ 0 & \boldsymbol{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} & 0 \\ \boldsymbol{sh_y} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

Source: Alyosha Efros

2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …

- Linear transformations, and
- Translations

Parallel lines remain parallel

Grauman

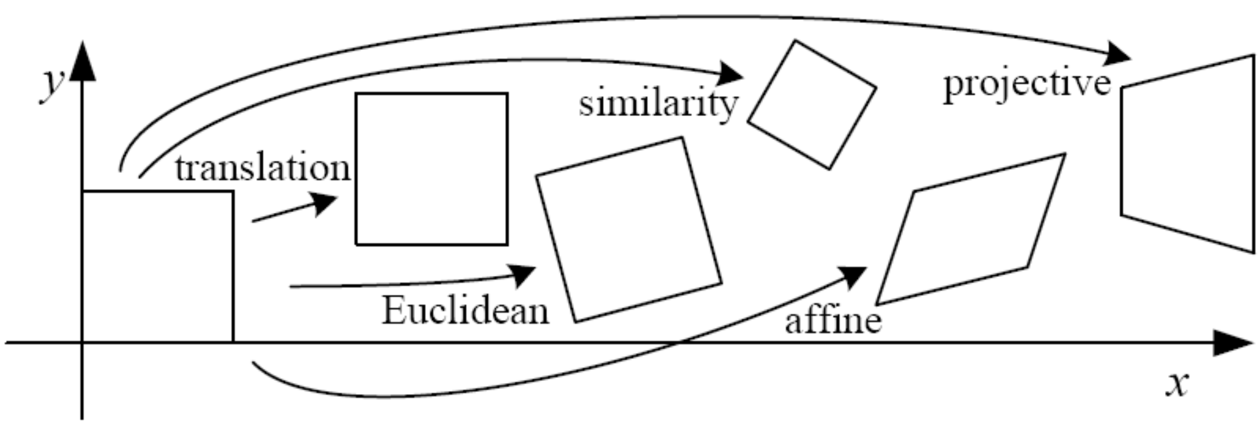
Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective transformations:

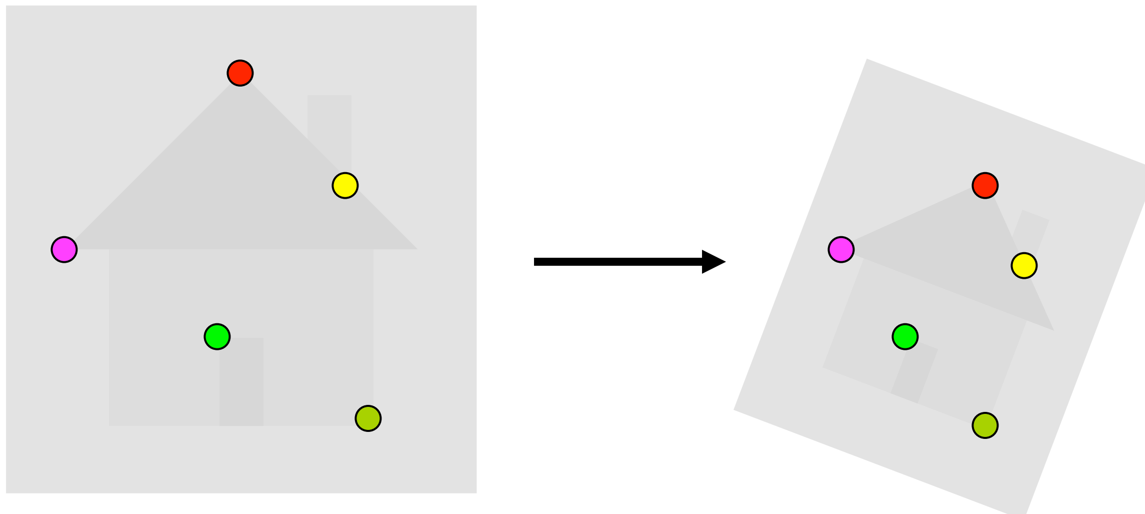
- Affine transformations, and
- Projective warps

Parallel lines do not necessarily remain parallel



Grauman

Image alignment



- Two broad approaches:
  - Direct (pixel-based) alignment
    - Search for alignment where most pixels agree
  - Feature-based alignment
    - Search for alignment where *extracted features* agree
    - Can be verified using pixel-based alignment

Source: L. Lazebnik

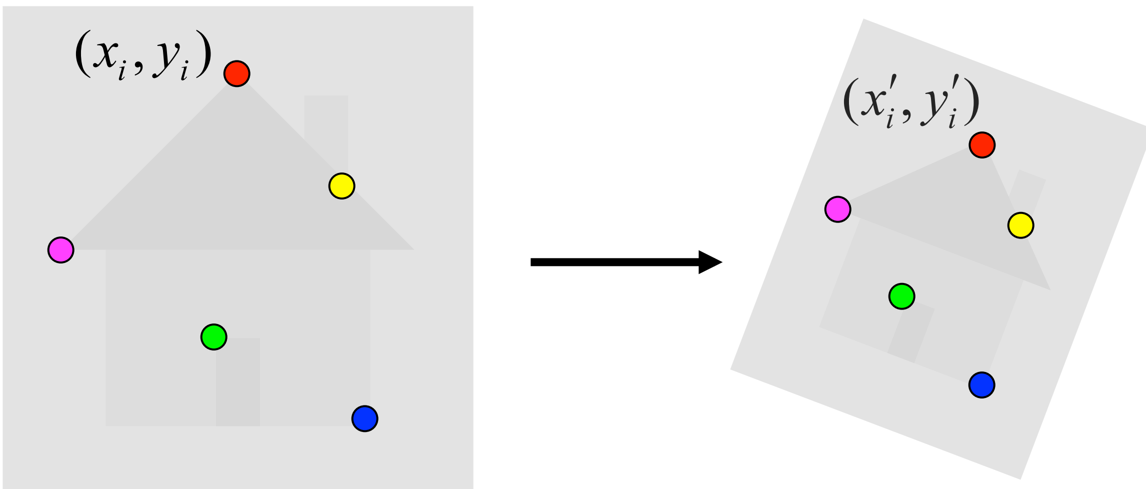
Fitting an affine transformation

Affine model approximates perspective projection of planar objects.

Figures from David Lowe, ICCV 1999

Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?

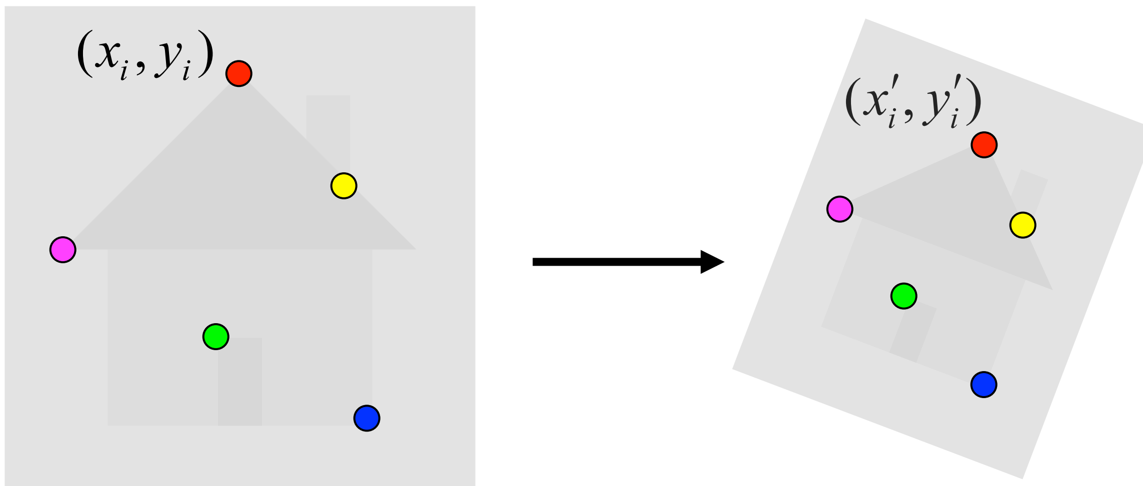


$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

Grauman

Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} \ \\ \ \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \ \\ \ \end{bmatrix}$$

Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- How many matches (correspondence pairs) do we need to solve for the transformation parameters?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for (x_{new}, y_{new}) ?

Grauman

Panoramas

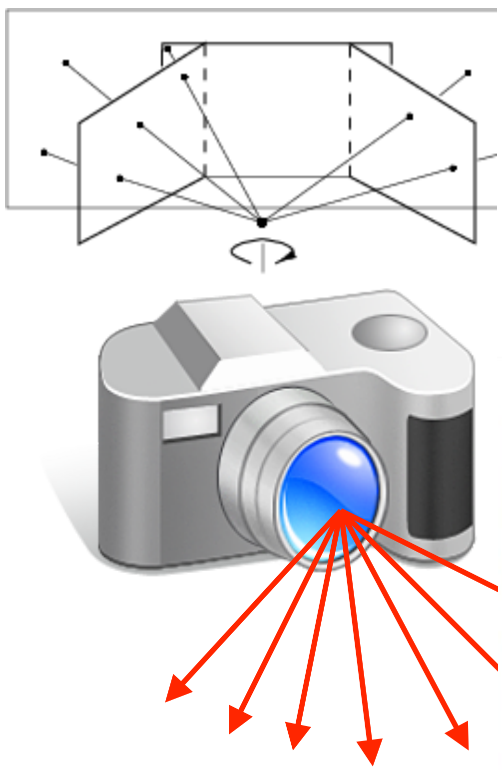


image from S. Seitz

Obtain a wider angle view by combining multiple images.

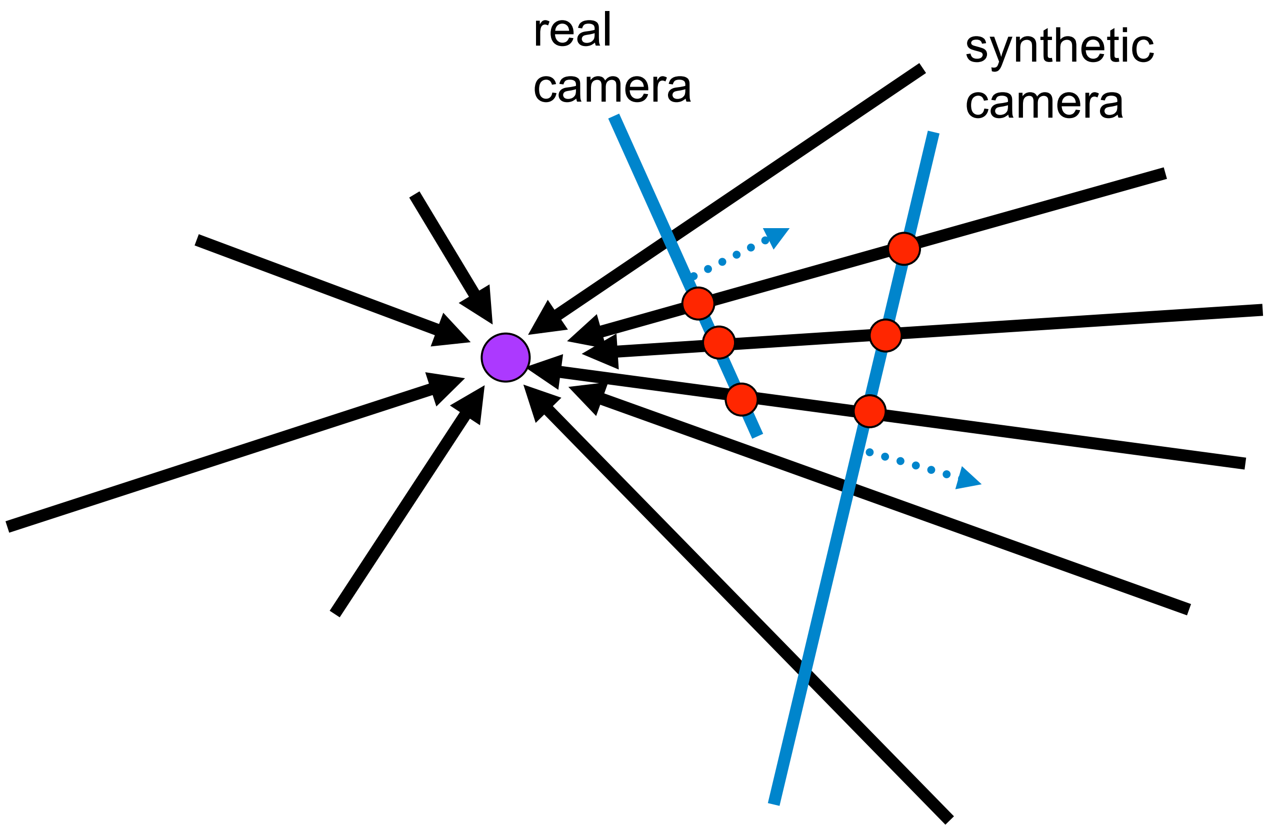
How to stitch together a panorama?

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation between second image and first
  - Transform the second image to overlap with the first
  - Blend the two together to create a mosaic
  - (If there are more images, repeat)

- …but **wait**, why should this work at all?
  - What about the 3D geometry of the scene?
  - Why aren’t we using it?

Source: Steve Seitz

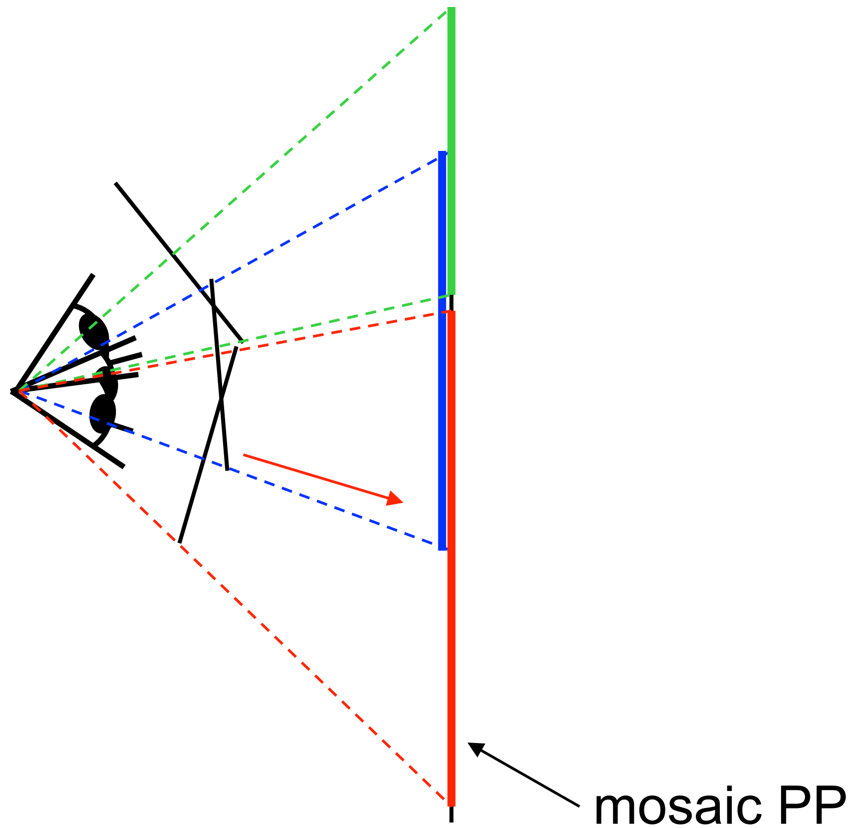
Panoramas: generating synthetic views



Can generate any synthetic camera view
as long as it has **the same center of projection**!

Source: Alyosha Efros

Image reprojection



The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

Source: Steve Seitz

Homography

How to relate two images from the same camera center?

– how to map a pixel from PP1 to PP2?

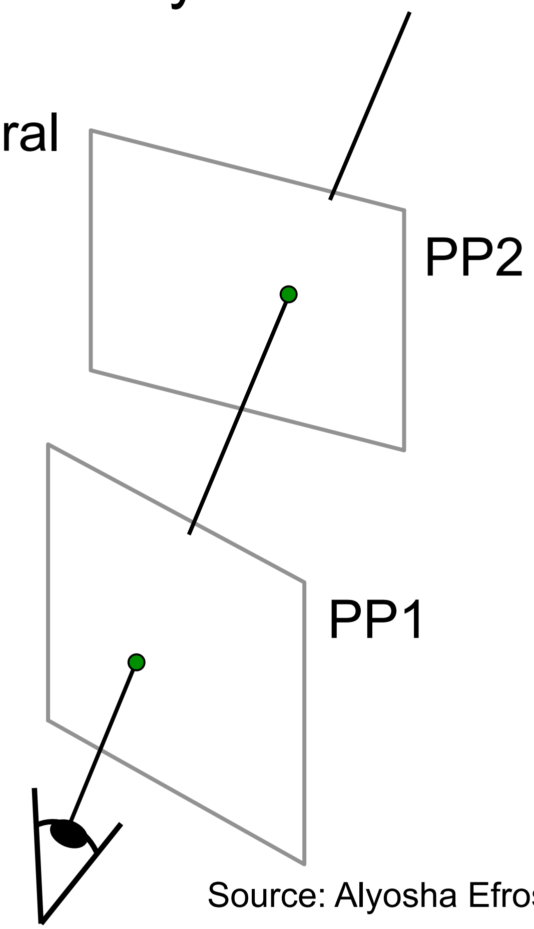
Think of it as a 2D **image warp** from one image to another.

A projective transform is a mapping between any two PPs with the same center of projection

- rectangle should map to arbitrary quadrilateral
- parallel lines aren't
- but must preserve straight lines

called **Homography**

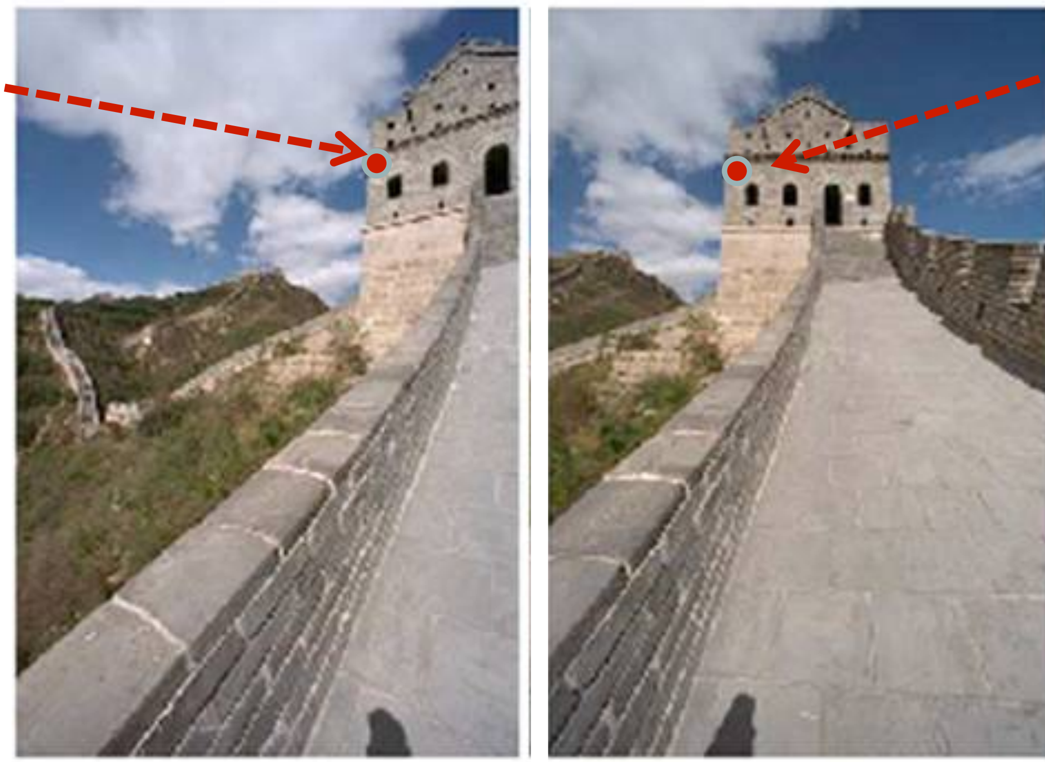
$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$



Source: Alyosha Efros

Homography

$$(x, y)$$



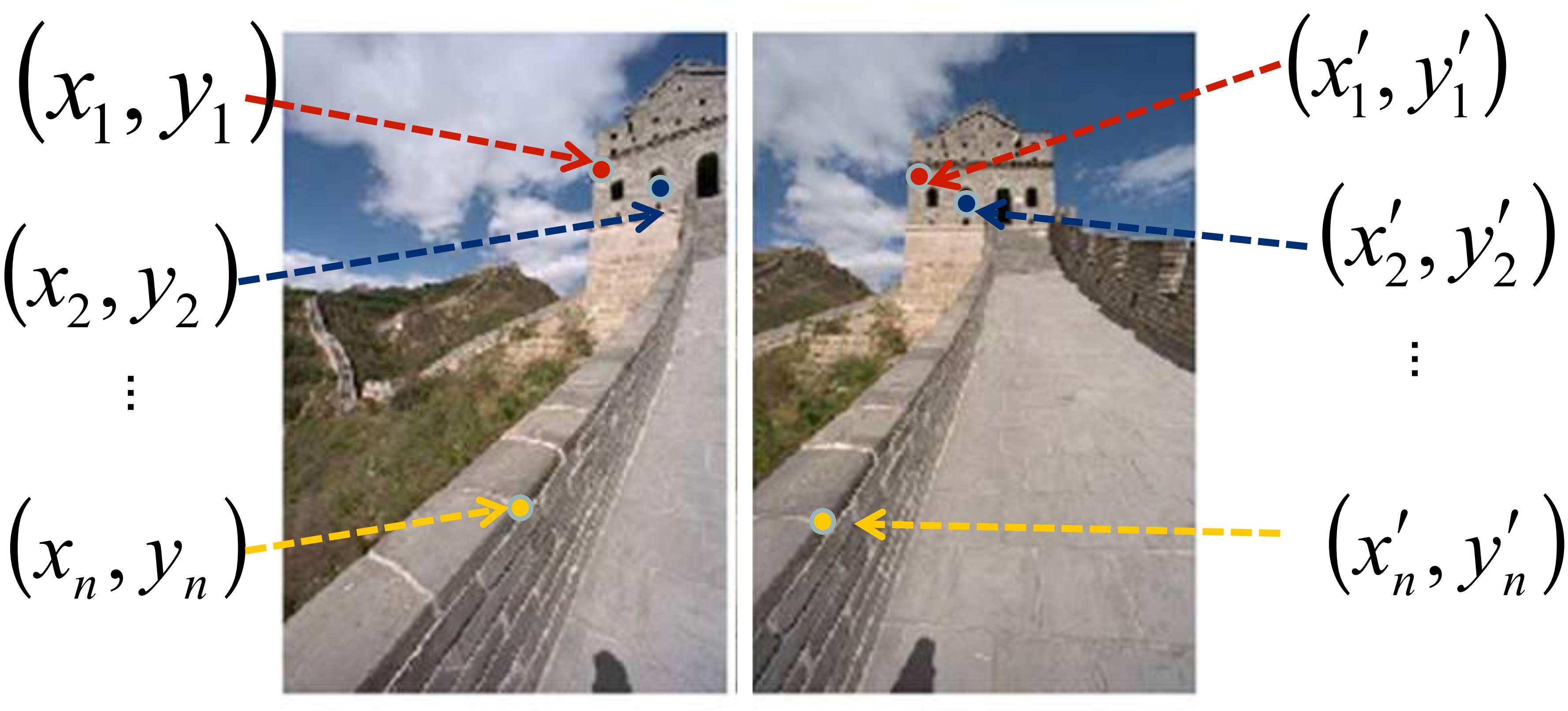
$$\left(\frac{wx'}{w}, \frac{wy'}{w} \right) = (x', y')$$

To **apply** a given homography **H**

- Compute **p'** = **Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$

Homography



To **compute** the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of **H** are the unknowns…

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Solving for homographies

p' = Hp

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Can set scale factor *i*=1. So, there are 8 unknowns.

Set up a system of linear equations:

Ah = b

where vector of unknowns h = $[a,b,c,d,e,f,g,h]^T$

Need at least 8 eqs, but the more the better…

Solve for h. If overconstrained, solve using least-squares:

$$\min \| Ah - b \|^2$$

>> **`help mldivide`**

Grauman

Recap: How to stitch together a panorama?

- Basic Procedure
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  - Compute transformation between second image and first
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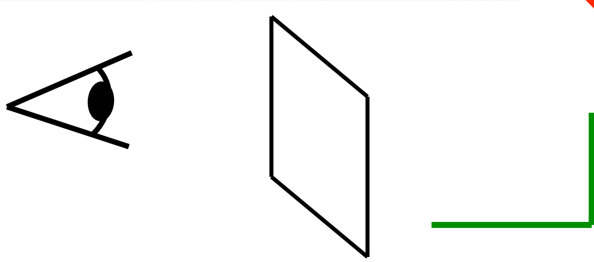
Source: Steve Seitz

Image warping with homographies



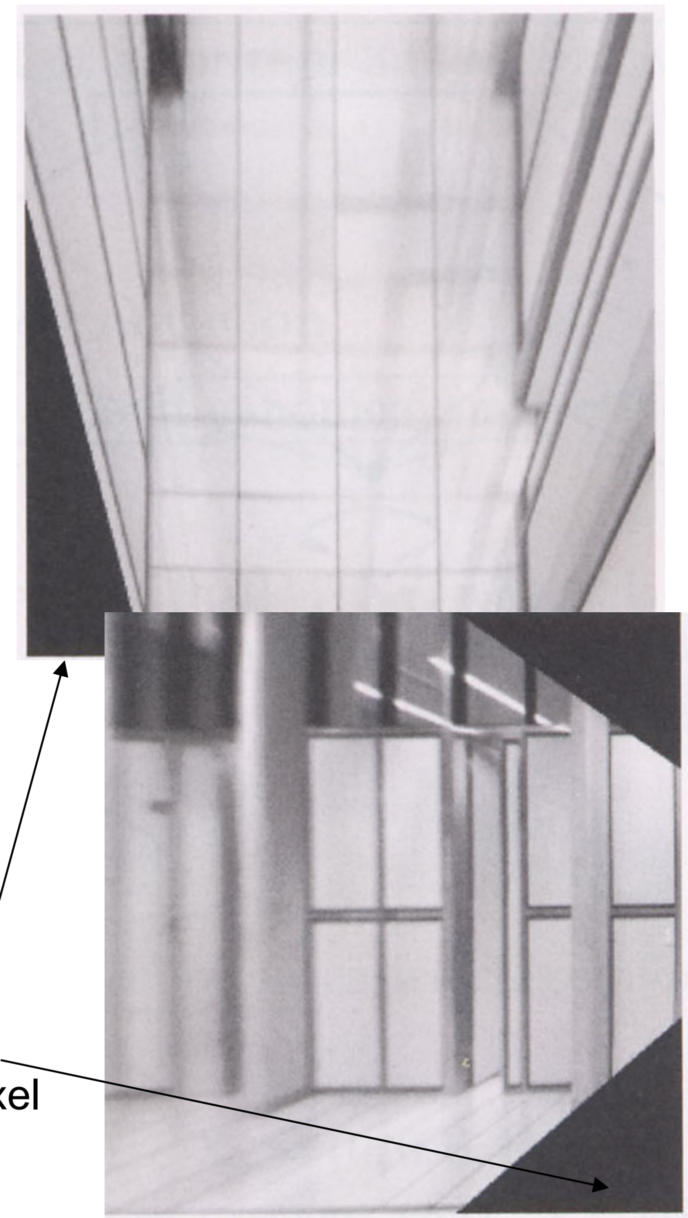
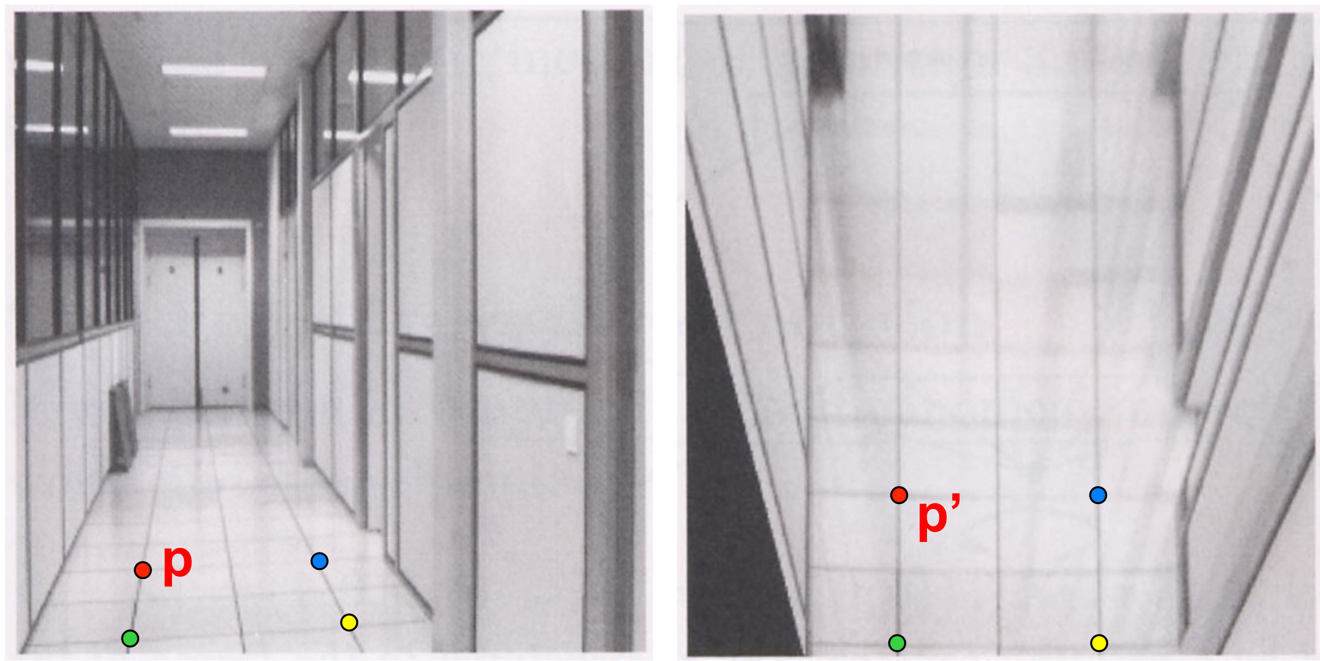


image plane in front

black area where no pixel maps to

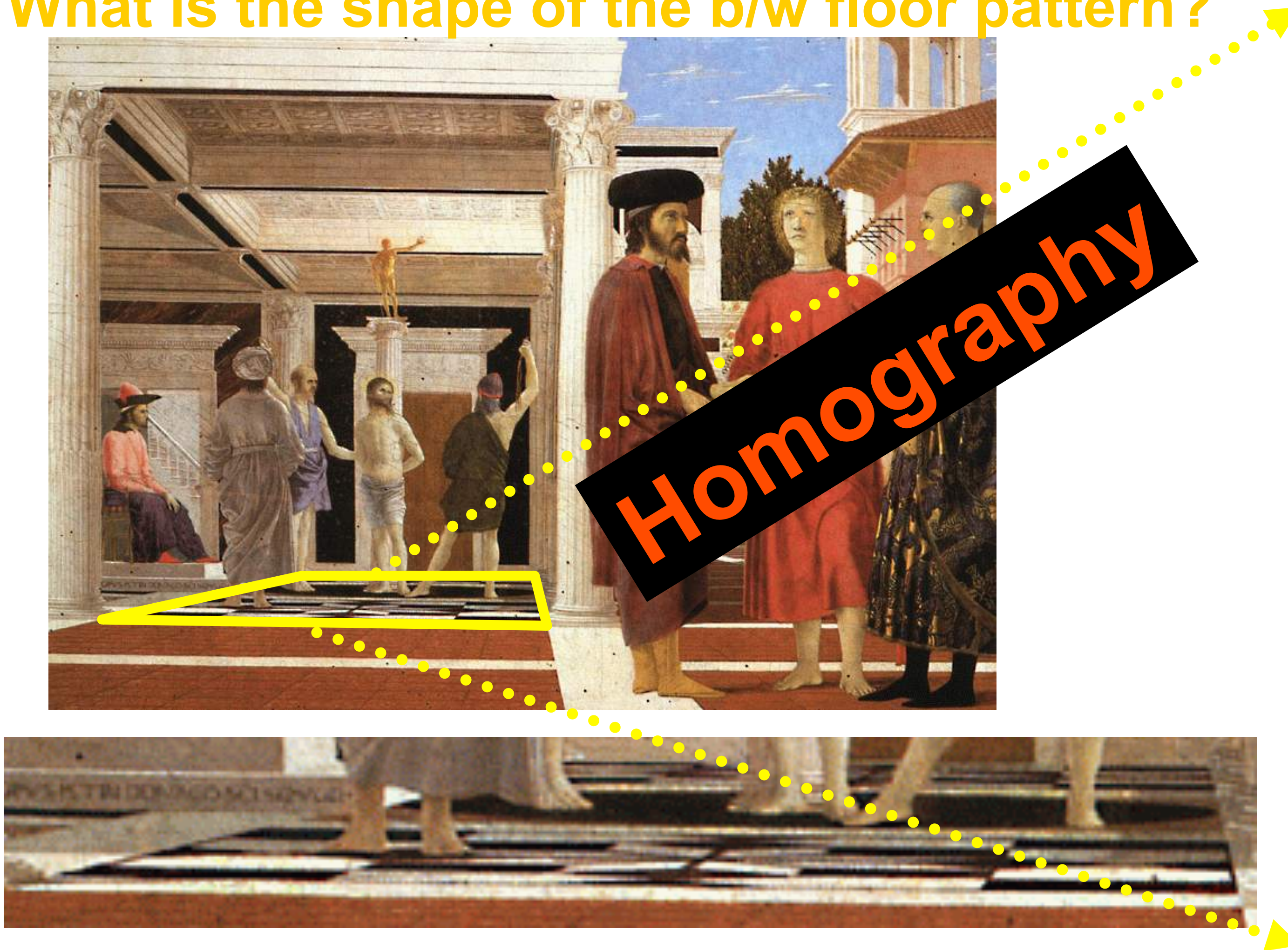
Source: Steve Seitz

Image rectification

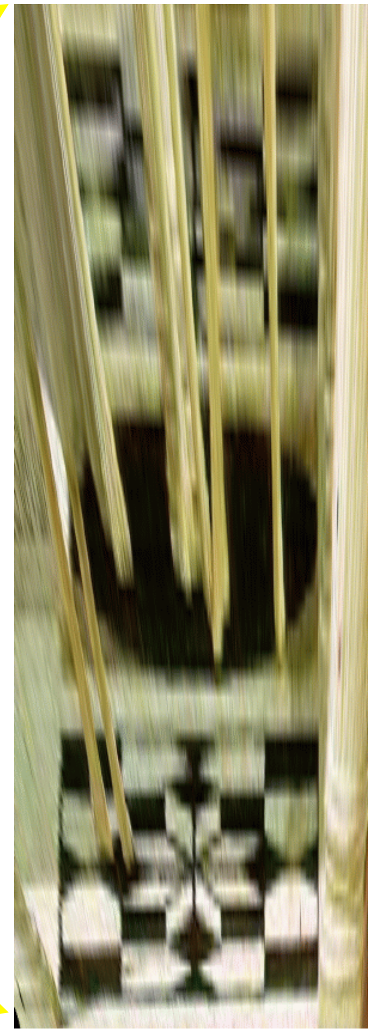


Analysing patterns and shapes

What is the shape of the b/w floor pattern?



The floor (enlarged)

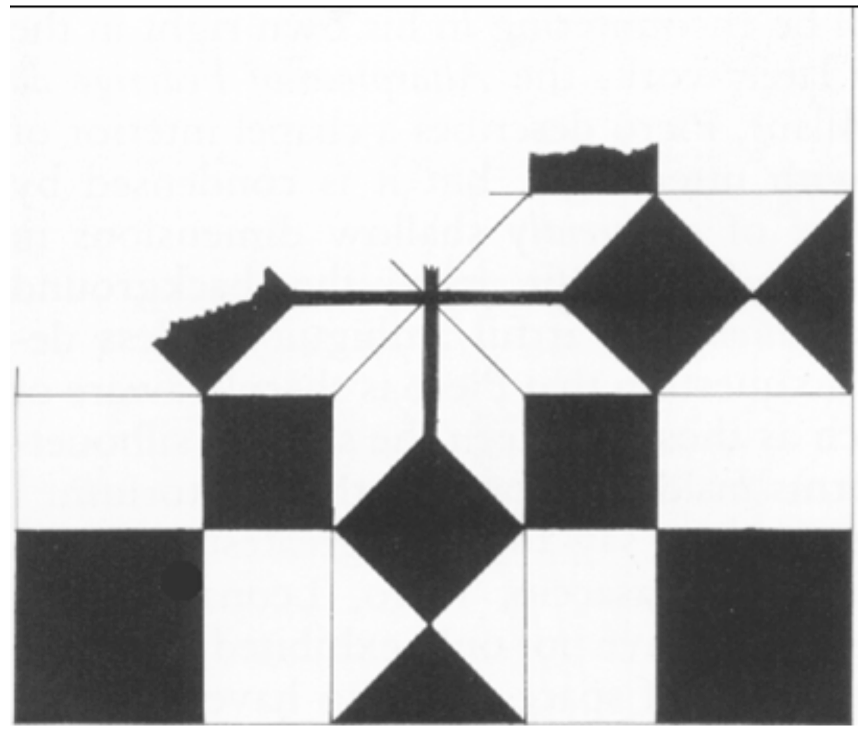


Automatically rectified floor

Slide from Criminisi

Analysing patterns and shapes

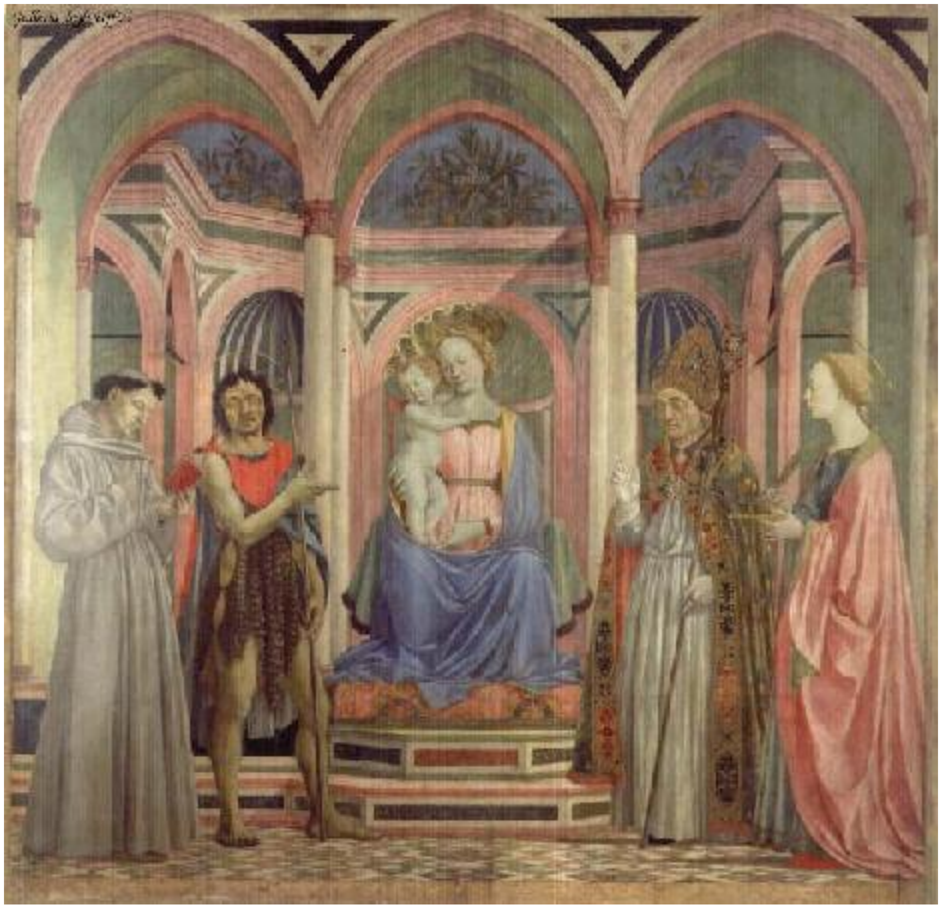
Automatic rectification



From Martin Kemp *The Science of Art*
(manual reconstruction)

Slide from Criminisi

Analysing patterns and shapes



***St. Lucy Altarpiece,* D. Veneziano**

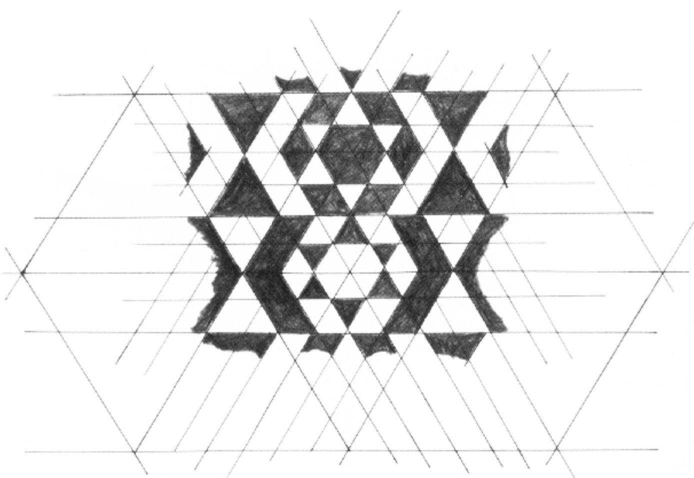
Slide from Criminisi

What is the (complicated) shape of the floor pattern?

Automatically rectified floor

Analysing patterns and shapes

Automatic rectification

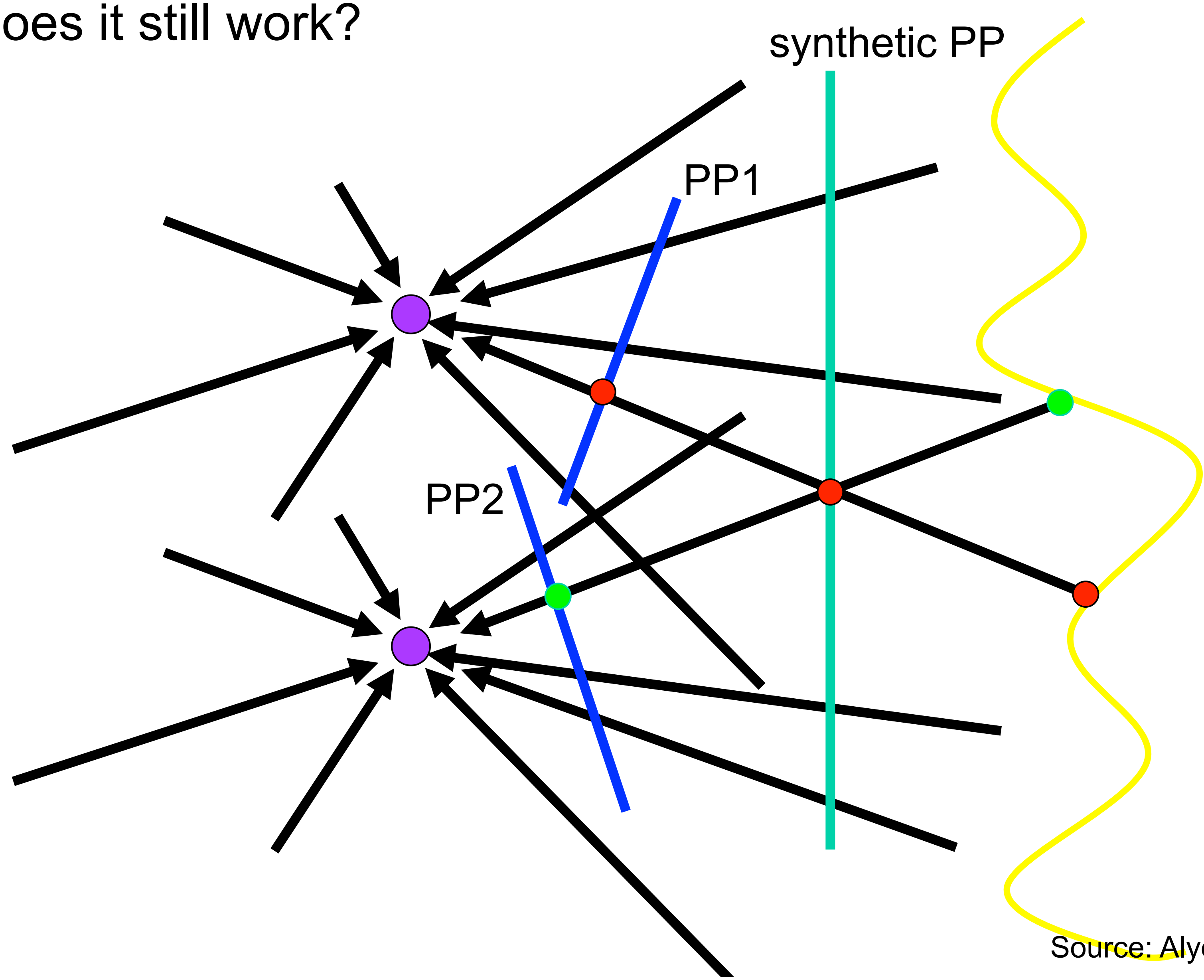


From Martin Kemp, *The Science of Art* *(manual reconstruction)*

Slide from Criminisi

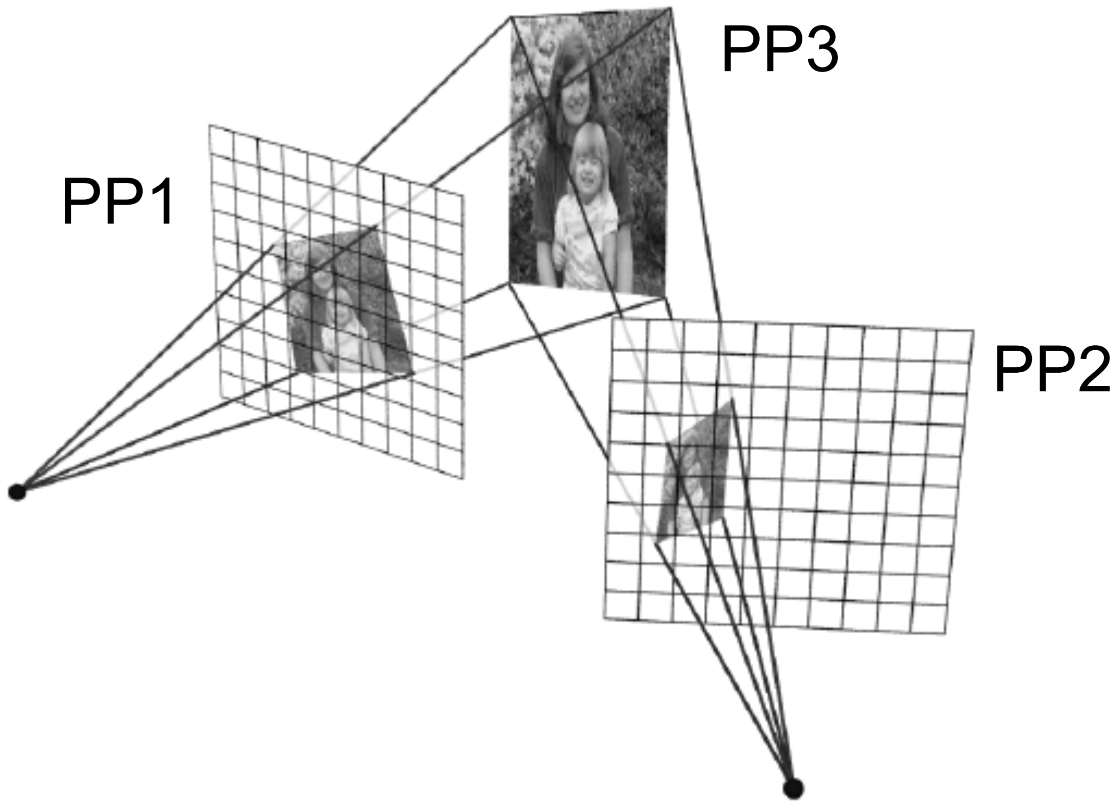
changing camera center

Does it still work?



Source: Alyosha Efros

Planar scene (or far away)



PP3 is a projection plane of both centers of projection, so we are OK!

This is how big aerial photographs are made

Source: Alyosha Efros

Grauman


Map
Satellite
Hybrid
200 ft
100 m
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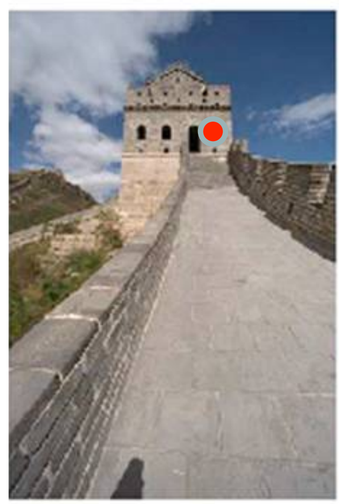
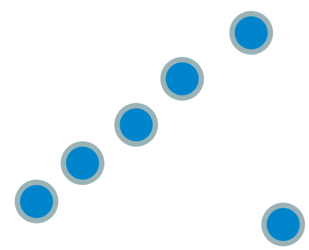
Outliers

- **Outliers** can hurt the quality of our parameter estimates, e.g.,
  - an erroneous pair of matching points from two images
  - an edge point that is noise, or doesn't belong to the line we are fitting.

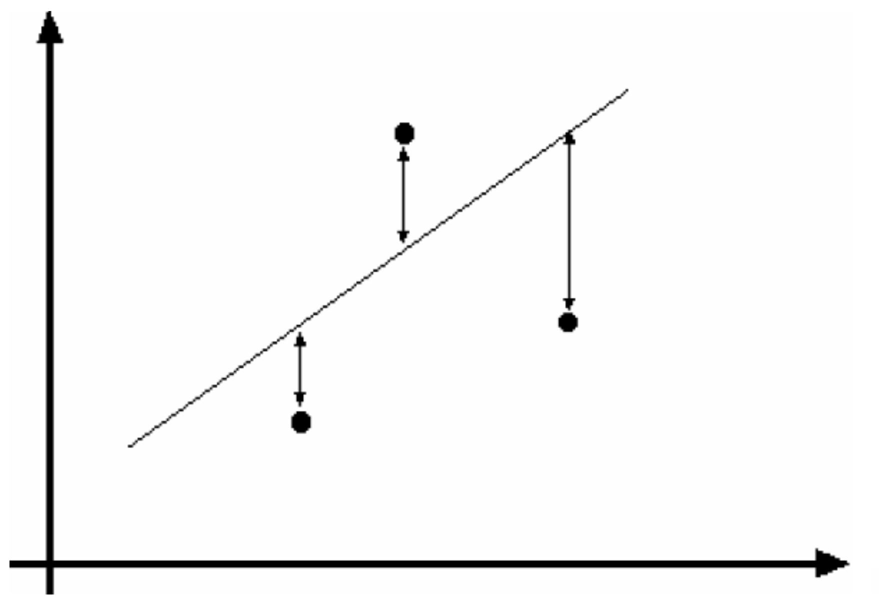




Grauman

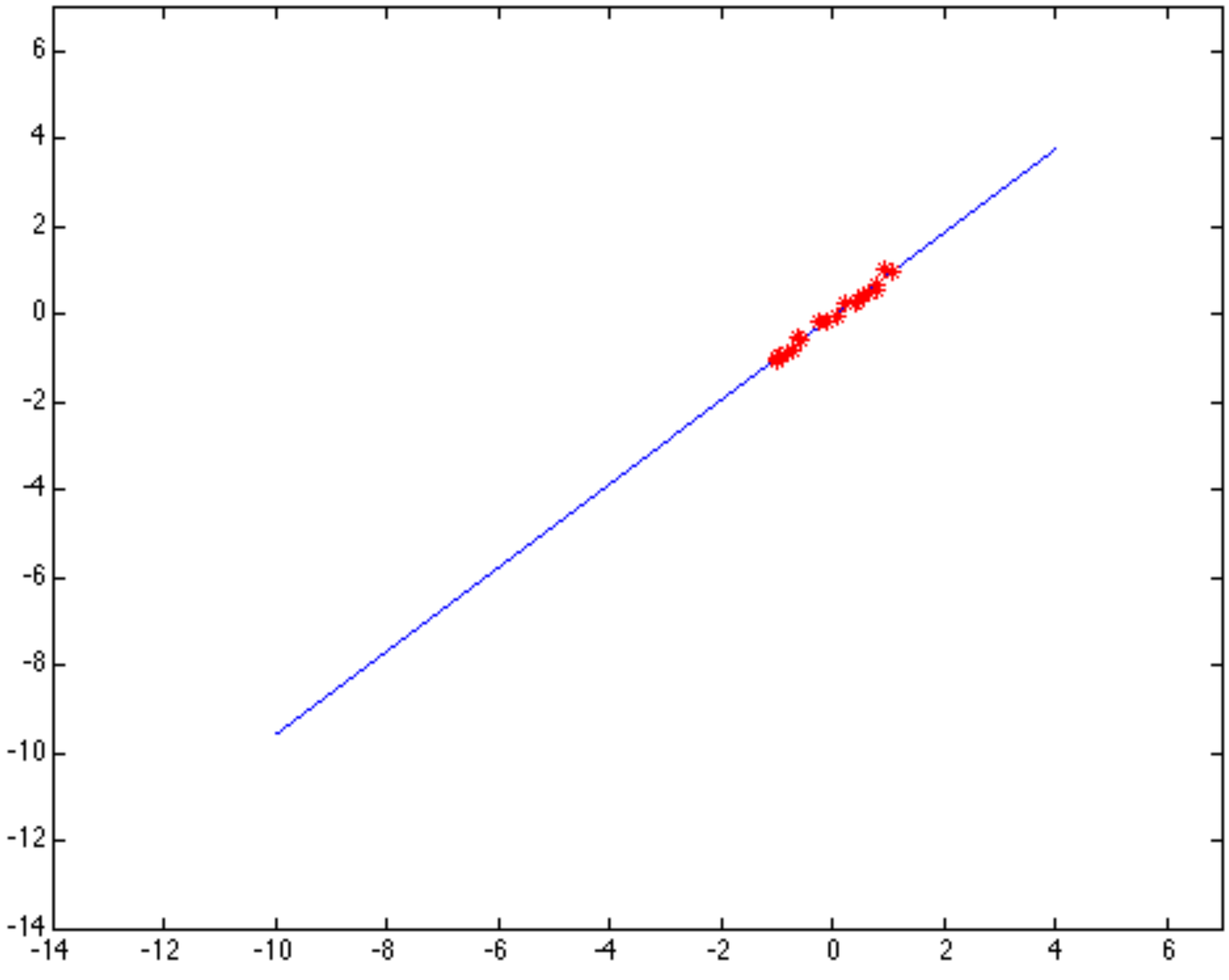
Example: least squares line fitting

- Assuming all the points that belong to a particular line are known

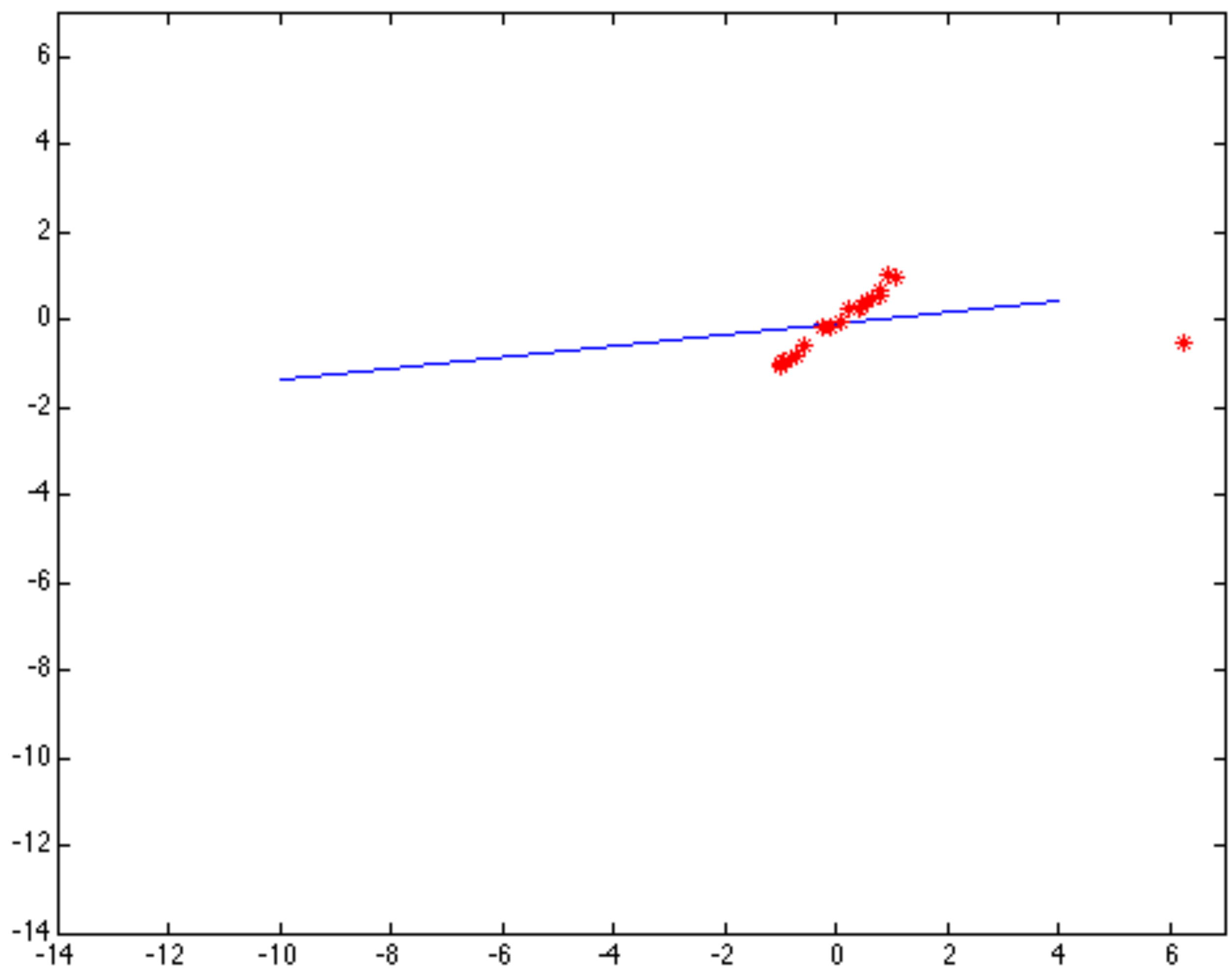


Grauman

Outliers affect least squares fit



Outliers affect least squares fit



RANSAC

- RANdom Sample Consensus

- Approach: we want to avoid the impact of outliers, so let’s look for “inliers”, and use those only.

- Intuition: if an outlier is chosen to compute the current fit, then the resulting line won’t have much support from rest of the points.

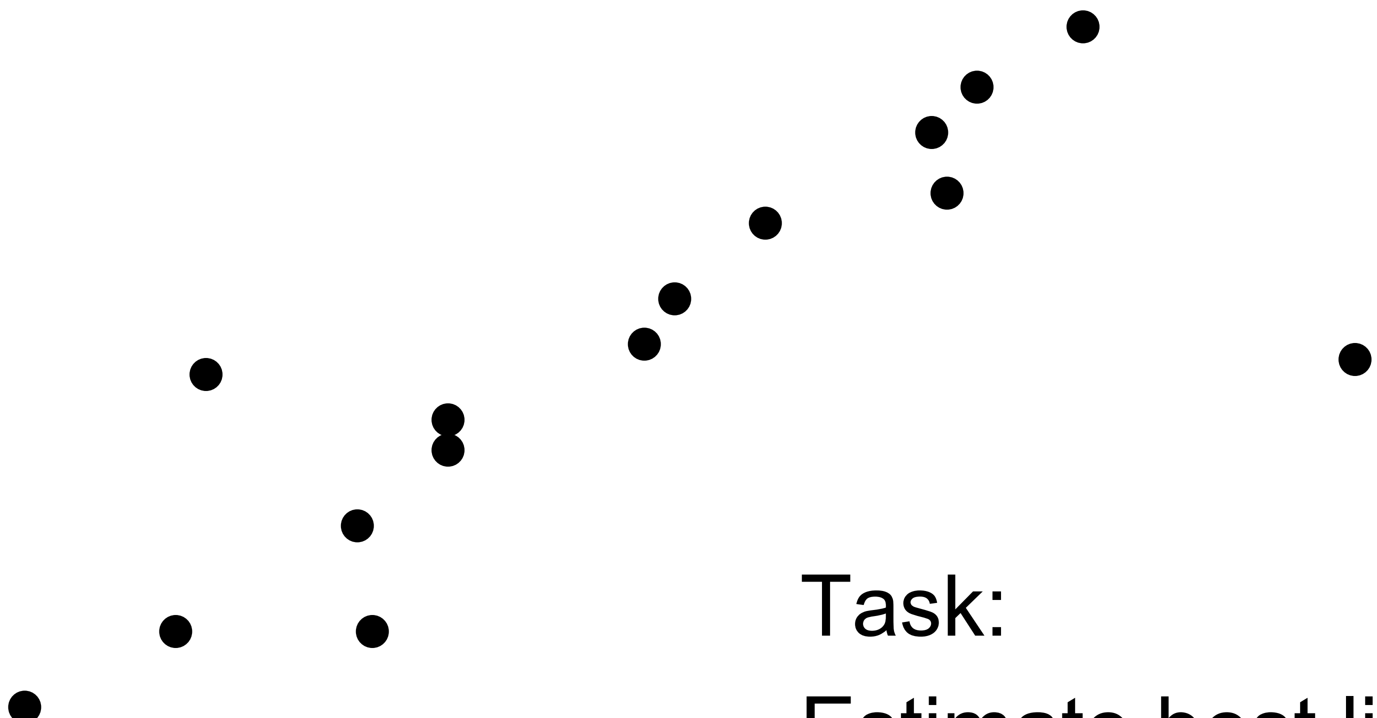
RANSAC

- <u>RANSAC loop</u>:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

- Keep the transformation with the largest number of inliers

RANSAC Line Fitting Example

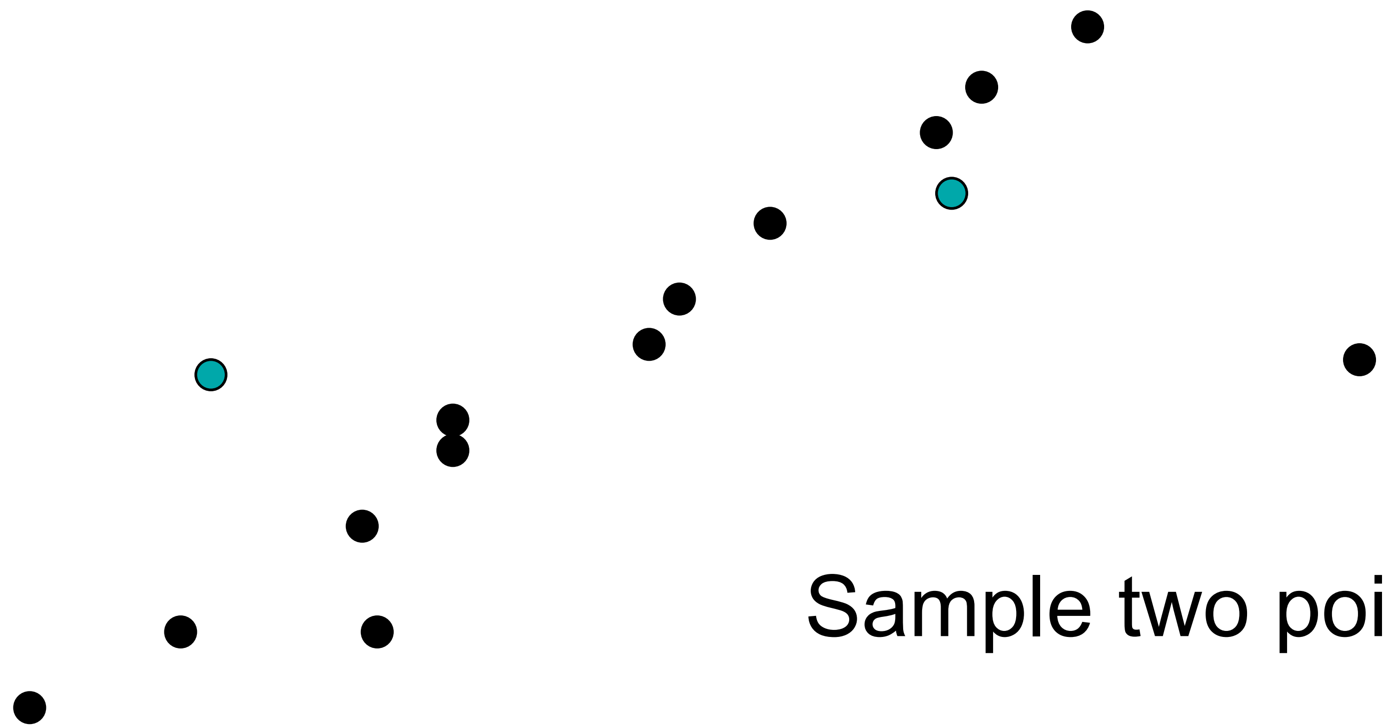


Task:

Estimate best line

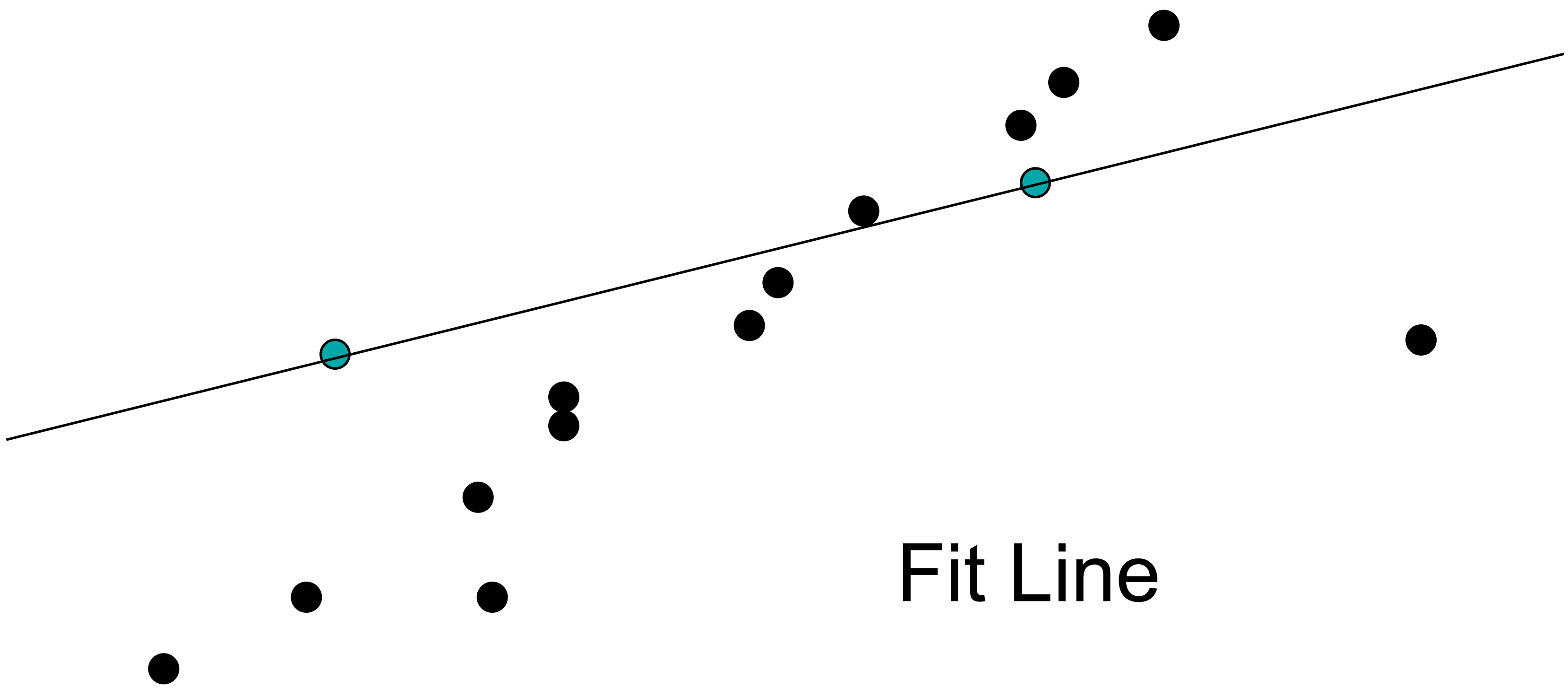
Slide credit: Jinxiang Chai, CMU

RANSAC Line Fitting Example

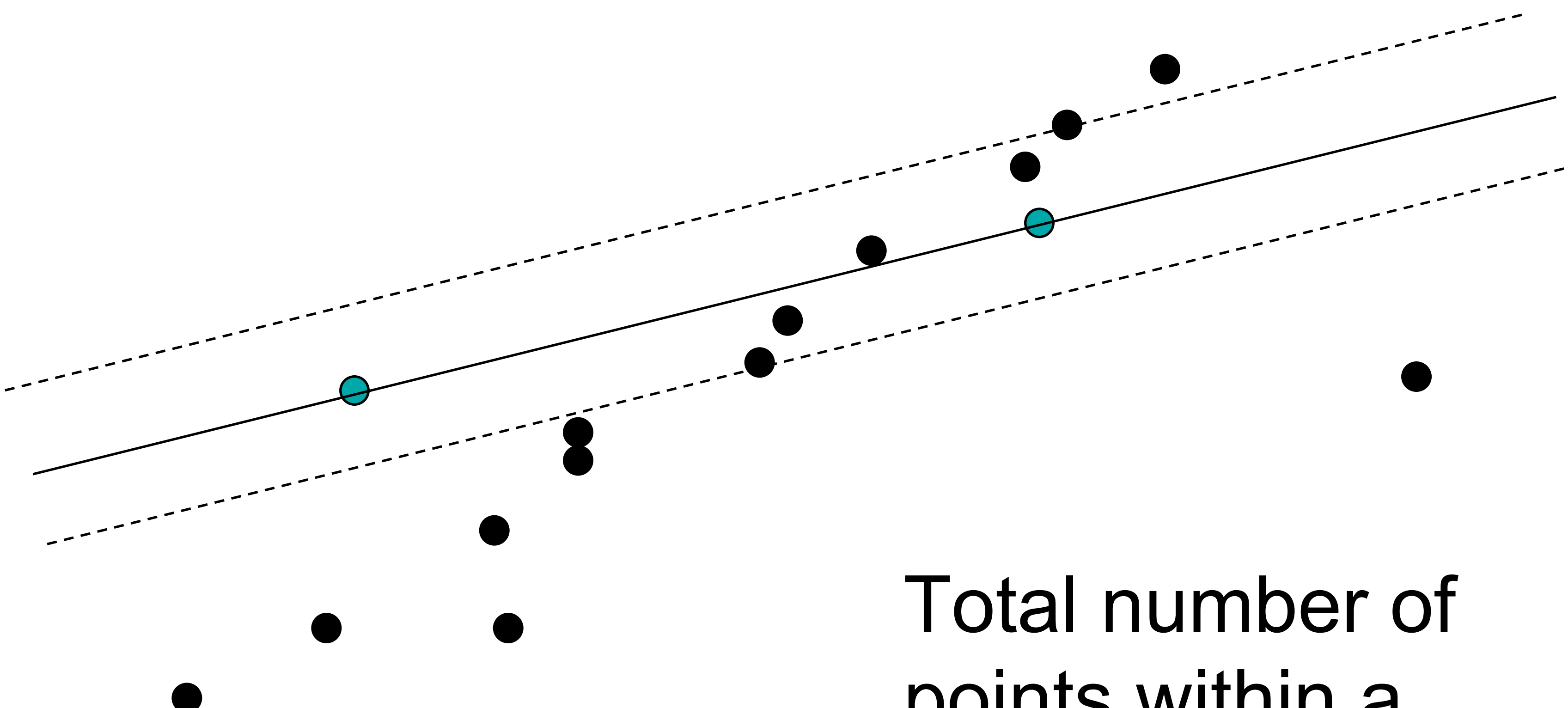


Sample two points

RANSAC Line Fitting Example



RANSAC Line Fitting Example

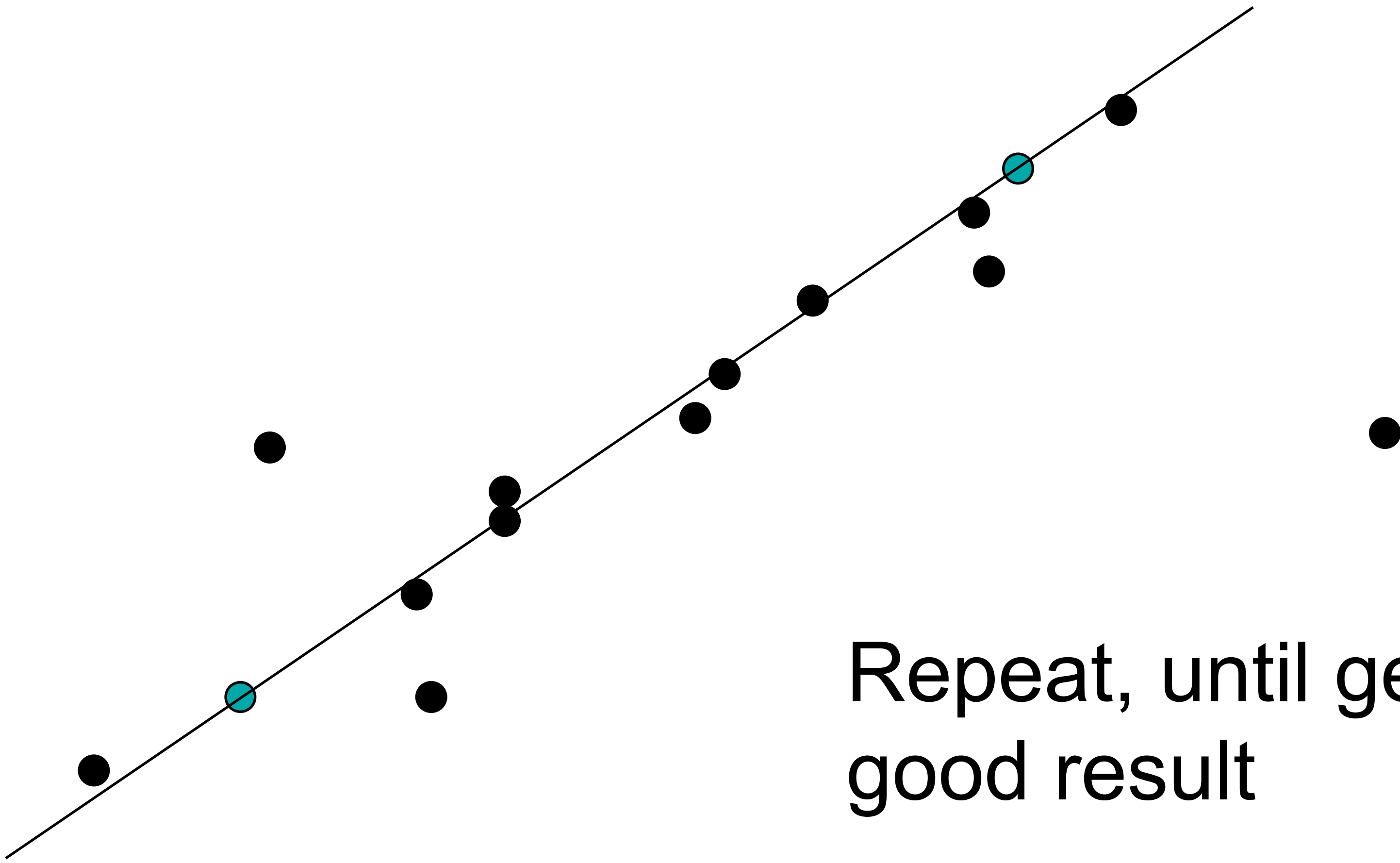


Total number of points within a threshold of line.

RANSAC Line Fitting Example

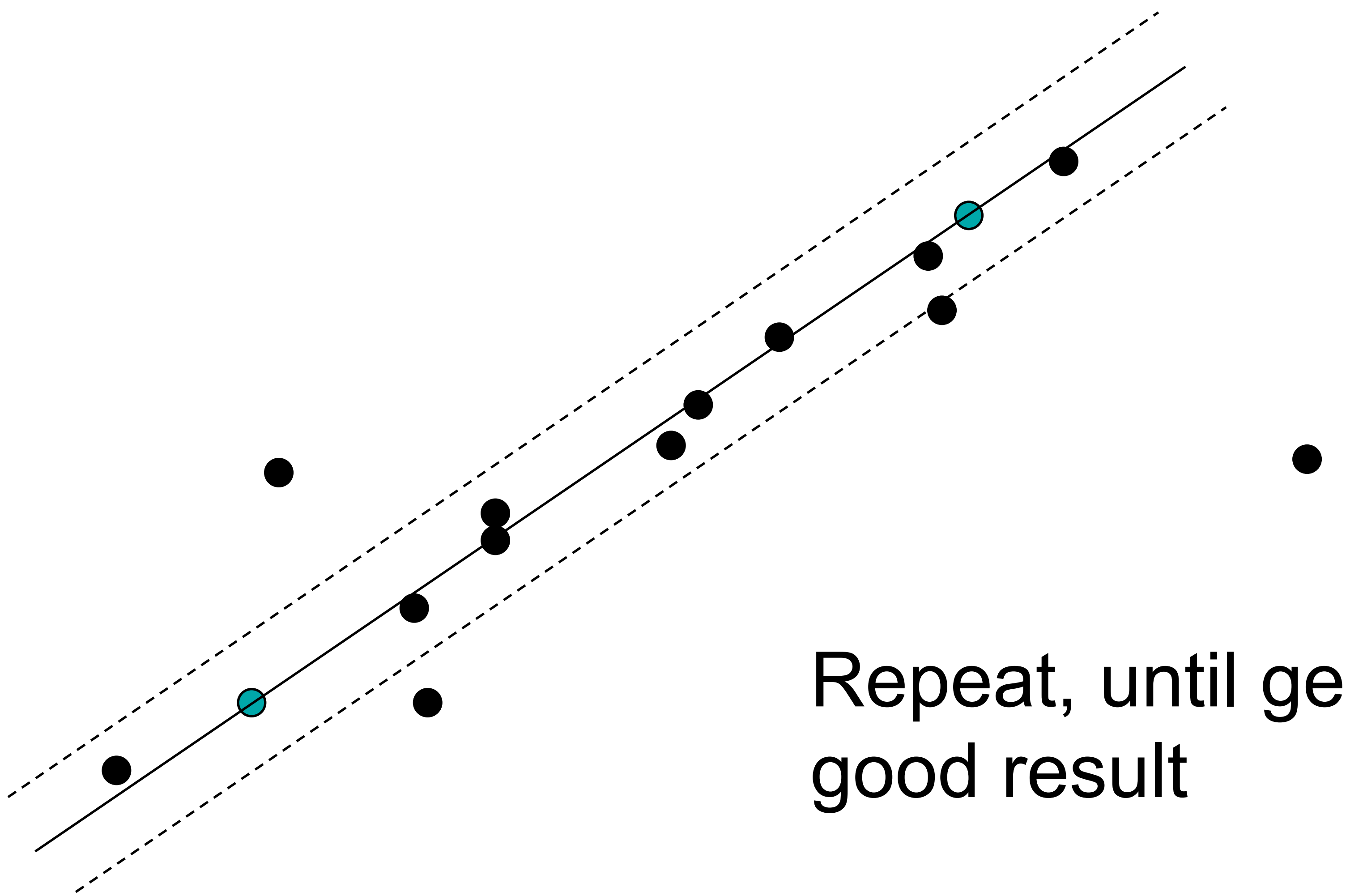
Repeat, until get a good result

RANSAC Line Fitting Example



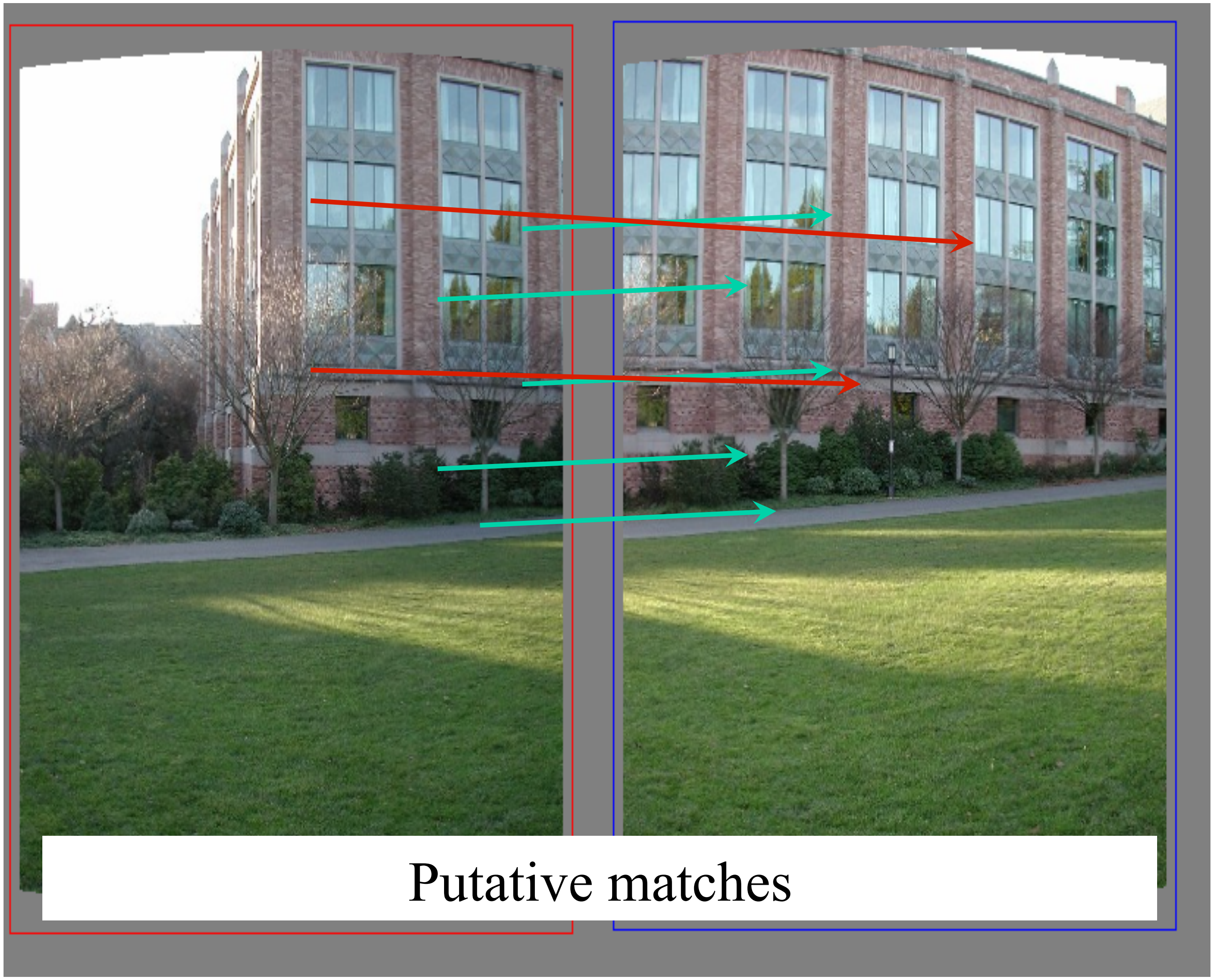
Repeat, until get a good result

RANSAC Line Fitting Example



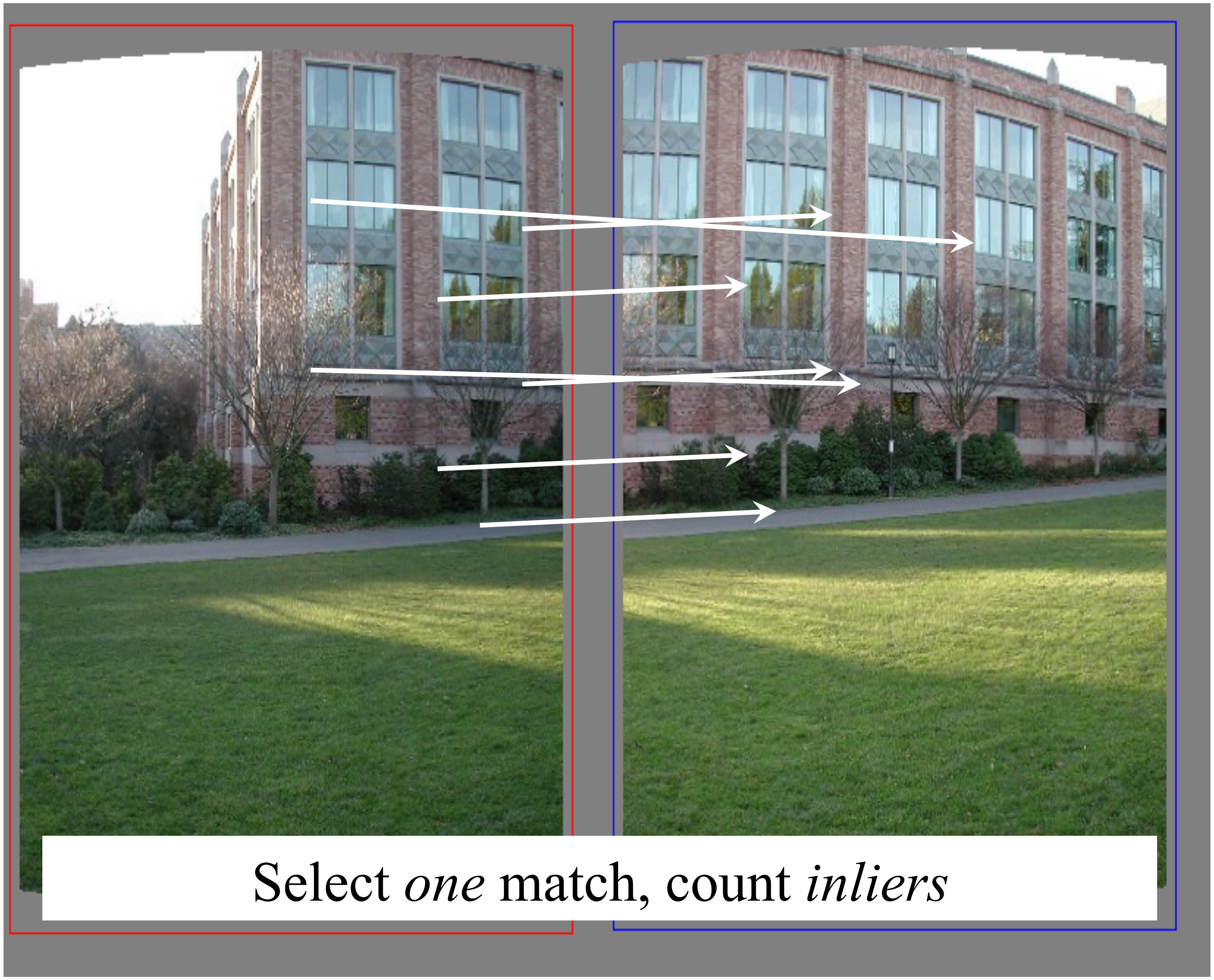
Repeat, until get a good result

RANSAC example: Translation

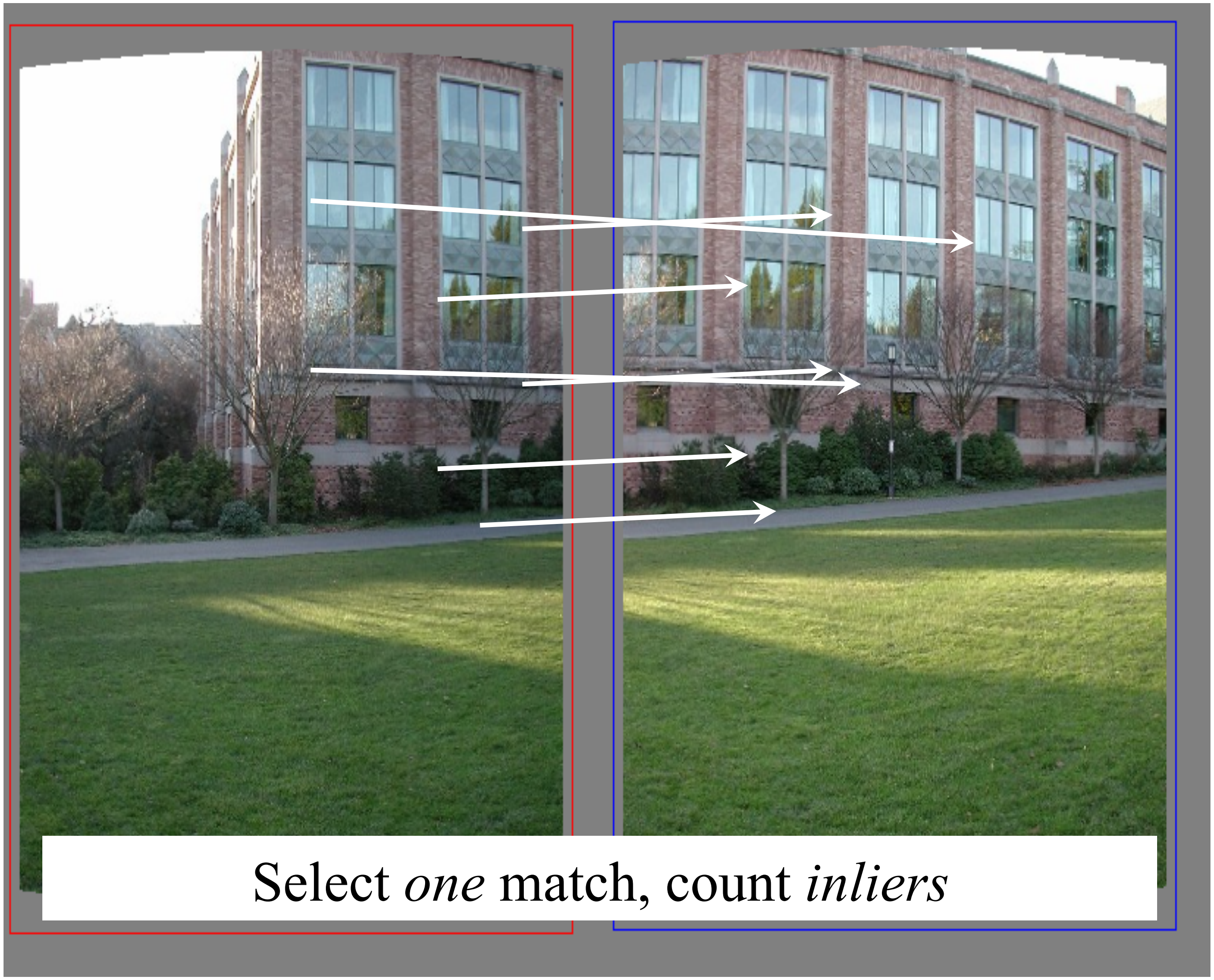


Source: Rick Szeliski

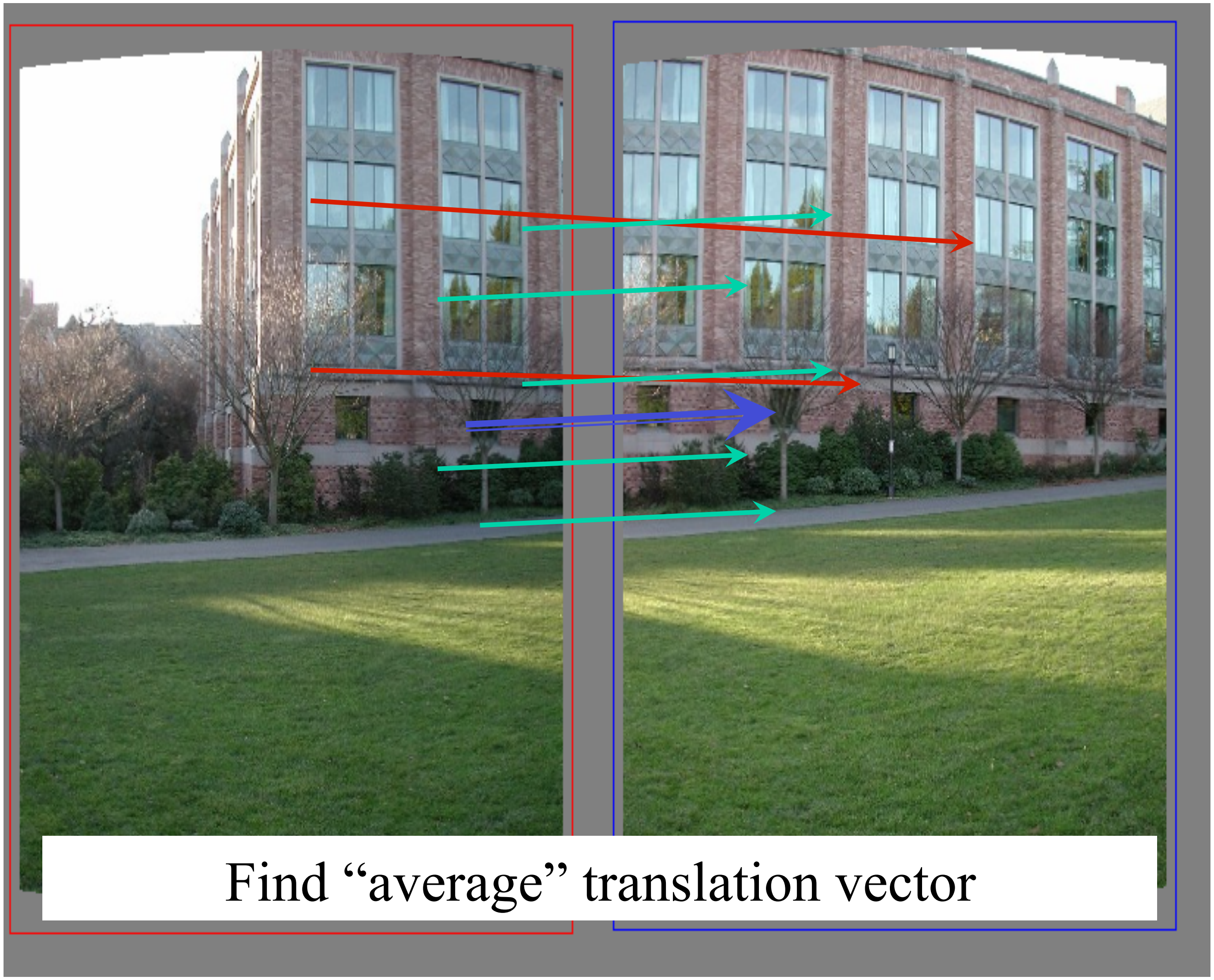
RANSAC example: Translation



RANSAC example: Translation



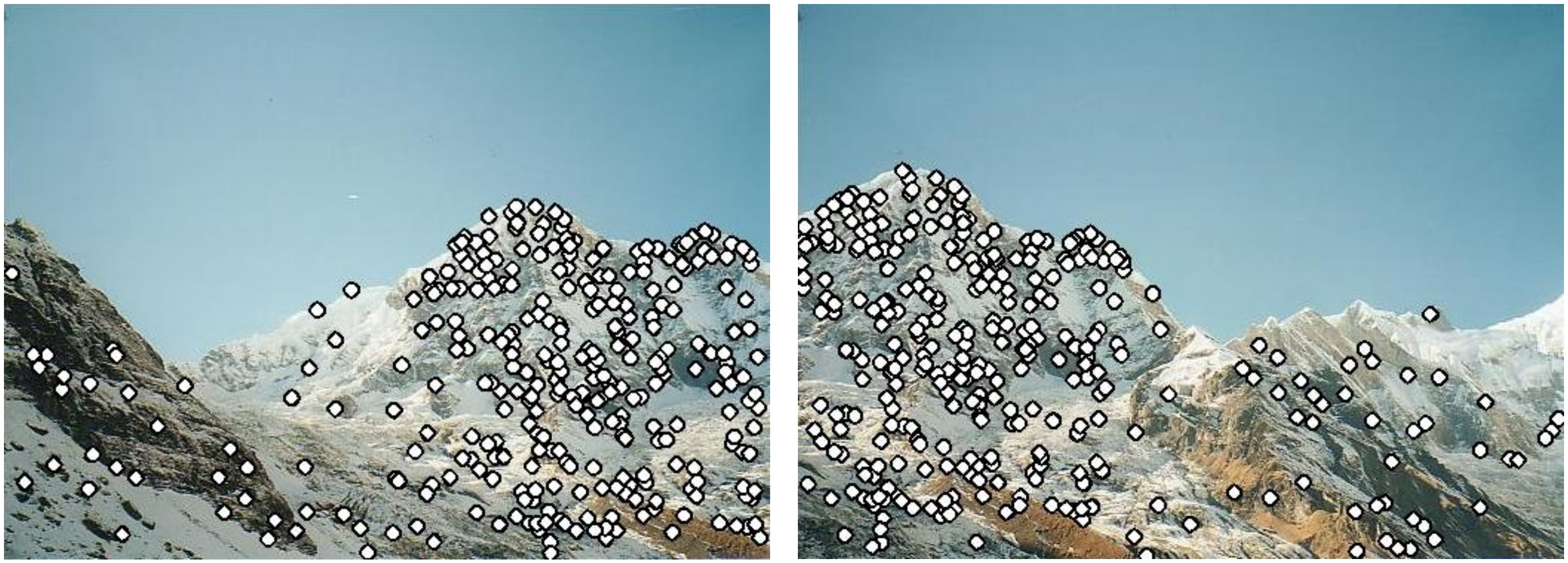
RANSAC example: Translation



Feature-based alignment outline

Source: L. Lazebnik

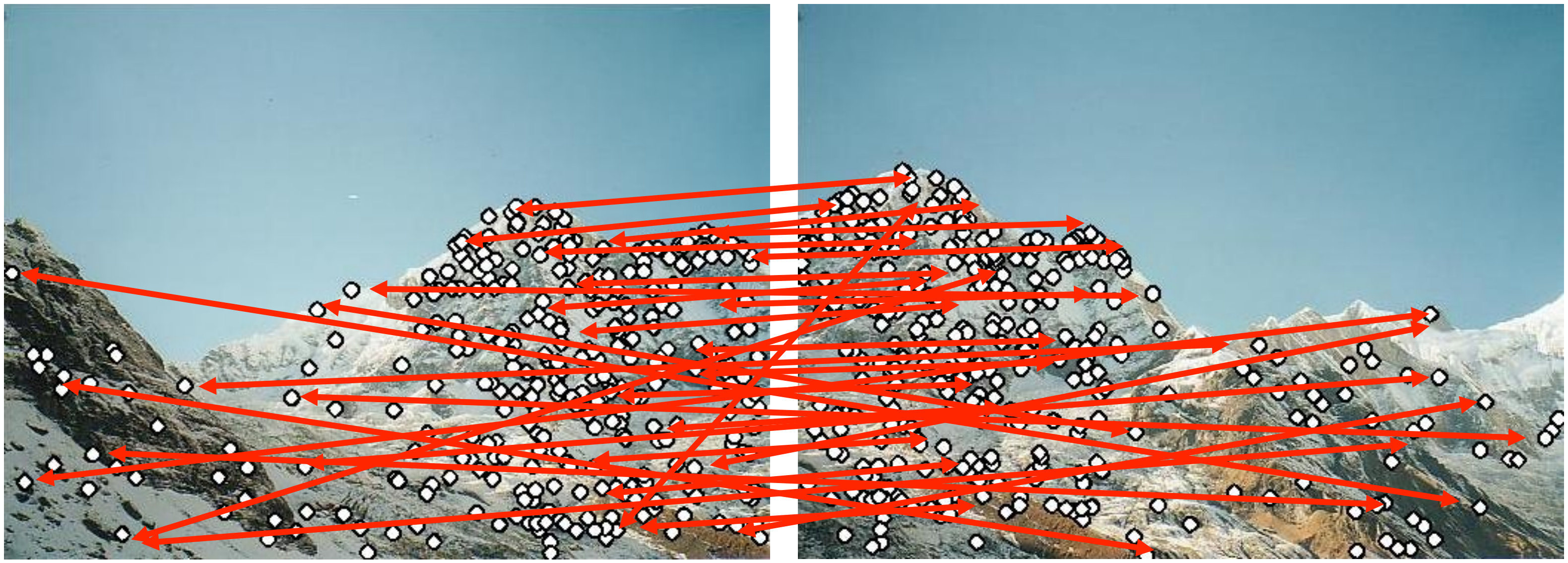
Feature-based alignment outline



- Extract features

Source: L. Lazebnik

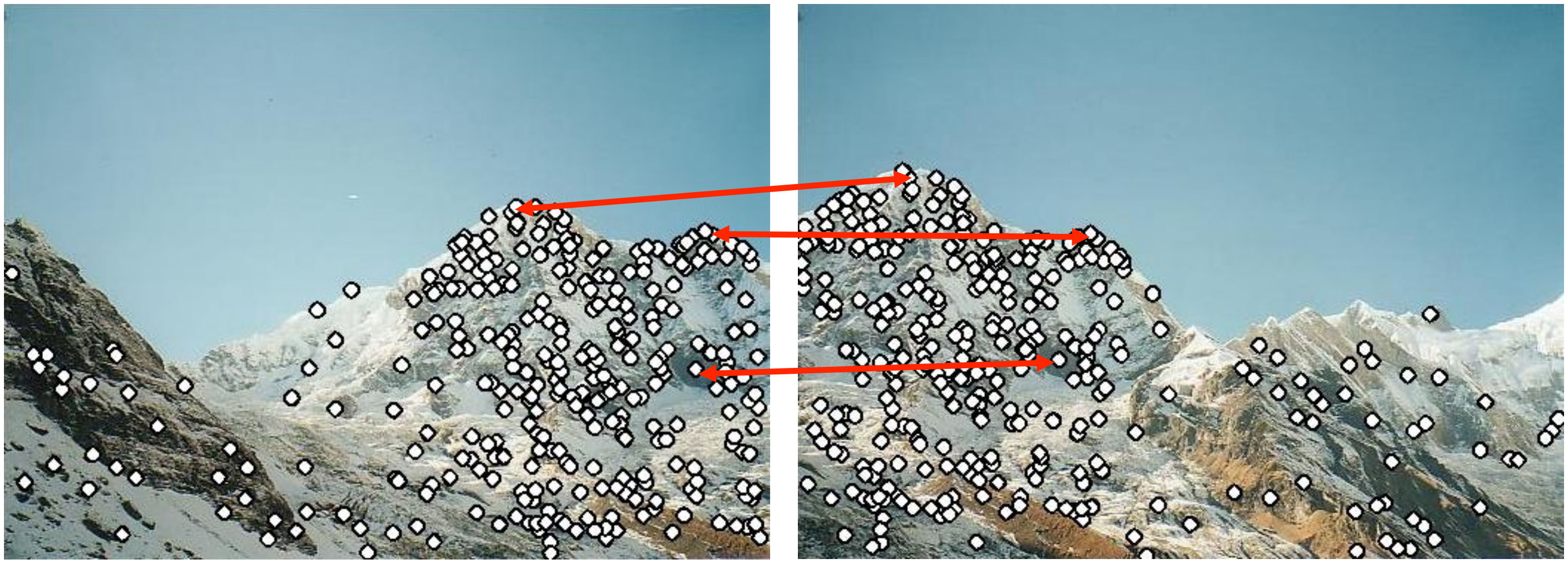
Feature-based alignment outline



- Extract features
- Compute *putative matches*

Source: L. Lazebnik

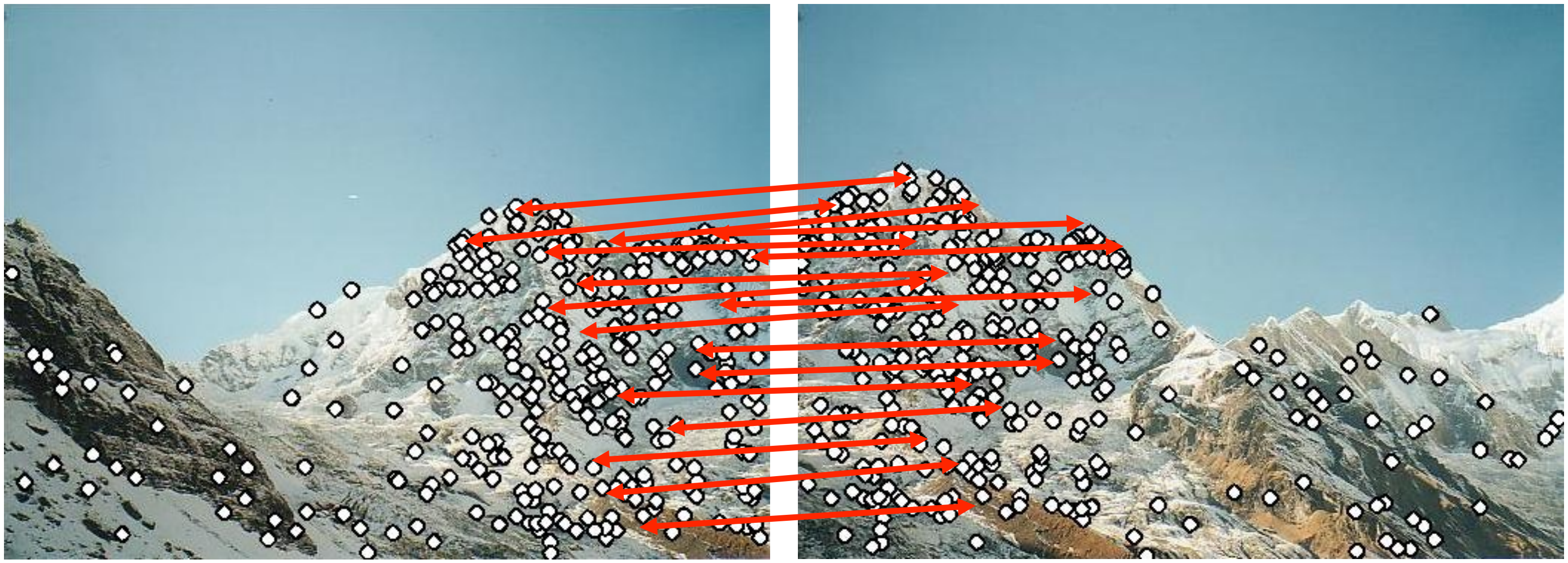
Feature-based alignment outline



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)

Source: L. Lazebnik

Feature-based alignment outline



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

Source: L. Lazebnik

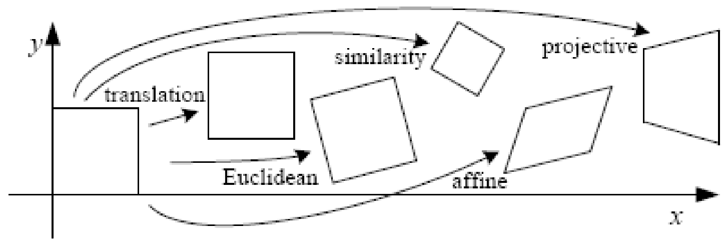
Feature-based alignment outline

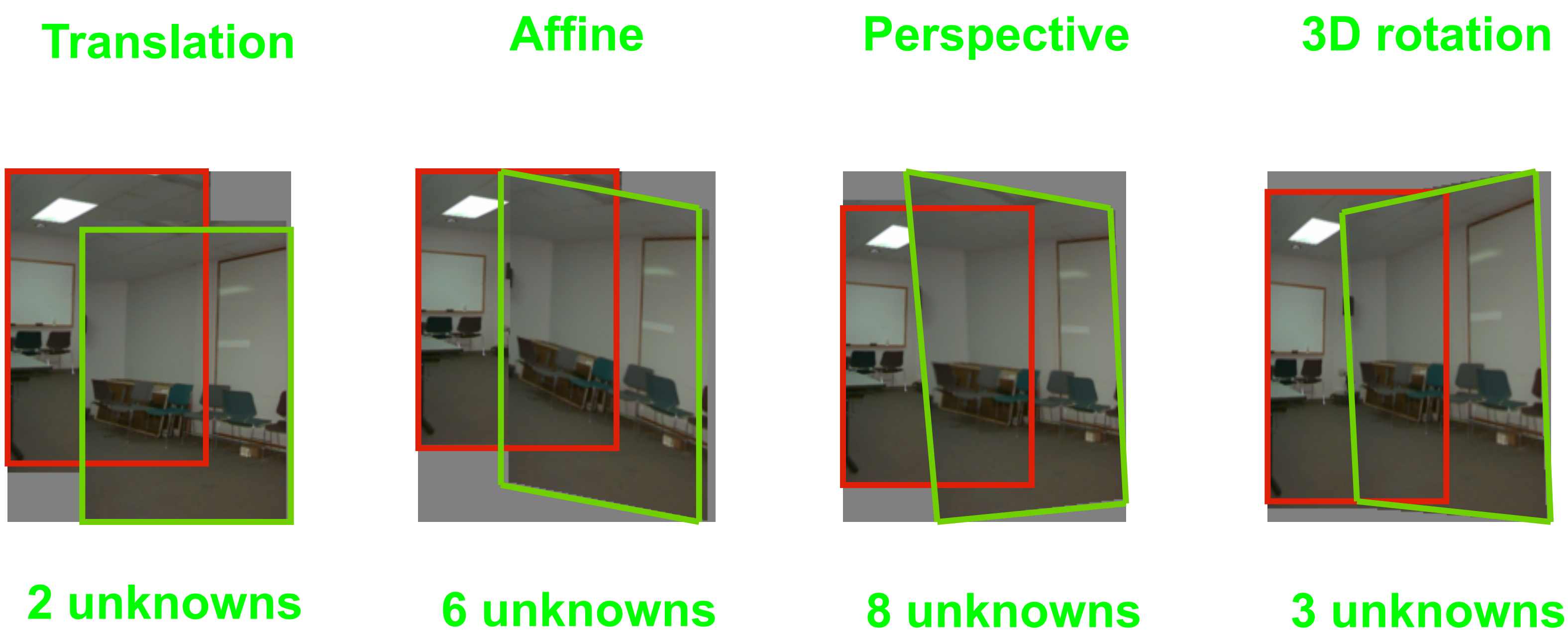
- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

Source: L. Lazebnik

Towards large-scale mosaics…

Motion models





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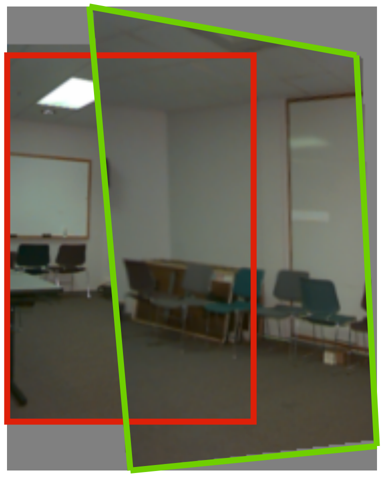
Plane perspective mosaics

- 8-parameter homographies
- Limitations:
  - local minima
  - slow convergence
  - difficult to control interactively



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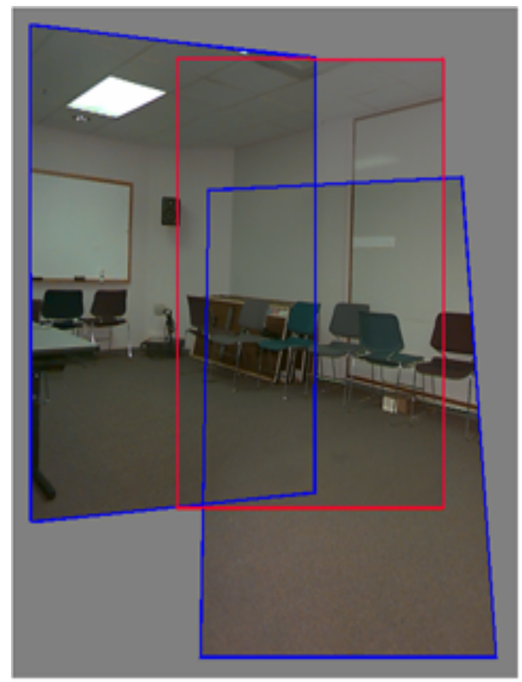
Rotational mosaics

– Directly optimize rotation and focal length

– Advantages:

- ability to build full-view panoramas
- easier to control interactively
- more stable and accurate estimates



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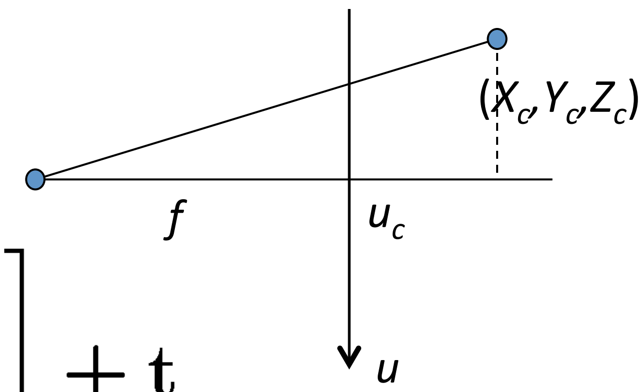
3D → 2D Perspective Projection



$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} \mathbf{R} \end{bmatrix}_{3\times 3} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \mathbf{t}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} f & 0 & u_c \\ 0 & f & v_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

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Rotational mosaic

- Projection equations

1. Project from image to 3D ray

- $(x_0,y_0,z_0) \quad = (u_0-u_c,v_0-v_c,f)$

2. Rotate the ray by camera motion

- $(x_1,y_1,z_1) \quad = \boldsymbol{R}_{01}\ (x_0,y_0,z_0)$

3. Project back into new (source) image

- $(u_1,v_1) \quad = (fx_1/z_1+u_c,fy_1/z_1+v_c)$

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Establishing correspondences

1. 'Direct' method: *(more next week)*
   - Use generalization of affine motion model [Szeliski & Shum '97]
2. Feature-based method
   - Extract features, match, find consistent *inliers* [Lowe ICCV'99; Schmid ICCV'98, Brown&Lowe ICCV'2003]
   - Compute $\boldsymbol{R}$ from correspondences (absolute orientation)

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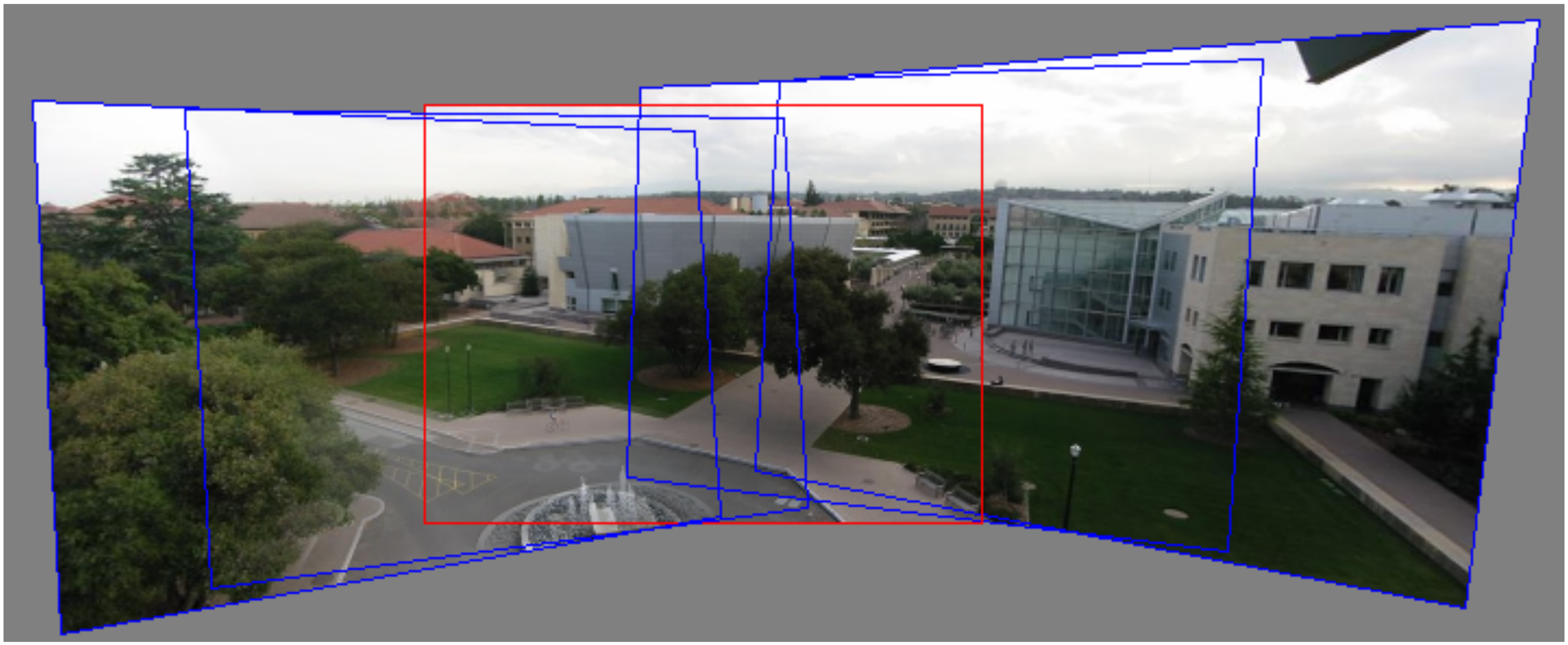
Absolute orientation

[Arun *et al.*, PAMI 1987] [Horn *et al.*, JOSA A 1988]
Procrustes Algorithm [Golub & VanLoan]

Given two sets of matching points, compute R

- $p_i' = \boldsymbol{R}\, p_i$ 3D rays
- $\boldsymbol{A} = \Sigma_{\mathbf{i}}\, p_i\, p_i'^T = \Sigma_{\mathbf{i}}\, p_i\, p_i^T\, \boldsymbol{R}^T = \boldsymbol{U}\, \boldsymbol{S}\, \boldsymbol{V}^T = (\boldsymbol{U}\, \boldsymbol{S}\, \boldsymbol{U}^T)\, \boldsymbol{R}^T$
- $\boldsymbol{V}^T = \boldsymbol{U}^T \boldsymbol{R}^T$
- $\boldsymbol{R} = \boldsymbol{V}\, \boldsymbol{U}^T$

Stitching demo



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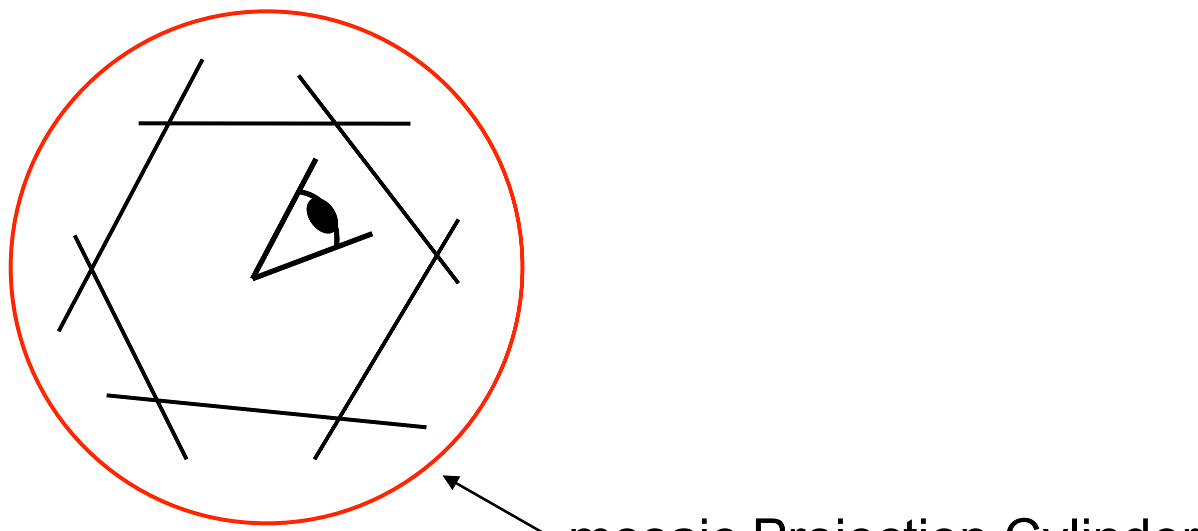
Panoramas

- What if you want a 360° field of view?



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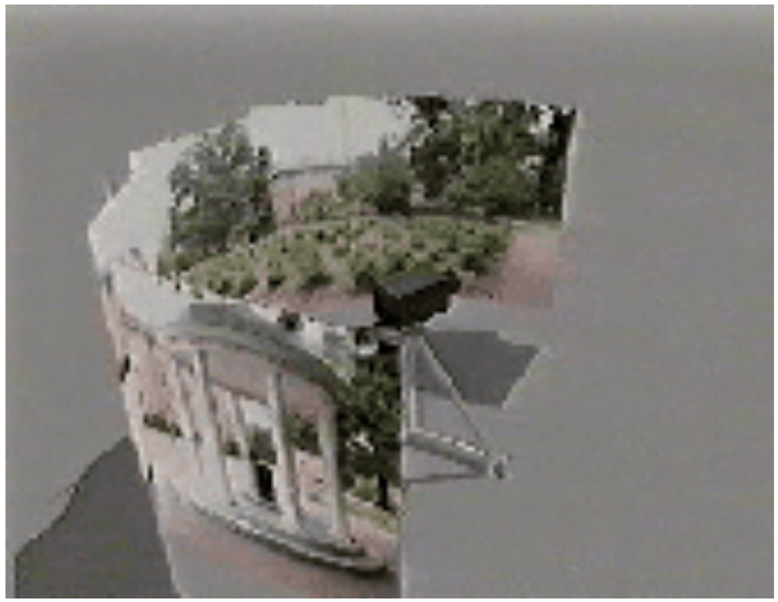
Cylindrical panoramas



- Steps
  - Reproject each image onto a cylinder
  - Blend
  - Output the resulting mosaic

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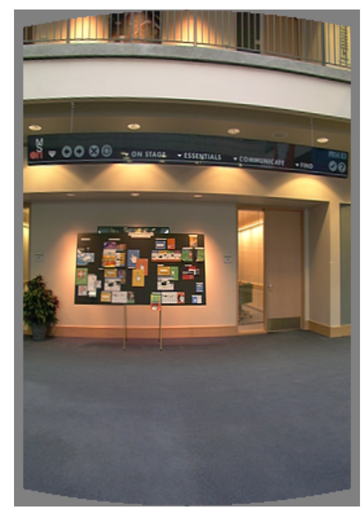
Cylindrical Panoramas

- Map image to cylindrical or spherical coordinates
  - need *known* focal length

Image 384x300

f = 180 (pixels)

f = 280



f = 380

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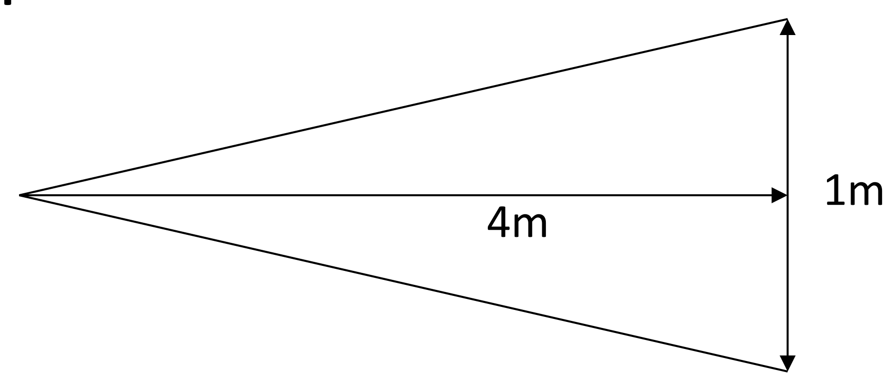
Determining the focal length

1. Initialize from homography $\boldsymbol{H}$ (see text or [SzSh'97])
2. Use camera's EXIF tags (approx.)
3. Use a tape measure



4. Ask your instructor

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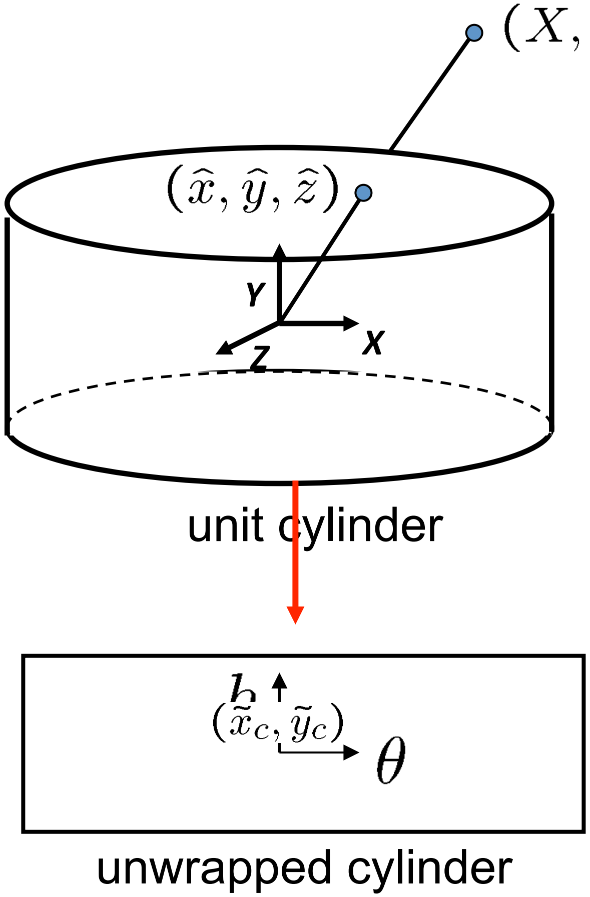
Cylindrical projection



(X, Y, Z) – Map 3D point (X,Y,Z) onto cylinder

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2 + Z^2}}(X, Y, Z)$$

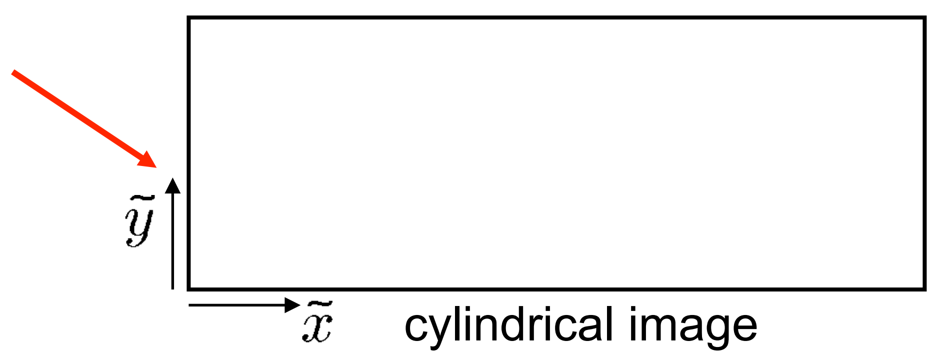
- Convert to cylindrical coordinates

$$(sin\theta, h, cos\theta) = (\hat{x}, \hat{y}, \hat{z})$$

- Convert to cylindrical image coordinates

$$(\tilde{x}, \tilde{y}) = (s\theta, sh) + (\tilde{x}_c, \tilde{y}_c)$$

  – s defines size of the final image



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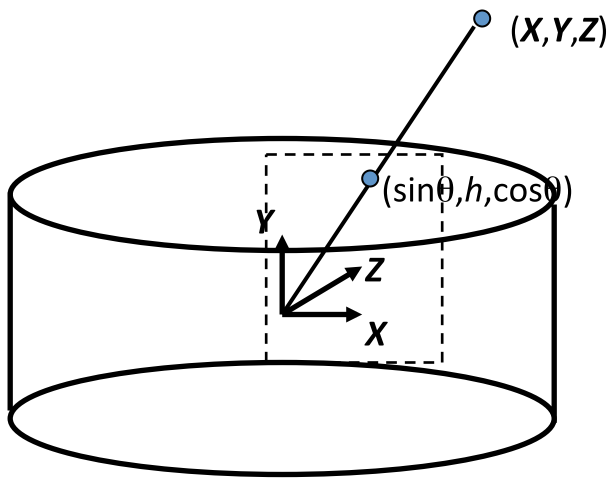
Cylindrical warping

- Given focal length *f* and image center (x_c, y_c)



$$
\begin{aligned}
\theta &= (x_{cyl} - x_c)/f \\
h &= (y_{cyl} - y_c)/f \\
\hat{x} &= \sin\theta \\
\hat{y} &= h \\
\hat{z} &= \cos\theta \\
x &= f\hat{x}/\hat{z} + x_c \\
y &= f\hat{y}/\hat{z} + y_c
\end{aligned}
$$

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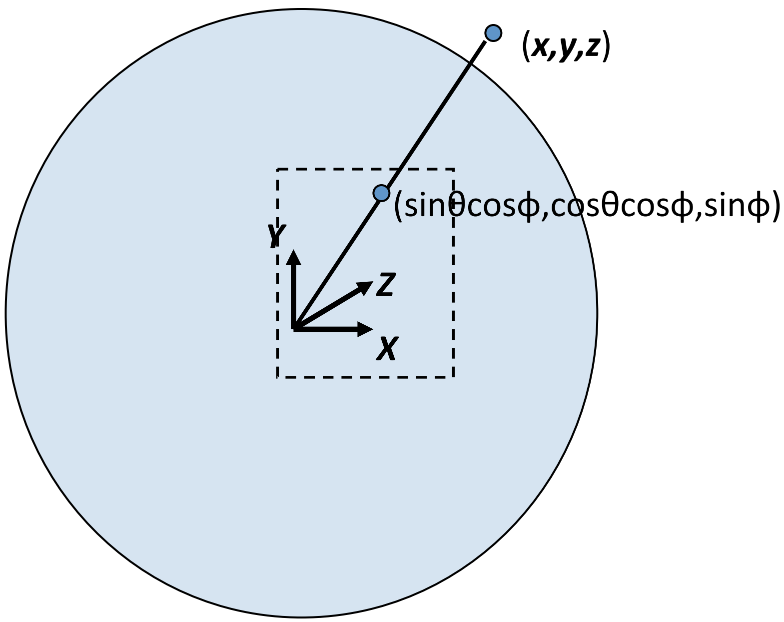
Spherical warping

- Given focal length *f* and image center (x_c, y_c)



$$
\begin{aligned}
\theta &= (x_{cyl} - x_c)/f \\
\varphi &= (y_{cyl} - y_c)/f \\
\hat{x} &= \sin\theta \cos\varphi \\
\hat{y} &= \sin\varphi \\
\hat{z} &= \cos\vartheta \cos \\
x &= f\hat{x}/\hat{z} + x_c \\
y &= f\hat{y}/\hat{z} + y_c
\end{aligned}
$$

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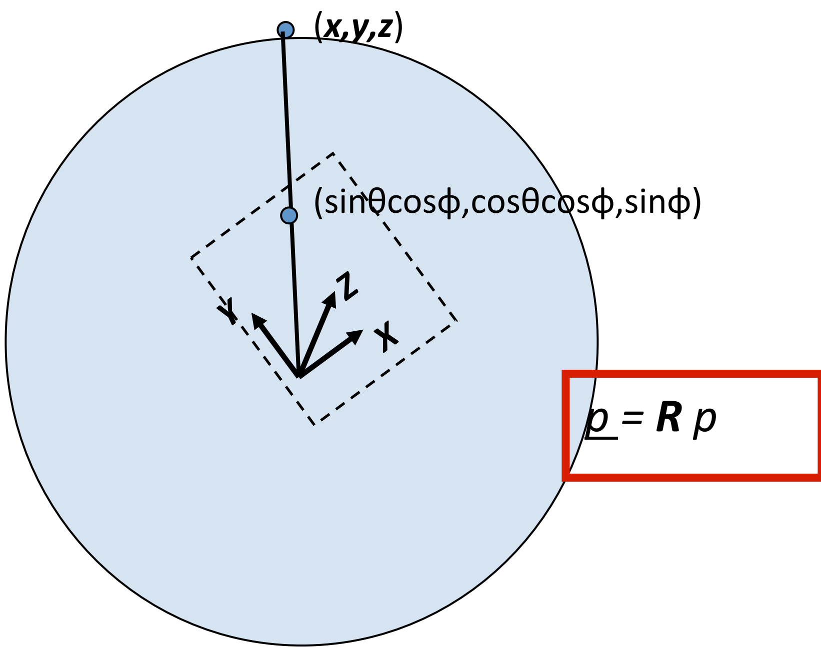
3D rotation

- Rotate image before placing on unrolled sphere



$$
\begin{aligned}
\theta &= (x_{cyl} - x_c)/f \\
\varphi &= (y_{cyl} - y_c)/f \\
\hat{x} &= \sin\theta \cos\varphi \\
\hat{y} &= \sin\varphi \\
\hat{z} &= \cos\vartheta \cos\varphi \\
x &= f\hat{x}/\hat{z} + x_c \\
y &= f\hat{y}/\hat{z} + y_c
\end{aligned}
$$

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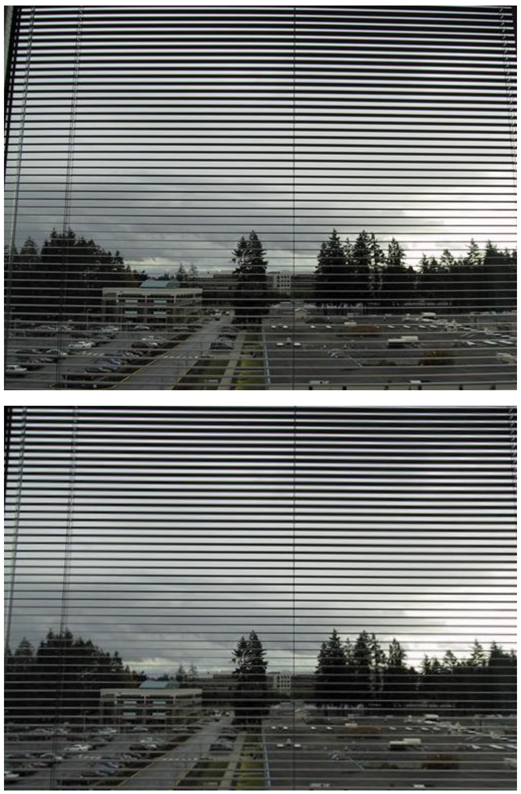
Radial distortion

- Correct for “bending” in wide field of view lenses



$$\begin{aligned}
\hat{r}^2 &= \hat{x}^2 + \hat{y}^2 \\
\hat{x}' &= \hat{x}/(1 + \kappa_1 \hat{r}^2 + \kappa_2 \hat{r}^4) \\
\hat{y}' &= \hat{y}/(1 + \kappa_1 \hat{r}^2 + \kappa_2 \hat{r}^4) \\
x &= f\hat{x}'/\hat{z} + x_c \\
y &= f\hat{y}'/\hat{z} + y_c
\end{aligned}$$

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Fisheye lens

- Extreme "bending" in ultra-wide fields of view

$$\hat{r}^2 = \hat{x}^2 + \hat{y}^2$$

$$(\cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi) = s\,(x, y, z)$$

uations become

$$x' = s\phi\cos\theta = s\frac{x}{r}\tan^{-1}\frac{r}{z},$$
$$y' = s\phi\sin\theta = s\frac{y}{r}\tan^{-1}\frac{r}{z},$$

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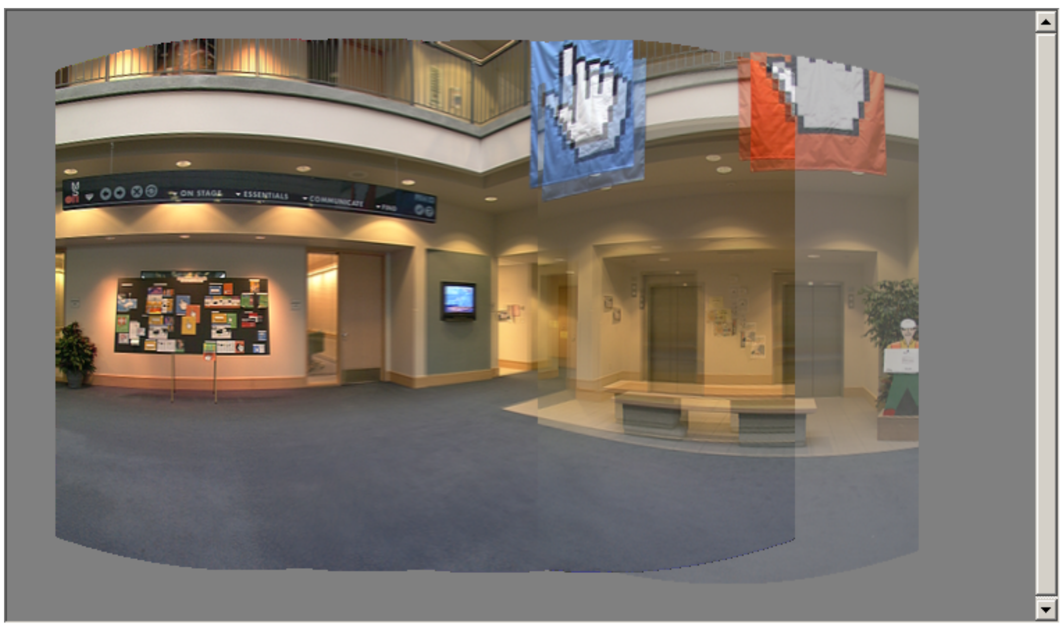
Image Stitching

1. Align the images over each other
   - camera pan ↔ translation on cylinder
2. Blend the images together



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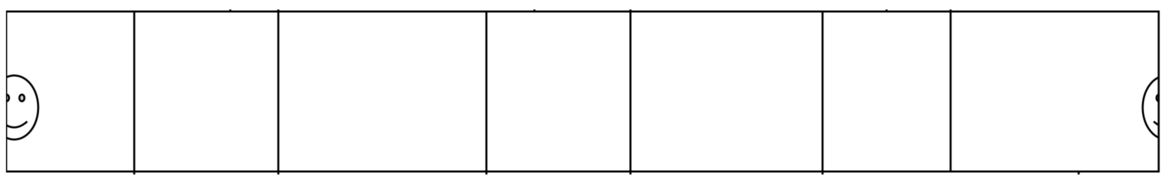
Assembling the panorama



- Stitch pairs together, blend, then crop

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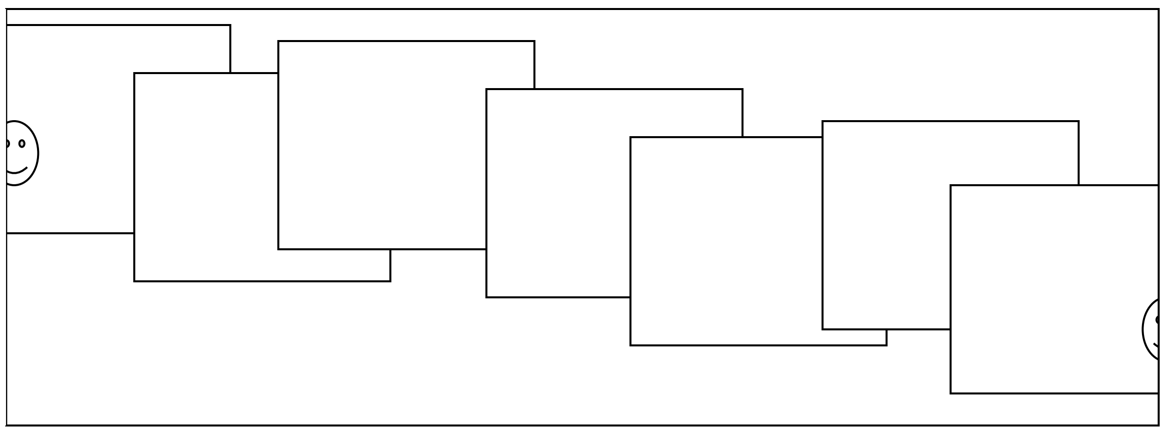
Problem: Drift



- Error accumulation
  - small (vertical) errors accumulate over time
  - apply correction so that sum = 0 (for 360° pan.)

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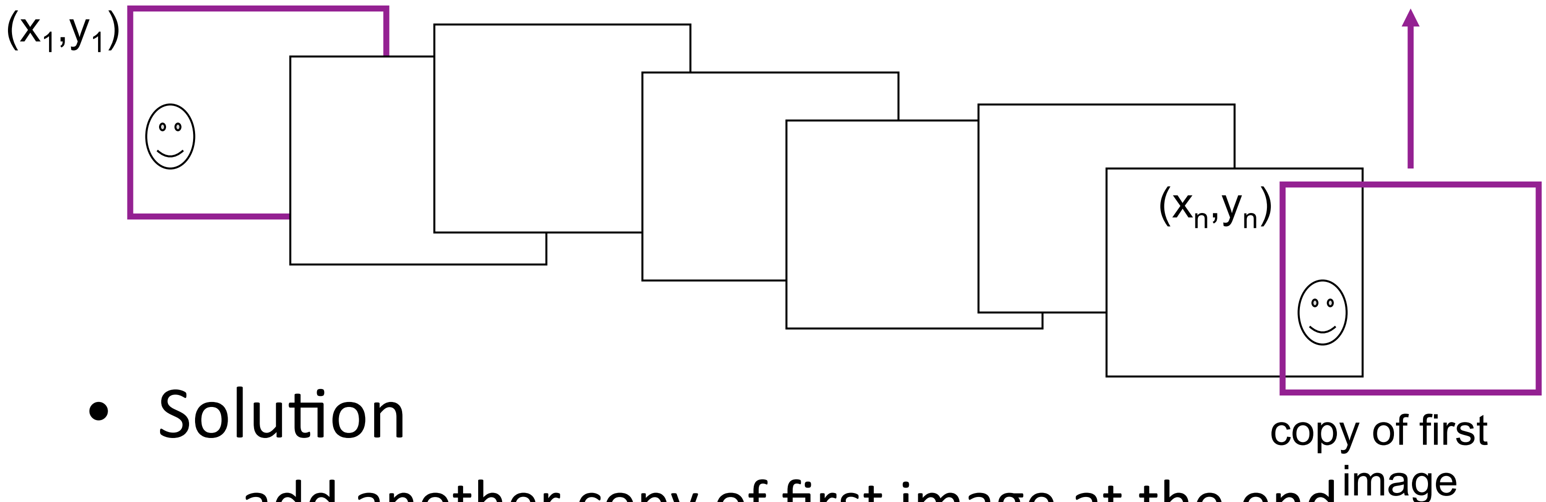
Problem: Drift



- Solution
  - add another copy of first image at the end
  - this gives a constraint: $y_n = y_1$
  - there are a bunch of ways to solve this problem
    - add displacement of $(y_1 - y_n)/(n - 1)$ to each image after the first
    - compute a global warp: $y' = y + ax$
    - run a big optimization problem, incorporating this constraint
      - best solution, but more complicated
      - known as "bundle adjustment"

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Full-view (360° spherical) panoramas

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Full-view Panorama

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Texture Mapped Model

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Global alignment

- Register *all* pairwise overlapping images
- Use a 3D rotation model (one R per image)
- Use direct alignment (patch centers) or feature based
- *Infer* overlaps based on previous matches (incremental)
- Optionally *discover* which images overlap other images using feature selection (RANSAC)

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Bundle adjustment formulations

Confidence / uncertainty of point i in image j

All pairs optimization:

$$E_{\text{all-pairs-2D}} = \sum_i \sum_{jk} c_{ij} c_{ik} \| \tilde{\boldsymbol{x}}_{ik}(\hat{\boldsymbol{x}}_{ij}; \boldsymbol{R}_j, f_j, \boldsymbol{R}_k, f_k) - \hat{\boldsymbol{x}}_{ik} \|^2, \quad (9.29)$$

Map 2D point i in image j to 2D point in image k

Full bundle adjustment, using 3-D point positions $\{\boldsymbol{x}_i\}$

$$E_{\text{BA-2D}} = \sum_i \sum_j c_{ij} \| \tilde{\boldsymbol{x}}_{ij}(\boldsymbol{x}_i; \boldsymbol{R}_j, f_j) - \hat{\boldsymbol{x}}_{ij} \|^2, \quad (9.30)$$

Map 3D point i in to 2D point in image i

Bundle adjustment using 3-D ray:

$$E_{\text{BA-3D}} = \sum_i \sum_j c_{ij} \| \tilde{\boldsymbol{x}}_i(\hat{\boldsymbol{x}}_{ij}; \boldsymbol{R}_j, f_j) - \boldsymbol{x}_i \|^2, \quad (9.31)$$

3-D ray from point i

All-pairs 3-D ray formulation:

$$E_{\text{all-pairs-3D}} = \sum_i \sum_{jk} c_{ij} c_{ik} \| \tilde{\boldsymbol{x}}_i(\hat{\boldsymbol{x}}_{ij}; \boldsymbol{R}_j, f_j) - \tilde{\boldsymbol{x}}_i(\hat{\boldsymbol{x}}_{ik}; \boldsymbol{R}_k, f_k) \|^2. \quad (9.32)$$

3-D ray from points i and j

Projected point → $\tilde{\boldsymbol{x}}_{ij} \sim \boldsymbol{K}_j \boldsymbol{R}_j \boldsymbol{x}_i$ and $\boldsymbol{x}_i \sim \boldsymbol{R}_j^{-1} \boldsymbol{K}_j^{-1} \tilde{\boldsymbol{x}}_{ij}$, ← *3-D ray from point*

Recognizing Panoramas

Matthew Brown & David Lowe

ICCV'2003

Recognizing Panoramas

[Brown & Lowe, ICCV'03]

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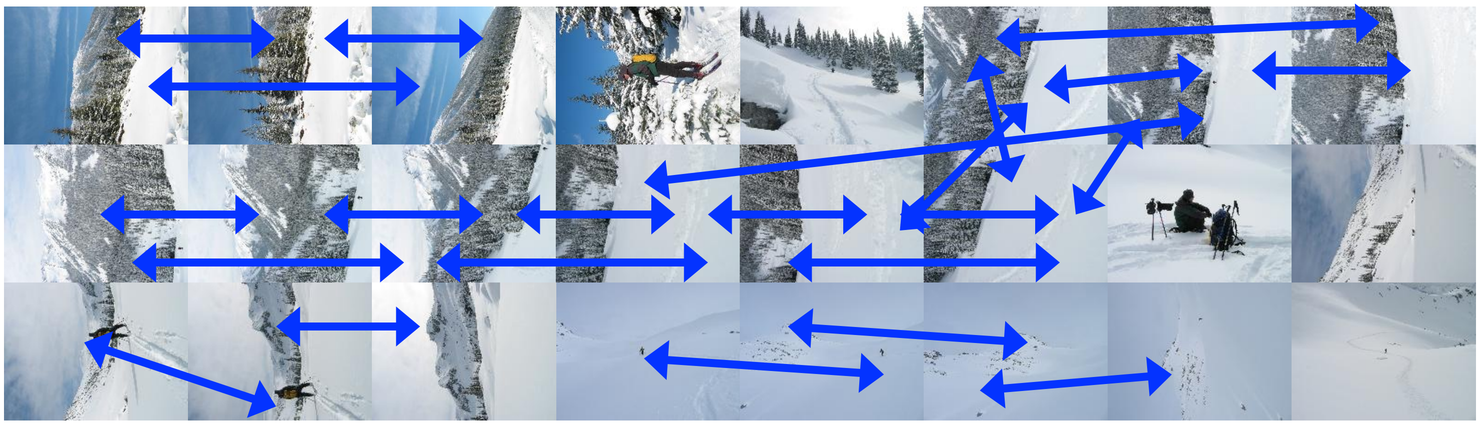
Finding the panoramas



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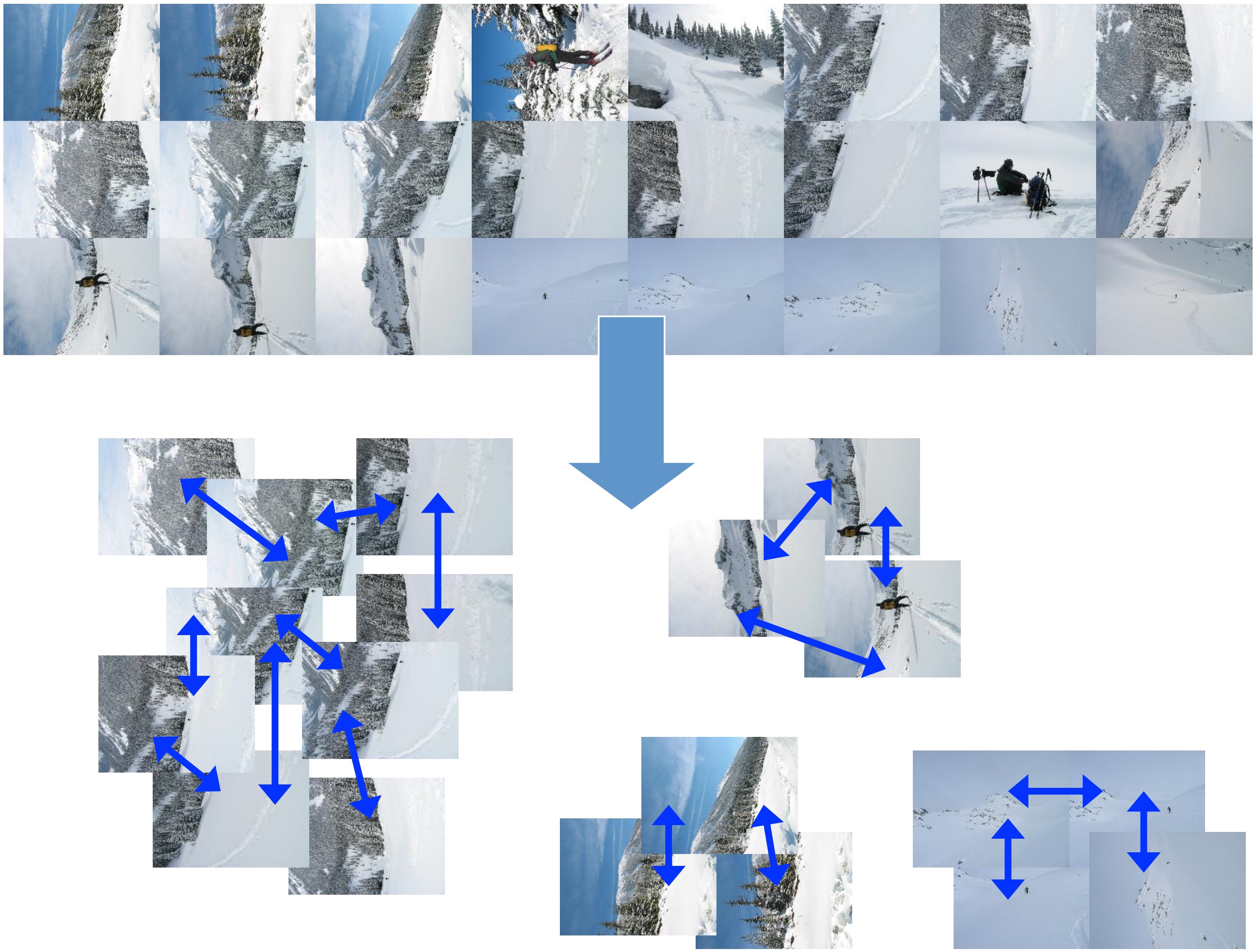
Finding the panoramas



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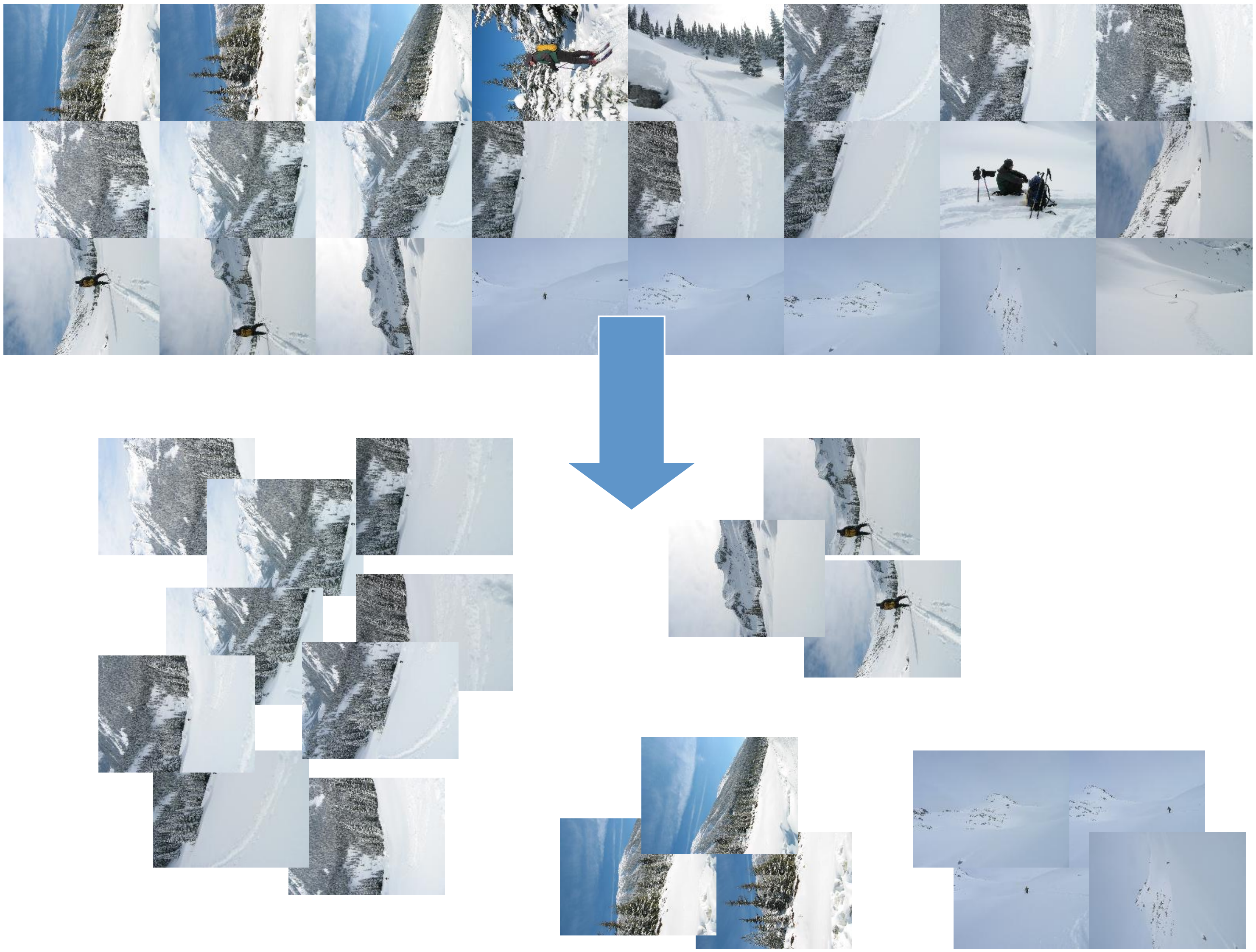
Finding the panoramas



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Finding the panoramas

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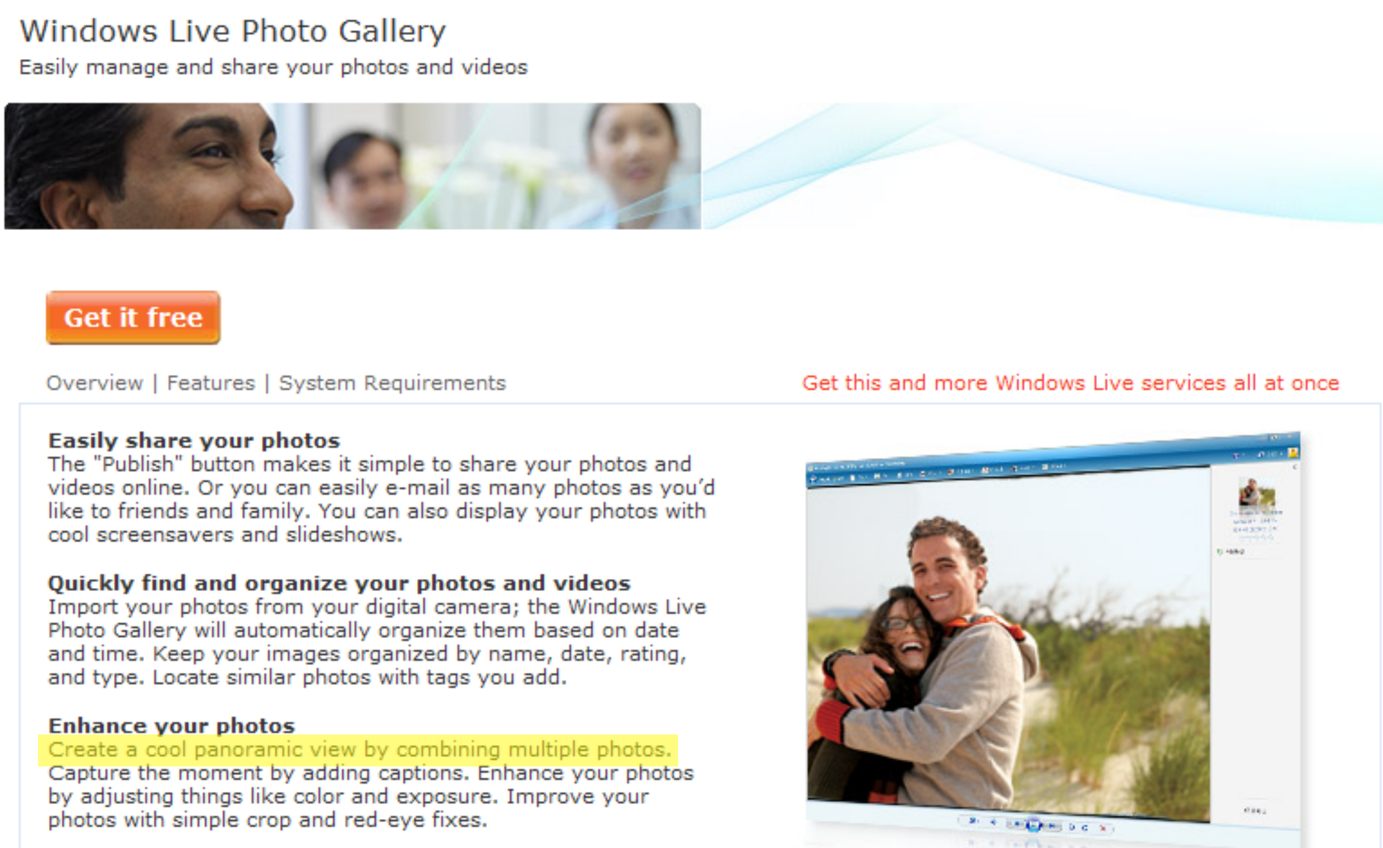
Fully automated 2D stitching demo



http://get.live.com/photogallery/overview

Rec.pano.: system components

1. Feature detection and description
   – more uniform point density
2. Fast matching (hash table)
3. RANSAC filtering of matches
4. Intensity-based verification
5. Incremental bundle adjustment

- [M. Brown, R. Szeliski, and S. Winder. Multi-image matching using multi-scale oriented patches, CVPR'2005]

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Multi-Scale Oriented Patches

- Interest points
  - Multi-scale Harris corners
  - Orientation from blurred gradient
  - Geometrically invariant to similarity transforms
- Descriptor vector
  - Bias/gain normalized sampling of local patch (8x8)
  - Photometrically invariant to affine changes in intensity

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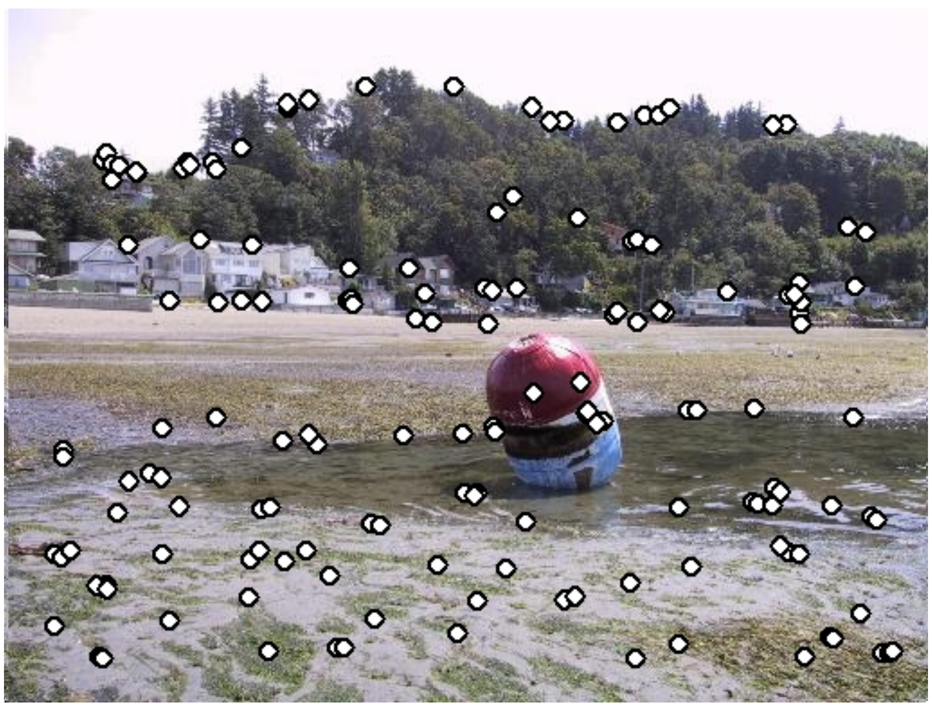
Features

- Distribute points evenly over the image



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Descriptor Vector

- Orientation = blurred gradient
- Similarity Invariant Frame
  - Scale-space position (x, y, s) + orientation (θ)

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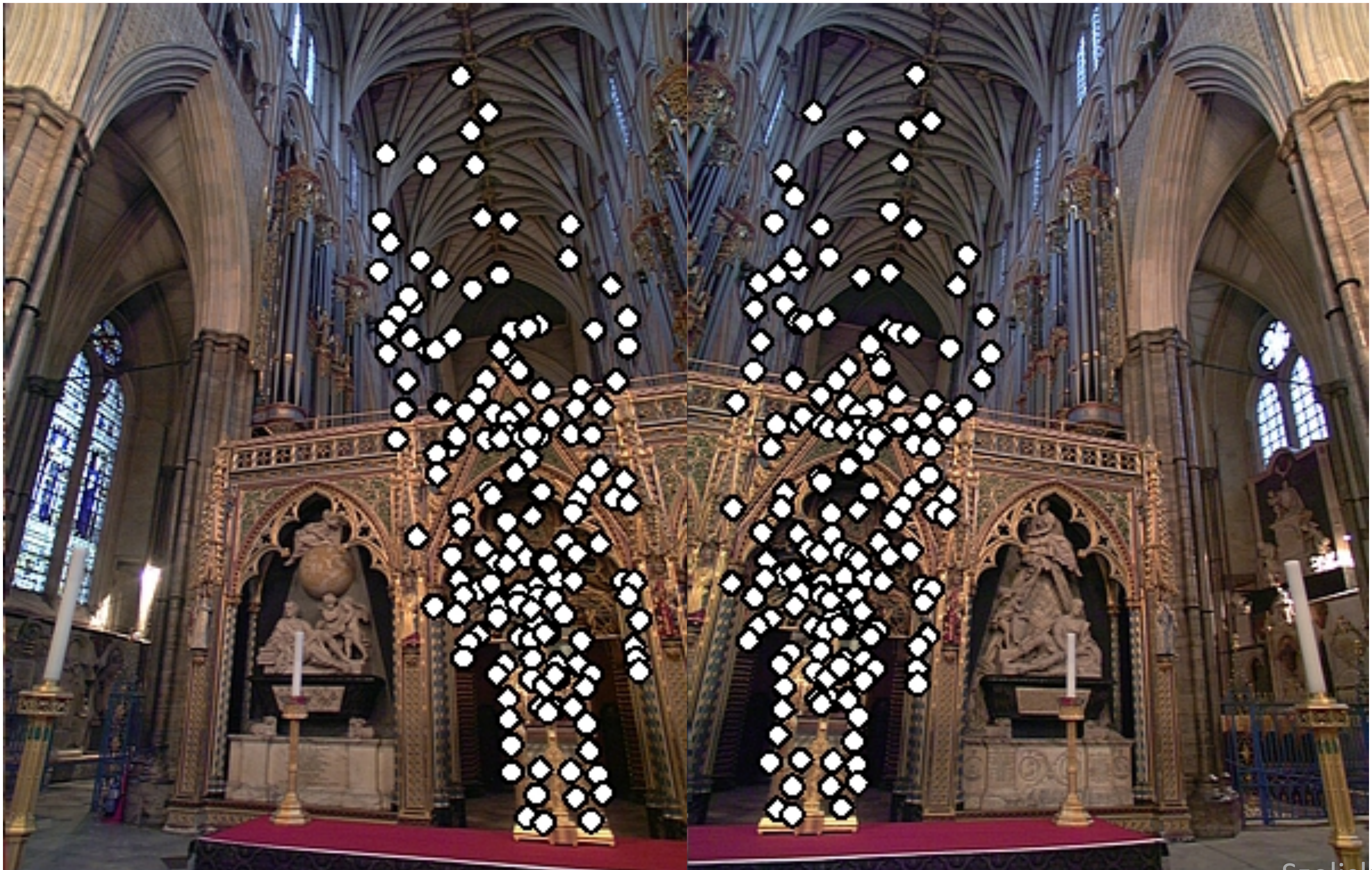
Probabilistic Feature Matching



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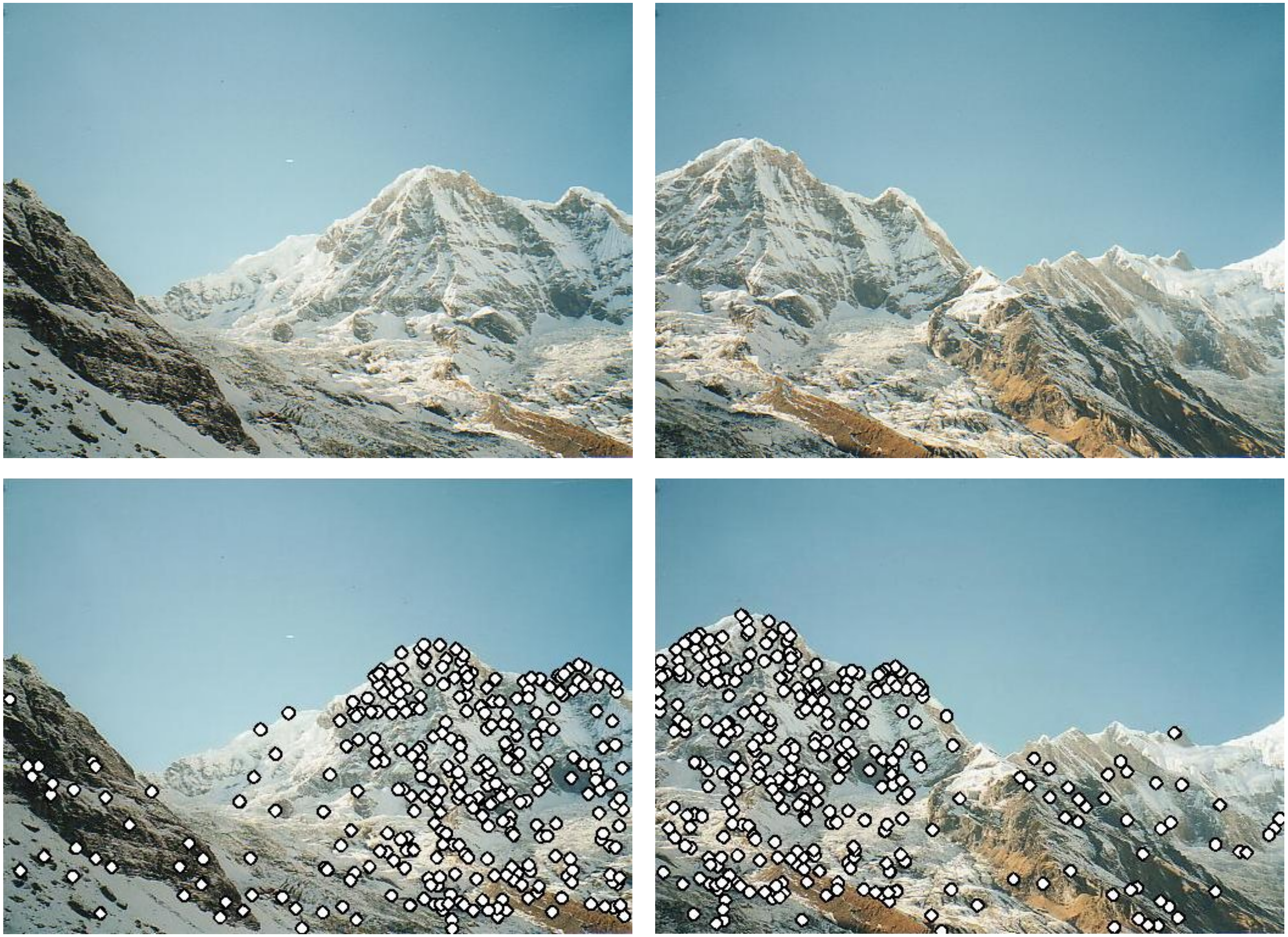
RANSAC motion model



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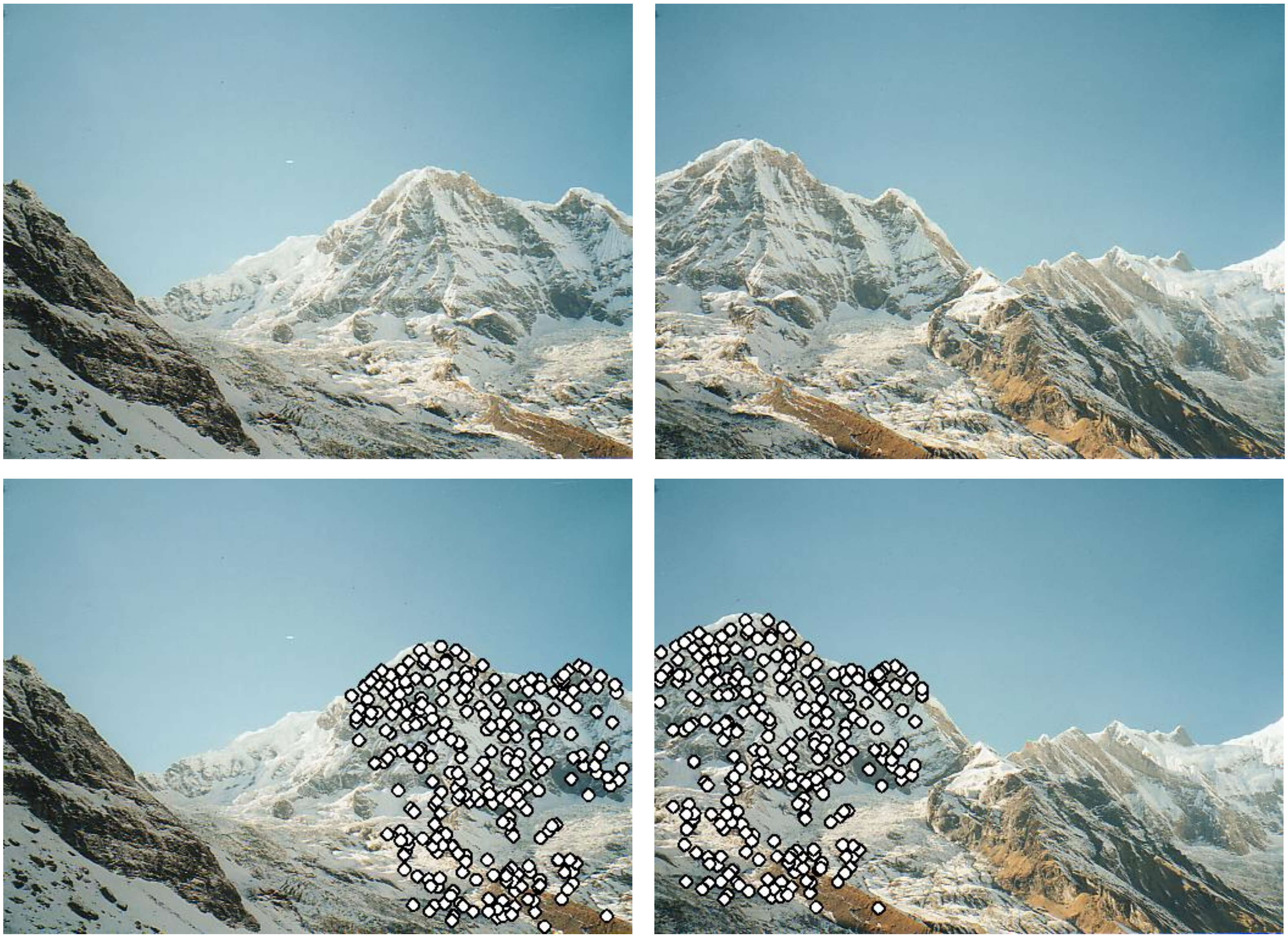
RANSAC motion model



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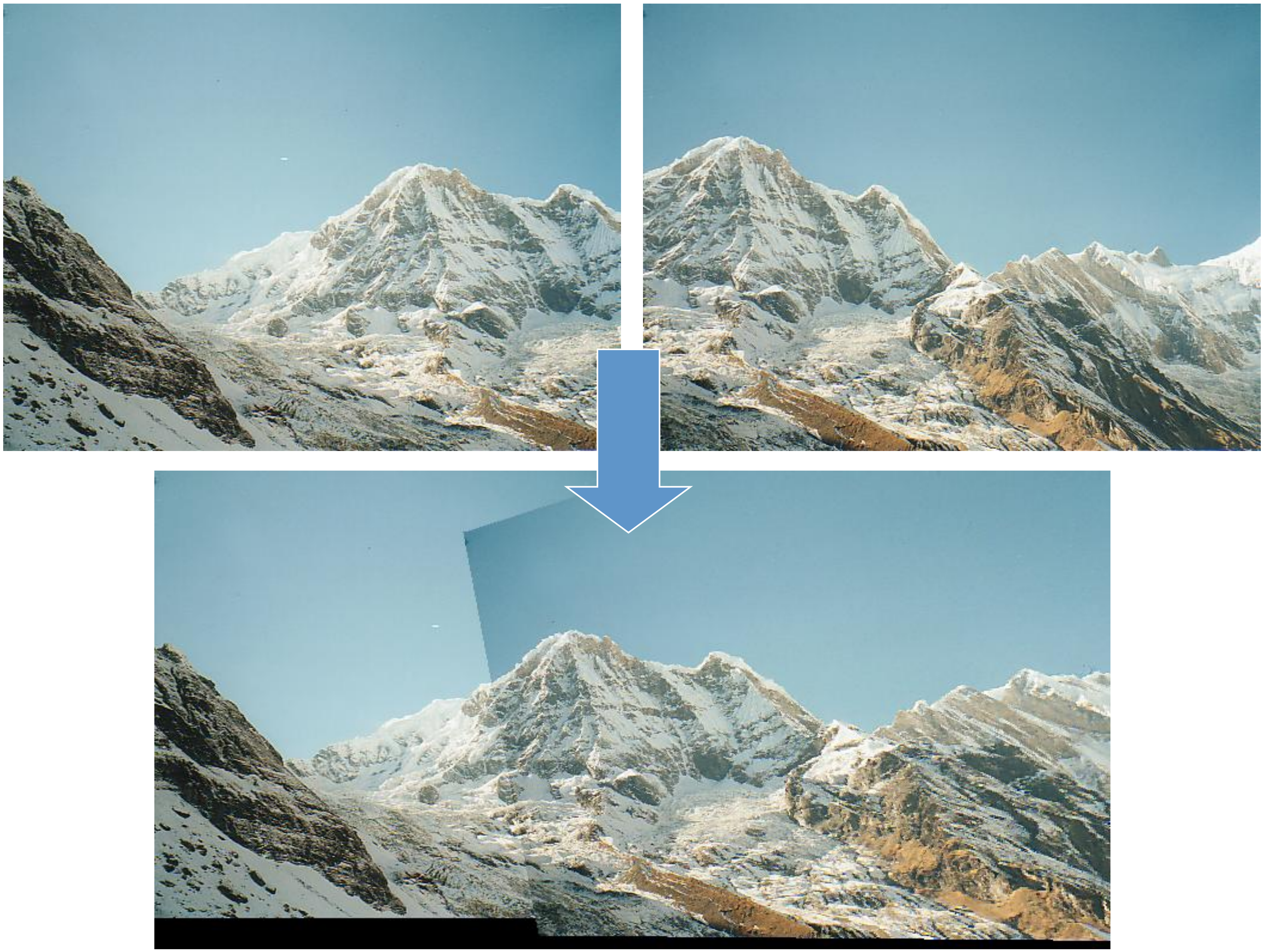
RANSAC motion model



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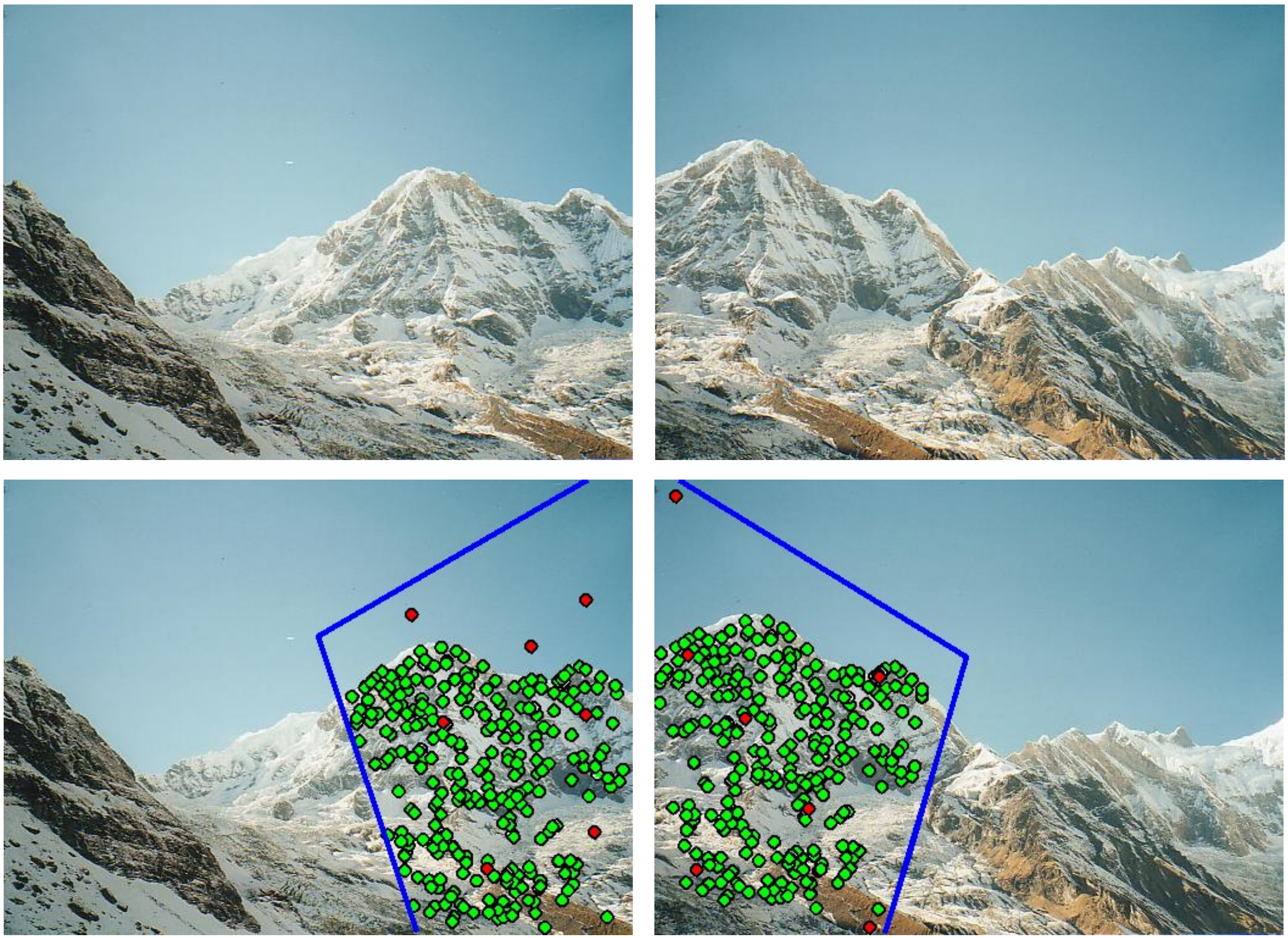
Probabilistic model for verification



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How well does this work?

Test on 100s of examples…

How well does this work?

Test on 100s of examples…

…still too many failures (5-10%)
for consumer application

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Matching Mistakes: False Positive

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Matching Mistakes: False Positive

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Matching Mistake: False Negative

- Moving objects: large areas of disagreement

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Matching Mistakes

- Accidental alignment
  - repeated / similar regions
- Failed alignments
  - moving objects / parallax
  - low overlap
  - “feature-less” regions (more variety?)
- No 100% reliable algorithm?

How can we fix these?

- Tune the feature detector
- Tune the feature matcher (cost metric)
- Tune the RANSAC stage (motion model)
- Tune the verification stage
- Use "higher-level" knowledge
  - e.g., typical camera motions

- → Sounds like a big "learning" problem
  - Need a large training/test data set (panoramas)

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Image Blending

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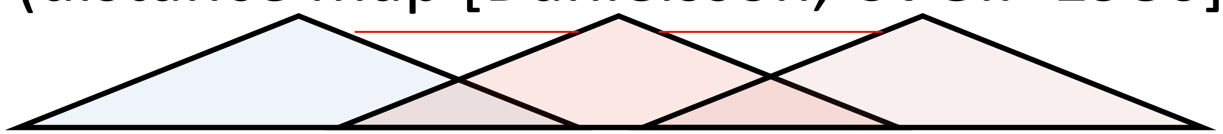
Image feathering

- Weight each image proportional to its distance from the edge
  (distance map [Danielsson, CVGIP 1980]



- 1. Generate *weight map* for each image
- 2. Sum up all of the weights and divide by sum: weights sum up to 1: $w_i' = w_i / (\sum_i w_i)$

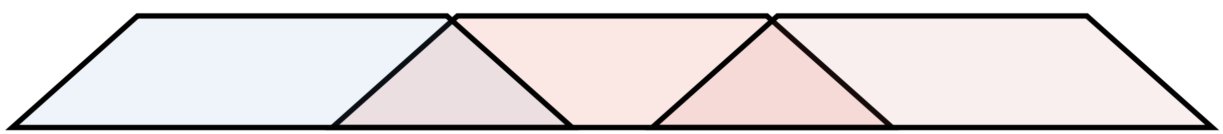


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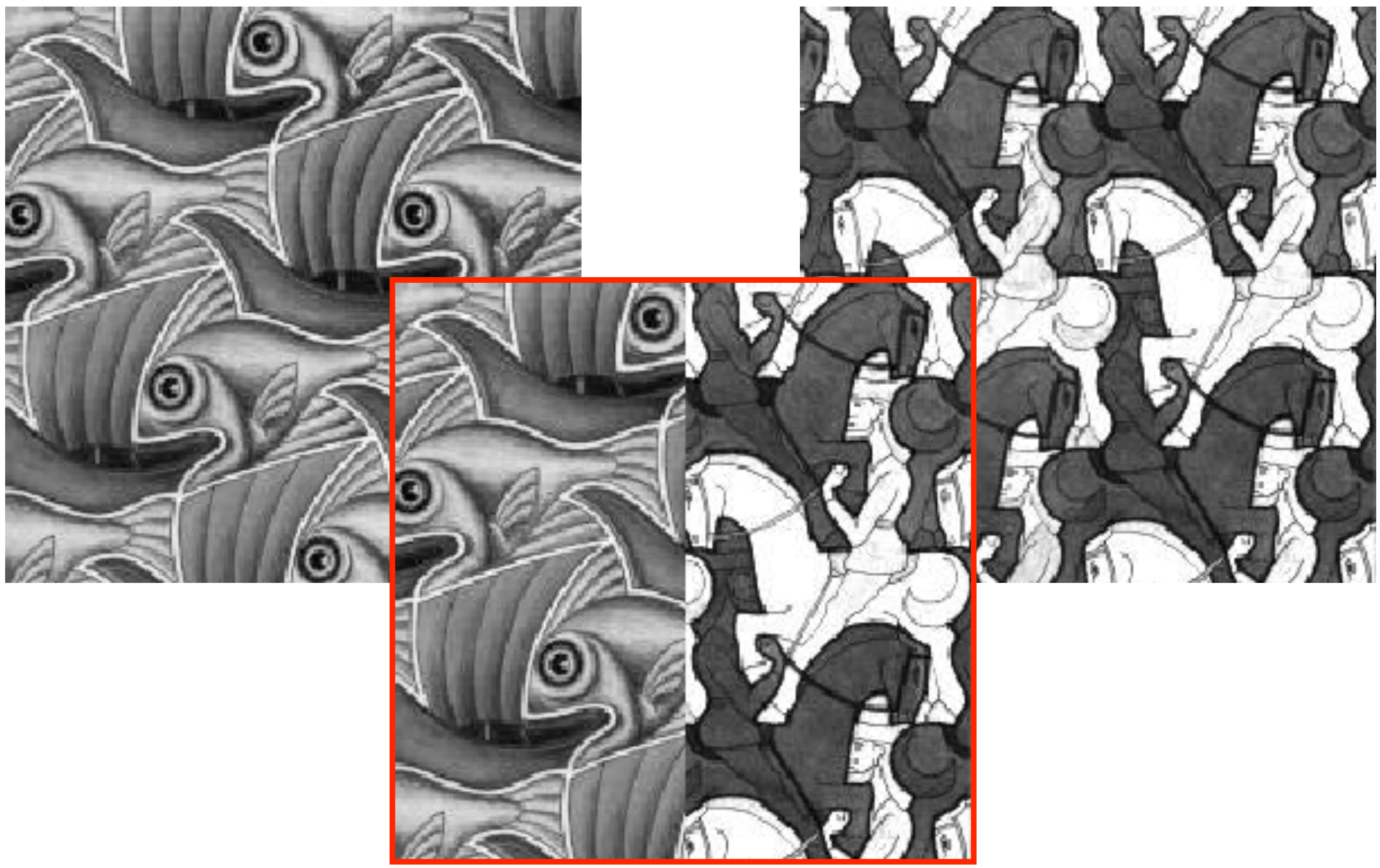
Image Feathering



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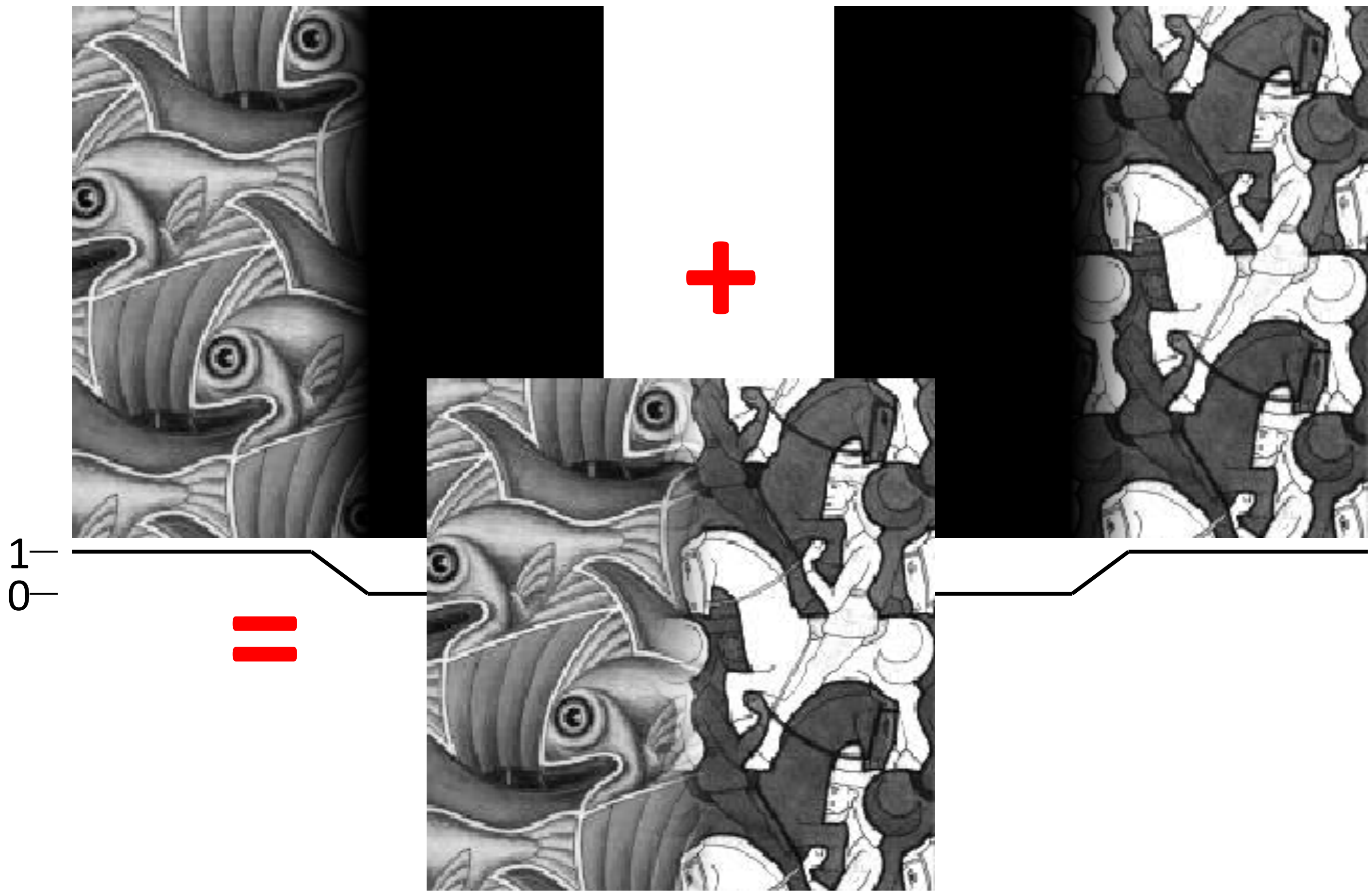
Feathering



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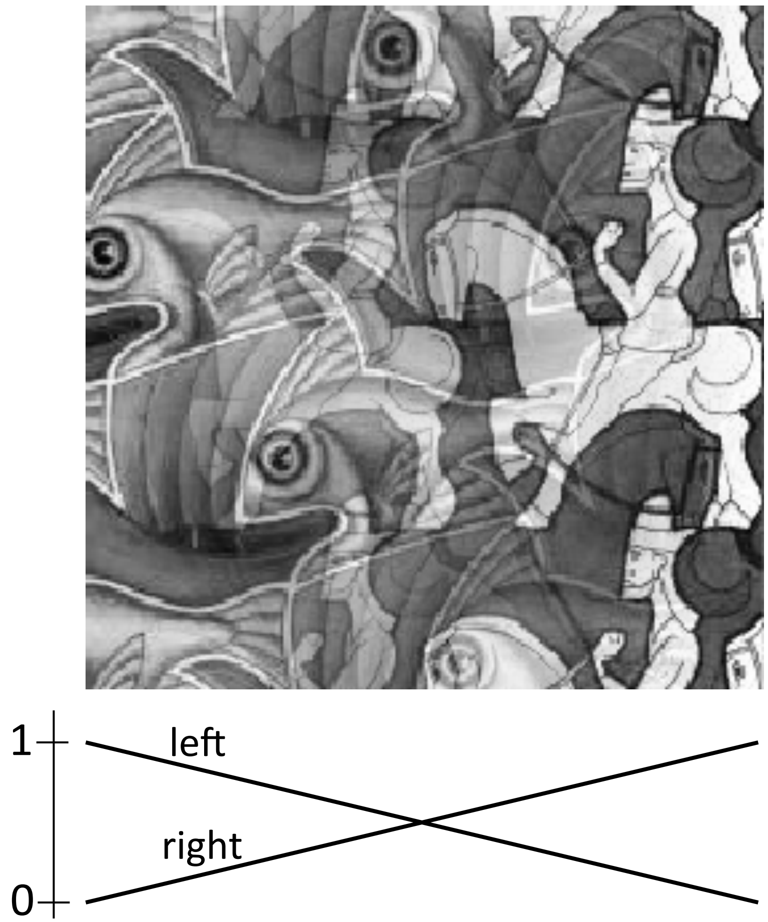
Effect of window size



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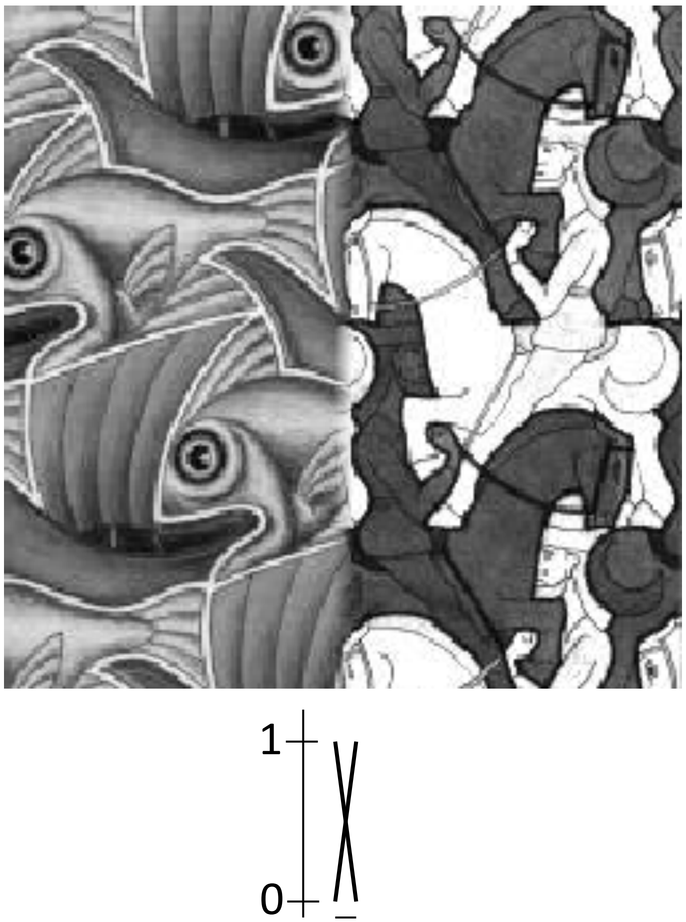
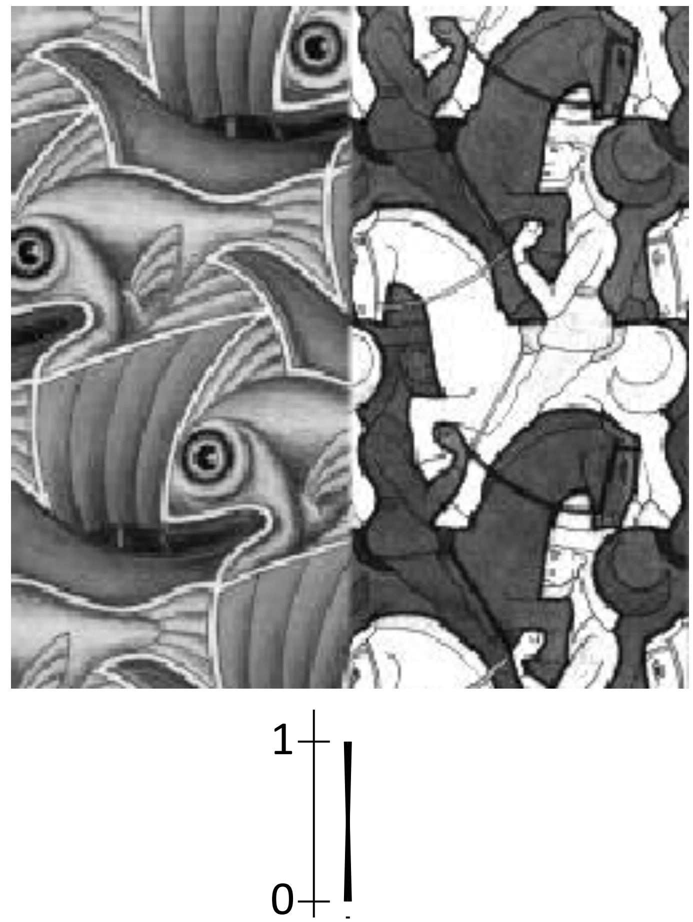
Effect of window size





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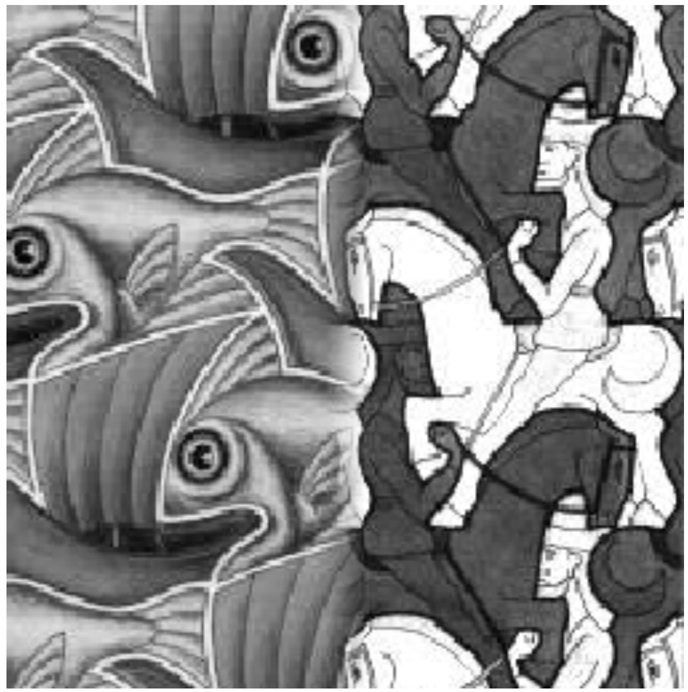
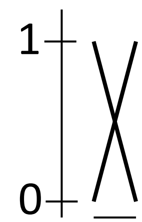
Good window size





"Optimal" window: smooth but not ghosted

- Doesn't always work...

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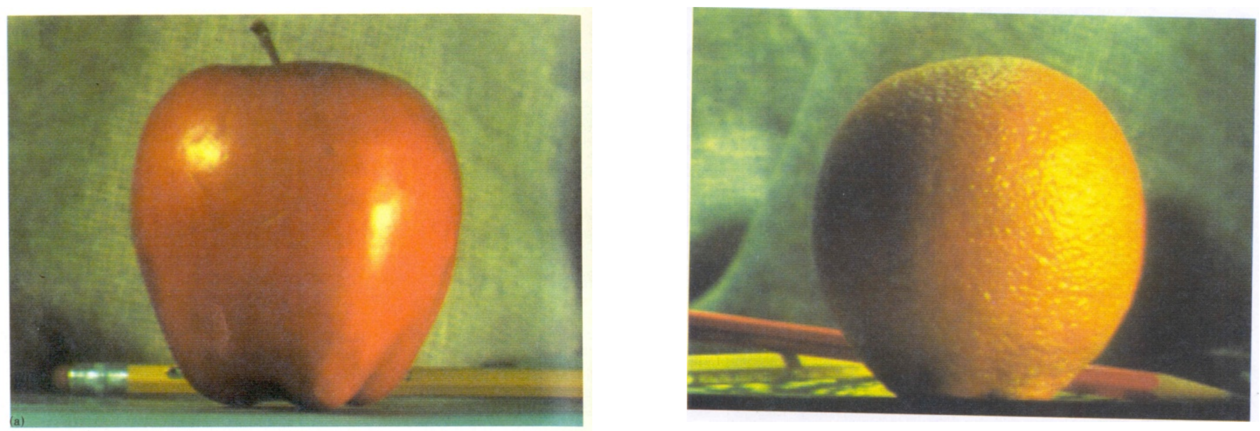
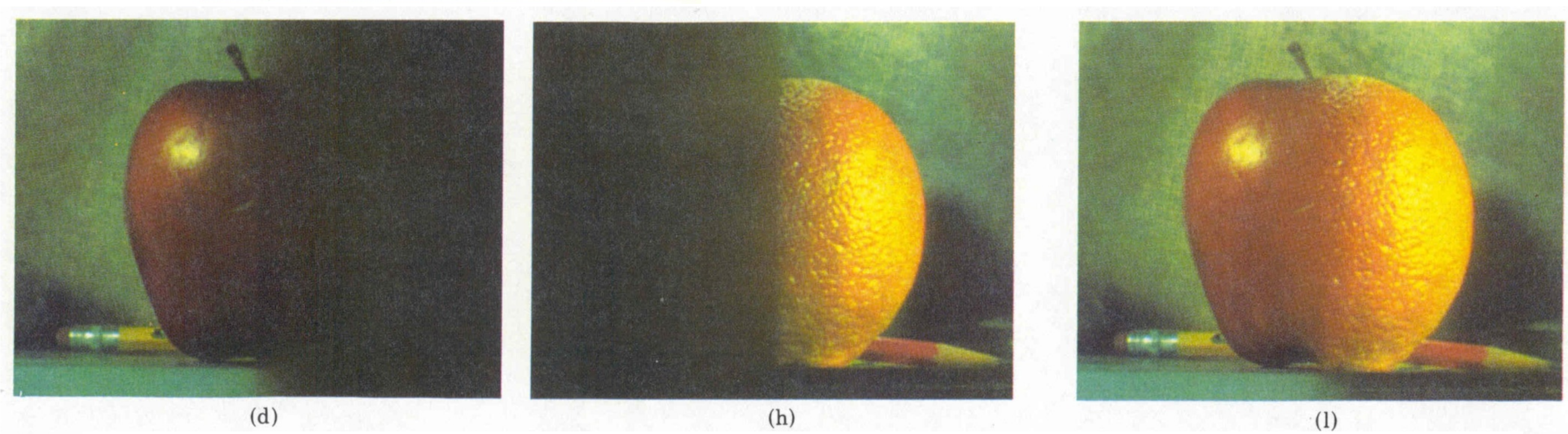
Pyramid Blending





Burt, P. J. and Adelson, E. H.,
A multiresolution spline with applications to image mosaics, ACM Transactions on Graphics, 42(4), October 1983, 217-236.

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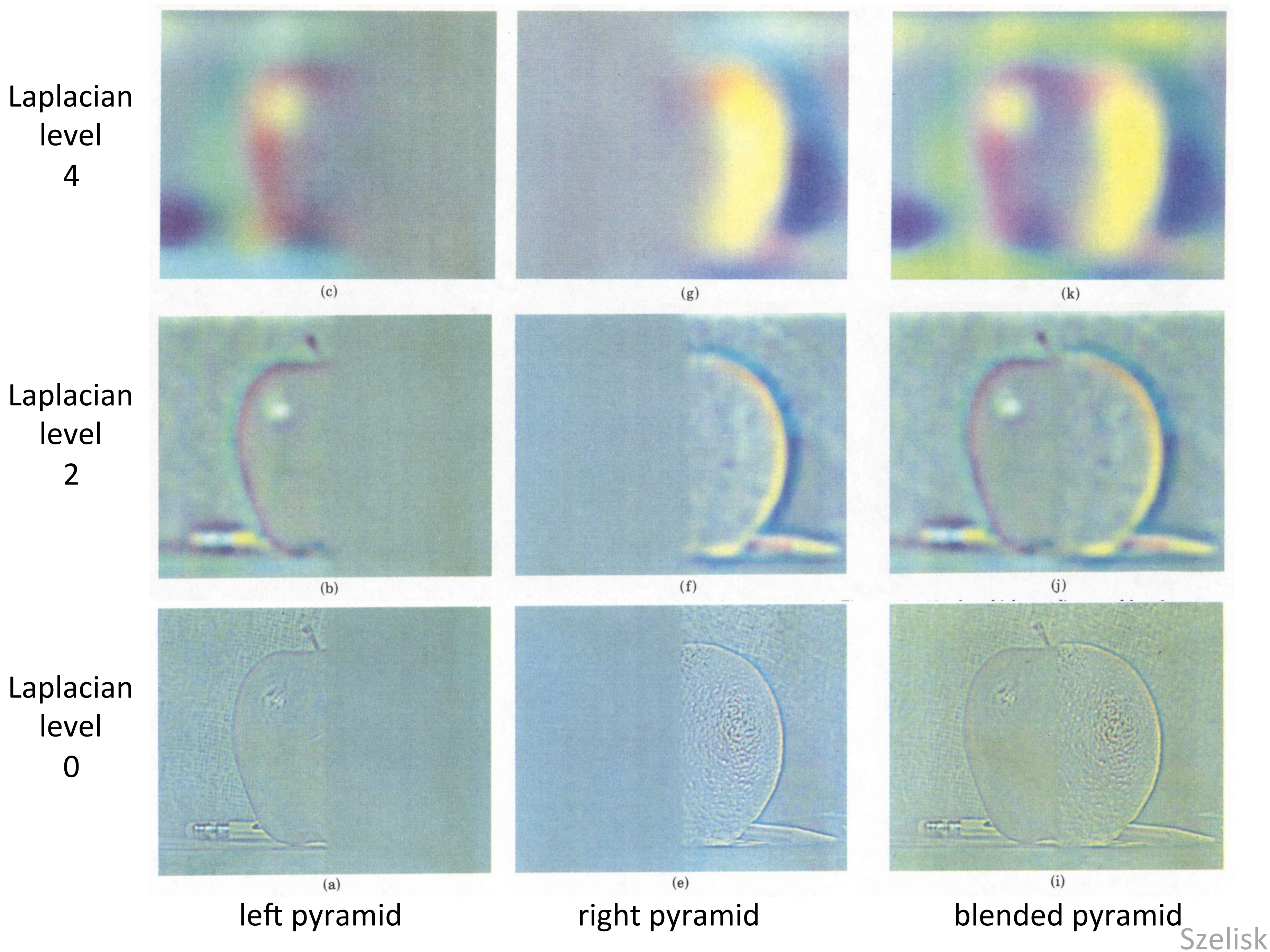
Laplacian level 4
Laplacian level 2
Laplacian level 0
(c)
(g)
(k)
(b)
(f)
(j)
(a)
(e)
(i)
left pyramid
right pyramid
blended pyramid
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Laplacian image blend

1. Compute Laplacian pyramid
2. Compute Gaussian pyramid on *weight* image (can put this in A channel)
3. Blend Laplacians using Gaussian blurred weights
4. Reconstruct the final image

- Q: How do we compute the original weights?
- A: For horizontal panorama, use *mid-lines*
- Q: How about for a general "3D" panorama?

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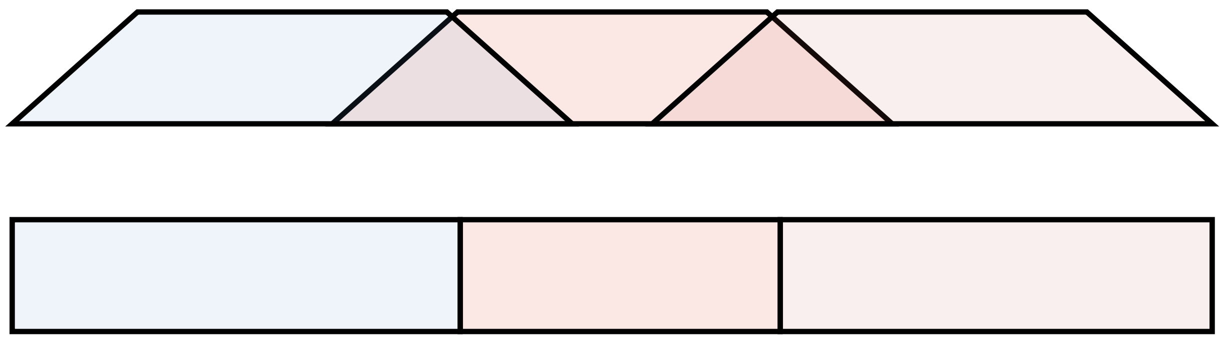
Weight selection (3D panorama)

- Idea: use original feather weights to select *strongest* contributing image



- Can be implemented using L-∞ norm: ($p = 10$)
- $$w_i' = [w_i^p / (\sum_i w_i^p)]^{1/p}$$

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Poisson Image Editing

- Blend the gradients of the two images, then integrate
- For more info: Perez et al, SIGGRAPH 2003

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De-Ghosting

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Local alignment (deghosting)

- Use local optic flow to compensate for small motions [Shum & Szeliski, ICCV'98]

Figure 3: Deghosting a mosaic with motion parallax: (a) with parallax; (b) after single deghosting step (patch size 32); (c) multiple steps (sizes 32, 16 and 8).

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Local alignment (deghosting)

- Use local optic flow to compensate for radial distortion [Shum & Szeliski, ICCV'98]

Figure 4: Deghosting a mosaic with optical distortion: (a) with distortion; (b) after multiple steps.

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Region-based de-ghosting

- Select only one image in *regions-of-difference* using weighted vertex cover [Uyttendaele *et al.*, CVPR'01]

(A) (B)

Figure 5 – (A) Ghosted mosaic. (B) Result of de-ghosting algorithm.

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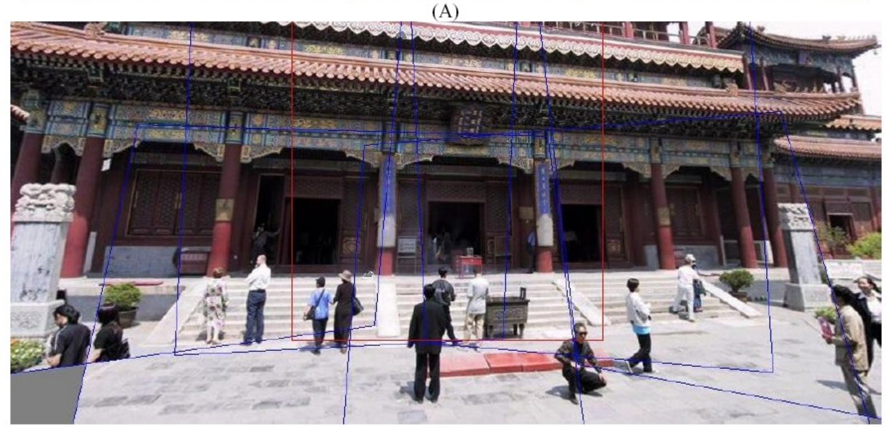
Region-based de-ghosting

- Select only one image in *regions-of-difference* using weighted vertex cover [Uyttendaele *et al.*, CVPR'01]

(A)



(B)

Figure 6 – (A) Ghosted mosaic. (B) Result of de-ghosting algorithm.

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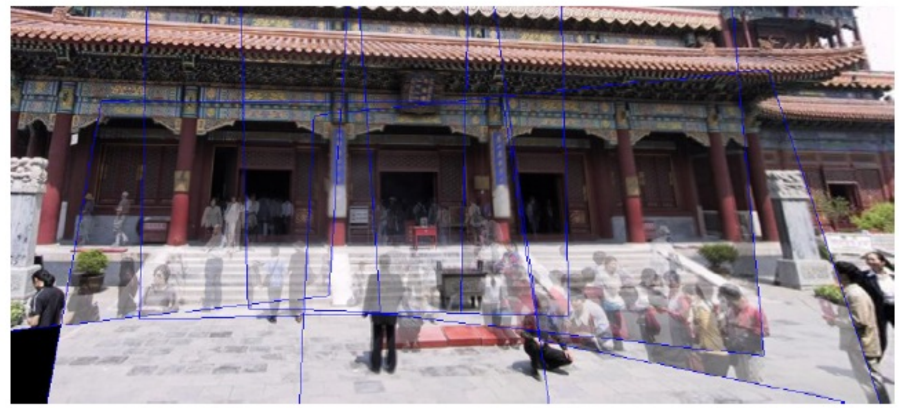
Cutout-based de-ghosting

- Select only one image per output pixel, using spatial continuity
- Blend across seams using gradient continuity ("Poisson blending")



[Agarwala *et al.*, SG'2004]

Cutout-based compositing

- Photomontage [Agarwala *et al.*, SG'2004]
- Interactively blend *different* images:
  group portraits

Figure 1 From a set of five source images (of which four are shown on the left), we quickly create a composite family portrait in which everyone is smiling and looking at the camera (right). We simply flip through the stack and coarsely draw strokes using the *designated source* image objective over the people we wish to add to the composite. The user-applied strokes and computed regions are color-coded by the borders of the source images on the left (middle).

Cutout-based compositing

- Photomontage [Agarwala *et al.*, SG'2004]
- Interactively blend *different* images:
  focus settings

Figure 2 A set of macro photographs of an ant (three of eleven used shown on the left) taken at different focal lengths. We use a global *maximum contrast* image objective to compute the graph-cut composite automatically (top left, with an inset to show detail, and the labeling shown directly below). A small number of remaining artifacts disappear after gradient-domain fusion (top, middle). For comparison we show composites made by Auto-Montage (top, right), by Haeberli's method (bottom, middle), and by Laplacian pyramids (bottom, right). All of these other approaches have artifacts; Haeberli's method creates excessive noise, Auto-Montage fails to attach some hairs to the body, and Laplacian pyramids create halos around some of the hairs.

Cutout-based compositing

- Photomontage [Agarwala *et al.*, SG'2004]
- Interactively blend *different* images:
  people's faces

Figure 6 We use a set of portraits (first row) to mix and match facial features, to either improve a portrait, or create entirely new people. The faces are first hand-aligned, for example, to place all the noses in the same location. In the first two images in the second row, we replace the closed eyes of a portrait with the open eyes of another. The user paints strokes with the *designated source* objective to specify desired features. Next, we create a fictional person by combining three source portraits. Gradient-domain fusion is used to smooth out skin tone differences. Finally, we show two additional mixed portraits.

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Other types of mosaics

- Can mosaic onto *any* surface if you know the geometry
  - See NASA's Visible Earth project for some stunning earth mosaics
    - http://earthobservatory.nasa.gov/Newsroom/BlueMarble/

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Final thought: What is a “panorama”?

- Tracking a subject
- Repeated (best) shots

- Multiple exposures

- “Infer” what photographer wants?

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Slide Credits

- Steve Seitz
- Kristen Grauman
- Alyosha Efros

Next time: Parametric Motion and Optic Flow

- The ‘Direct Motion’ analogue to today…