

Let us prove that $L = \{a^n b^n c^n, n \geq 0\}$ is not a regular language.

For this, let us consider a pumping length of p (i.e., unspecified).

Now for any such p , let us show that we can produce a string s of length longer than p , such that there is no way to decompose s in $s = xyz$, where $|xy| \leq p$ and $|y| > 0$, such that $\forall i \geq 0$, the new string $xy^i z \in L$.

In other words, for any such p , let us show that we can produce a string s of length longer than p , such that for any decomposition of s as $s = xyz$ where $|xy| \leq p$ and $|y| > 0$, $\exists i \geq 0$, such that $xy^i z \notin L$.

Let us take s as $s = a^p b^p c^p$. Decompositions of s such that $|xy| \leq p$ and $|y| \geq 0$ can only be as follows: $s = a^j a^k a^{p-j-k} b^p c^p$ where $x = a^j$, $y = a^k$, and $z = a^{p-j-k} b^p c^p$, with $j + k \leq p$.

As a result, the new string $xy^0 z = a^j a^{p-j-k} b^p c^p$ where $p - k$ is obviously $\neq p$. Similarly, the new string $xy^2 z = a^j (a^k)^2 a^{p-j-k} b^p c^p = a^{p+k} b^p c^p$ where $p + k$ is obviously $\neq p$.

Consequently, we can affirm that, for any decomposition of s that follows the Pumping Lemma conditions ($|xy| \leq p$ and $|y| > 0$), there exists an $i \geq 0$ such that the new string $xy^i z \notin L$.
