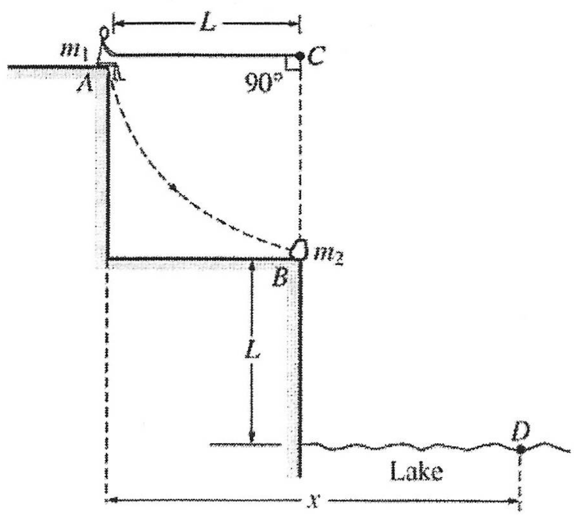
10: Energy and Momentum Combined Problems #2

2. A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that its horizontal and taut, as shown above. The person then drops off the ledge and swings down the rope toward position B on the lower ledge where an object with mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.



a. The speed of the person just before the collision with the object.

$U_g = K$

$m_1 g L = \frac{1}{2} m_1 v^2$

$v = \sqrt{2gL}$

b. The tension in the rope just before the collision.

Circular motion

$F_{net} = ma$

$F_T - F_w = \frac{mv^2}{r}$

$F_T - mg = \frac{m\,2gL}{L}$

$F_T = 3mg$

c. the speed of the person and the object just after the collision.

inelastic Collision

$$m_1 v_i = (m_1 + m_2) v_f$$

$$m_1 \sqrt{2gL} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1}{(m_1 + m_2)} \sqrt{2gL}$$

d. The ratio of the kinetic energy of the person-object system before the collision to the kinetic energy after the collision.

$$\frac{K_i}{K_f} = \frac{\frac{1}{2} m_1 \left(\sqrt{2gL}\right)^2}{\frac{1}{2}(m_1 + m_2)\left[\frac{m_1}{m_1 + m_2}\sqrt{2gL}\right]^2}$$

$$= \frac{m_1 \, 2gL}{m_1 + m_2 \left[\frac{m_1^2}{(m_1 + m_2)^2} 2gL\right]}$$

$$\frac{K_i}{K_f} = \frac{m_1 + m_2}{m_1}$$

e. The magnitude of the total velocity of person just before they land in the water at point D.

$$V_{1f} = \sqrt{2gL}$$

$$V_{yf} = \sqrt{2gL}$$

$$V_f = \sqrt{2gL + 2gL}$$

$$V_f = 2\sqrt{gL}$$

$$\theta = 45^\circ$$