Exercise 3.3.1

i) Indicate all the BCNF violations. Don't forget FD's that follow from the set of given FD's.

ii) Decompose the relations into collections of relations that are in BCNF.

a) R(A,B,C,D) with FD's AB->C, C->D, and D->A

Following algorithm 3.20

1. **Check whether R is in BCNF.**

   Definition: Relation R is in BCNF iff whenever there is a non trivial FD $A_1 A_2 \ldots A_n \rightarrow B_1 B_2 \ldots B_n$ for R $\{ A_1 A_2 \ldots A_n \}$ is a superkey.

   AB->C: The set closure of AB^+ is { A, B, C, D } therefore the left hand side of this FD is a superkey.

   C->D: The set closure of C^+ is { C, D, A } therefore the left hand side is not a superkey. **BCNF Violation**

   D-> A: The set closure of D^+ is { D, A } therefore the left hand side is not a superkey. **BCNF Violation**

2. **If there are BCNF violations, let one be X->Y. Use Algorithm 3.7 to compute X^+. Choose $R_1 = X^+$ as one relation schema and let R_2 have attributes X and those attributes of R that are not in X^+.**

   The BCNF violation we will use is C->D. C^+ is { C, D, A }
   R_1 = { C, D, A } and R_2 = { C, B }

3. **Use Algorithm 3.12 to compute the sets of FD's for R_1 and R_2; let these be S_1 and S_2, respectively.**

   S_1 = { C->D, C->A, D->A } and $S_{2\,=}$ { empty }

4. **Recursively decompose R_1 and R_2 using this algorithm. Return the union of the results of these decompositions.**
   R_2 is in BCNF because B,C is the key R_2={**C,B**}.

   Now check R_1:
   C->D, C->A: The set closure of C^+ is { C, D, A }
   D->A: The set closure of D^+ is { D, A } therefore the left hand side is not a superkey. **BCNF Violation**

   Step 2:

$R_{1.1}$ = { D, A } and $R_{2.1}$ = { D, C }

Recursion ends because each of these sets now contains only two attributes. Return the union of all R sets.

Answer:
$R_{1.1}$ = { D, A }
$R_{2.1}$ = { D, C }
R_2 = { C, B }

b) R(A,B,C,D) with FD's B->C and B->D

Following algorithm 3.20

1. **Check whether R is in BCNF.**
Relation R is in BCNF iff whenever there is a non trivial FD A_1 A_2 … A_n -> B_1 B_2 … B_n for R { A_1 A_2 … A_n }is a superkey.

B->C and B->D: The set closure of B^+ is { B, C, D } therefore neither of the left side of these FD's are superkeys. **BCNF Violations**

2. **If there are BCNF violations, let one be X->Y. Use Algorithm 3.7 to compute X^+. Choose $R_1 = X^+$ as one relation schema and let R_2 have attributes X and those attributes of R that are not in X^+.**

The BCNF violation we will use is B->C. B^+ is { B, C, D }
R_1 = { B, C, D }and R_2 = { B, A }

3. **Use Algorithm 3.12 to compute the sets of FD's for R_1 and R_2; let these be S_1 and S_2, respectively.**

S_1 = { B->C, B->D } and S_2 = { empty }

4. **Recursively decomposes R_1 and R_2 using this algorithm. Return the union of the results of these decompositions.**

R_2 is in BCNF because it only has two attributes and is the key.
Now check R_1:
B->C, B->D: The set closure of B^+ is { B, C, D } therefore the left hand sides of these FD's are superkeys.

Answer:
R_1 = { B, C, D }
R_2 = { B, A }

Exercise 3.5.1

i) Indicate all the 3NF violations.
ii) Decompose the relations, as necessary, into collections of relations that are in 3NF.

Relation R is in 3NF if whenever there is a non trivial FD $A_1 A_2 \ldots A_n$ -> $B_1 B_2 \ldots B_n$ for R either { $A_1 A_2 \ldots A_n$ }is a superkey, or B is a member of some key.

a) R(A,B,C,D) with FD's AB->C, C->D, and D->A

Find all the keys in this relation by finding the set closures of the combinations of attributes.

$A^+ = \{ A \}$	**$AB^+ = \{ A,B,C,D \}$**	**$BC^+ = \{ B,C,D,A \}$**	$CD^+ = \{ C,D,A \}$
$B^+ = \{ B \}$	$AC^+ = \{ A,C,D \}$	**$BD^+ = \{ B,D,A,C \}$**	
$C^+ = \{ C,D,A \}$	$AD^+ = \{ A,D \}$		
$D^+ = \{ D,A \}$			

Check all the FD's for a violation of the 3NF condition.

AB->C: AB is a superkey, C is prime because it is a member of key BC.
C->D: D is prime because it is a member of key BD.
D->A: A is prime because it is a member of key AB.

Answer:
No 3NF violations. No decomposition needed.

b) R(A,B,C,D) with FD's B->C and B->D

Find all the keys in this relation by finding the set closures of the combinations of attributes.

$A^+ = \{ A \}$	**$AB^+ = \{ A,B,C,D \}$**	$BC^+ = \{ B,C,D \}$	$CD^+ = \{ C,D \}$
$B^+ = \{ B,C,D \}$	$AC^+ = \{ A,C \}$	$BD^+ = \{ B,D \}$	
$C^+ = \{ C \}$	$AD^+ = \{ A,D \}$		
$D^+ = \{ D \}$			

Check all the FD's for a violation of the 3NF condition.

B->C: B is not a superkey, C is not prime. **3NF Violation**
B->D: B is not a superkey. D is not prime. **3NF Violation**

Decompose on violation B->C.
$R_1 = \{ B,C \}$ and $R_2 = \{ B,A,D \}$

Now recursively check for 3NF violations on R_2

Find all the keys in R_2 by find the set of closures of the combinations of attributes that exist in R_2. The only FD that exists for R_2 is B->D.

$B^+ = \{ B,D \}$
$A^+ = \{ A \}$
$D^+ = \{ D \}$

<u>$BA^+ = \{ B,A,D \}$</u>
$BD^+ = \{ B,D \}$

$AD^+ = \{ A,D \}$

Again check all FD's for a violation of the 3NF condition. In this case only one FD exists.

B->D: B is not a superkey and D is not prime. **3NF Violation**

Decompose into two relations:
$R_{2.1} = \{ B,D \}$ and $R_{2.2} = \{ B,A \}$

Answer:
$R_1 = \{ B,C \}$
$R_{2.1} = \{ B,D \}$
$R_{2.2} = \{ B,A \}$

Now your turn!

C) **R(A,B,C,D) with FD's AB->C, BC-> D, CD->A, and AD->B**
D) **R(A,B,C,D) with FD's A->B,B->C,C->D, and D->A.**
E) **R(A,B,C,D,E) with FD's AB->C, DE->C and B->D**
F) **R(A,B,C,D,E) with FD's AB->C,C->D,D->B, and D->E.**