Restate each premise and the conclusion of the following argument using logical notation. Give a list of each basic logical variable you define. Then determine whether the argument is valid. State each of your steps using logical notation, and explain each step carefully.

1. She is a Math Major or a Computer Science Major.
2. If she does not know discrete math, she is not a Math Major.
3. If she knows discrete math, she is smart.
4. She is not a Computer Science Major.
5. Therefore, she is smart.

Solution:

She is a Math Major	m
She is a Computer Science Major	c
She knows discrete math	k
She is smart	s
1. She is a Math Major or a Computer Science Major.	$m \vee c$
2. If she does not know discrete math, she is not a Math Major.	$\neg k \to \neg m$
3. If she knows discrete math, she is smart.	$k \to s$
4. She is not a Computer Science Major.	$\neg c$
5. Therefore, she is smart.	s

Direct proof: starting with $\neg c$ and $m \vee c$ we can conclude m. From m and the contrapositive of $\neg k \to \neg m$ we can conclude k. From k and $k \to s$ we can conclude s.

Thus, the argument is valid.

- (part a) Determine whether the following propositions are true or false:

  1. $1 + 1 = 3$ if and only if $2 + 2 = 3$
     Solution: TRUE
  2. If it is raining, then it is raining.
     Solution: TRUE
  3. If $1 < 0$ then $3 = 4$.
     Solution: TRUE
  4. If $1 + 1 = 2$ or $1 + 1 = 3$, then $2 + 2 = 3$ and $2 + 2 = 4$
     Solution: FALSE.

- (part b) Use logical equivalences to show that that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

  Solution: You don't need to state the reasons. This is done below just to explain to you what is happening at each step.

$p \leftrightarrow q$	
$(p \to q) \wedge (q \to p)$	defn of $\leftrightarrow$
$[\neg p \vee q] \wedge [\neg q \vee p]$	defn of $\to$
$[(\neg p \vee q) \wedge \neg q] \vee [(\neg p \vee q) \wedge p]$	distribution
$[(\neg p \wedge \neg q) \vee (q \wedge \neg q)] \vee [(\neg p \wedge p) \vee (q \wedge p)]$	distribution
$[(\neg p \wedge \neg q) \vee F] \vee [F \vee (p \wedge q)]$	
$(\neg p \wedge \neg q) \vee (p \wedge q)$	

- (part a) Write out the truth table for the proposition $s \equiv \neg(r \rightarrow \neg q) \vee (p \wedge \neg r)$.

  Solution: intermediate steps are helpful as you do the problem, but they are not required.

p	q	r	$\neg q$	$\neg r$	$r \rightarrow \neg q$	$\neg(r \rightarrow \neg q)$	$p \wedge \neg r$	$\neg(r \rightarrow \neg q) \vee (p \wedge \neg r)$
T	T	T	F	F	F	T	F	T
T	T	F	F	T	T	F	T	T
T	F	T	T	F	T	F	F	F
T	F	F	T	T	T	F	T	T
F	T	T	F	F	F	T	F	T
F	T	F	F	T	T	F	F	F
F	F	T	T	F	T	F	F	F
F	F	F	T	T	T	F	F	F

- (part b) Suppose you know that q and r in part (a) are both true. Can you conclude anything about the proposition s? Prove your result using two methods: with your truth table above, and by using a logical argument (or logical equivalences).

  Solution: Yes, you can conclude that s is true. This correponds to row 1 and 5 of the truth table, above. For the logical equivalences:

$$
\begin{array}{c|l}
\neg(r \rightarrow \neg q) \vee (p \wedge \neg r) & \\
\neg(T \rightarrow \neg T) \vee (p \wedge \neg T) & \text{plug in } q \text{ and } r \\
\neg(T \rightarrow F) \vee (p \wedge F) & \\
\neg F \vee F & \\
T \vee F & \\
T &
\end{array}
$$

- (part a) Find all possible counterexamples, if any, to these universally quantified statements, where the domain for all variables consists of all real numbers.

  1. $\forall x(x^2 \neq x)$: **Solution:** $x = 1$ and $x = 0$ are the only counter-examples.
  2. $\forall x(x^2 \neq 2)$: **Solution:** $x = \sqrt{2}$ and $x = -\sqrt{2}$ are the only counter-examples.
  3. $\forall x(|x| > 0)$: **Solution:** $x = 0$ is the only counter-example.

- (part b) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. **For the solution: only true / false required to be stated. No reason need be given. I give the reason for clarity.**

  1. $\forall x \exists y(x^2 = y)$: **Solution:** this is true. For any given x, we can select the specific value y such that $y = x^2$.
  2. $\forall x \exists y(x = y^2)$: **Solution:** this is false, since no such y can exist if $x < 0$.
  3. $\exists x \forall y(xy = 0)$: **Solution:** this is true. We can let $x = 0$.
  4. $\exists x \forall y((y \neq 0) \rightarrow (xy = 1))$: **Solution:** this is false. It attempts to say that there is an x that is the reciprocal of all nonzero values y, which does not exist.