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Physics 356F: Problem Set #1
assigned 18 September 2013
(due the week of October 1, at the start of your tutorial section)

1. An arbitrary polarization state of a photon may be written $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle \equiv \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$.

   Since an overall phase has no measurable effect, we can take $\alpha = a$ and $\beta = be^{i\phi}$, where a, b, and ϕ (and hence α) are all real.

   (a) Suppose that the probability P_R of this photon being transmitted by a right-circular polarizer is equal to the probability P_L of it being transmitted by a left-circular polarizer:

   $$\begin{aligned} |\langle R|\psi\rangle|^2 &= |\langle L|\psi\rangle|^2 \text{ , or} \\ \left| \begin{pmatrix} 1/\sqrt{2} & -i/\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ be^{i\phi} \end{pmatrix} \right|^2 &= \left| \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ be^{i\phi} \end{pmatrix} \right|^2 \end{aligned}$$

   What can you conclude about a, b, and ϕ?

   (b) Suppose *additionally* that the probability P_{45} of this photon being transmitted by a 45° polarizer is equal to the probability P_{-45} of it being transmitted by a −45° polarizer:

   $$\begin{aligned} |\langle 45°|\psi\rangle|^2 &= |\langle -45°|\psi\rangle|^2 \text{ , or} \\ \left| \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ be^{i\phi} \end{pmatrix} \right|^2 &= \left| \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} a \\ be^{i\phi} \end{pmatrix} \right|^2 \end{aligned}$$

   What more can you conclude about a, b, and ϕ?

   (c) What can you say about the polarization of the photon?

2. Problem 1.7 from the textbook

3. Problem 1.8 from the textbook

4. Problem 2.9 from the textbook

5. Problem 2.10 from the textbook

6. Suppose an interferometer has two interfering paths to a given final event, and the probability of taking path 1 is P_1, while the probability of taking path 2 is P_2. Assume that the two paths are indistinguishable.

   (a) What can you say in general about the probability *amplitudes* for path 1 and path 2 (making no assumptions about phase)?

   (b) Show that the probability $P_{1\text{or}2}$ of the final event occuring depends on P_1 and P_2 and on the *relative* phase of the two amplitudes but not on their *absolute* phases.

   (c) Find the *average* value of $P_{1\text{or}2}(\Delta\phi)$, averaged over all possible values of the phase difference $\Delta\phi$:

$$\overline{P_{1\text{or}2}(\Delta\phi)} = \int_0^{2\pi} \frac{d\Delta\phi}{2\pi} P_{1\text{or}2}(\Delta\phi) \ . \qquad (1)$$

7. Consider a Mach-Zehnder interferometer as described in class, but with beam-splitters whose reflectivities are 36% and whose transmissivities are 64% (instead of the 50-50 case we discussed).

   (a) What are the *probability amplitudes* for reflection and transmission at these beam splitters?

   (b) Suppose that the path-length difference between the two arms of the interferometer is some fixed ΔL. Calculate the probability of reaching output port 1 (which can be reached if there is one reflection and one transmission, in either order), as a function of the wavelength λ.

   (c) Calculate the probability of reaching output port 2 (which requires either two transmissions or two reflections), as a function of λ.

   (d) Sketch P_1 and P_2 over the range of wavelengths from $\Delta L/4$ to ΔL, labelling the relevant points on the sketches (i.e., the positions and values of the minima and maxima).

8. Suppose we have constructed a *three*-path interferometer, i.e., a system where a photon can take any of three different paths to reach our detector. The probability amplitude of this detection event occurring via path 1 is $\frac{1}{3}e^{i\phi_1}$; via path 2 it is $\frac{1}{3}e^{i\phi_2}$; and via path 3 it is $\frac{1}{3}e^{i\phi_3}$.
   Further suppose that some evil spy has measured whether or not our photon takes path 3 (e.g., by inserting a bomb which would be triggered by path 3, although without absorbing the photon). On the other hand, there is no possible way for any one to tell whether path 1 or path 2 is followed.
   What is the probability of us detecting the photon, as a function of ϕ_1, ϕ_2, and ϕ_3? Sketch this as a function of the relevant phase variable(s).