Prof. Aephraim Steinberg

Physics 356F: Problem Set #2 assigned 25 October 2013 (due the week of 5 November 2013 at the start of your tutorial)

1. problem 6.17
2. problem 6.19
3. (a) Show from the definition of the commutator that $[AB, C] = A[B, C] + [A, C]B$.
   (b) Using the result in a and the similar identity for $[A, BC]$, evaluate $[P^2, X^2]$.
   (c) To check this result, evaluate the commutator directly in the position basis by calculating $P^2X^2\psi(x) - X^2P^2\psi(x)$ for an arbitrary function $\psi(x)$.
4. Consider a state $\psi(x) = e^{ikx} + e^{-ikx}$.
   (a) Find the formula for $\Pr(p')$, the probability of finding this particle to have momentum p'. Note that the state as written is not normalized, so you need to explicitly normalize the probability distribution. Also note that this is an easy question (i.e., don't start making tortured loops just yet).
   (b) What are $\langle P \rangle$ and ΔP? (Note that this should also be easy.)
   (c) Assuming that this is a free particle, what are $\langle \mathcal{H} \rangle$ and ΔE, the mean energy and the energy uncertainty?
   (d) Is this an eigenstate of $\mathcal{H}$ or not? What about P?
   (e) Should this be surprising, given your knowledge of $[\mathcal{H}, P]$? Why or why not? If it is at first surprising, why shouldn't we be overly disturbed?
5. Suppose that a particle goes through two slits and finds itself in the state $|\text{slit1}\rangle + |\text{slit2}\rangle$ (I will not bother writing out the normalization). An evil spy attempts to measure which slit the particle traverses, but he is not entirely successful. He succeeds in entangling the particle with his measuring device, creating a final state $|\text{slit1}\rangle|A\rangle + |\text{slit2}\rangle|B\rangle$, where A and B are the two possible final states of his device.

(a) The probability $\Pr(x)$ of finding the particle at some position x on a screen can be calculated by evaluating the expectation value of the projection operator $|x\rangle\langle x|$. Let us make the simplification that $|x\rangle$ can be approximated by $|\text{slit1}\rangle + |\text{slit2}\rangle e^{i\phi}$, where ϕ serves as a reparametrization of x. The visibility V of an interference fringe is defined as (max-min)/(max+min), where "max" and "min" refer to the maxima and minima of $\Pr(x)$.
Find the visibility as a function of $\langle A|B\rangle$.

(b) If the spy had been entirely successful in making his measurement, what would that have implied about $|A\rangle$ and $|B\rangle$ and why? Using this result in conjunction with part a, what do you conclude?