Prof. Aephraim Steinberg

Physics 356F: Problem Set #5
assigned 20 November 2013
(to be handed in at the start of lecture or at the start of tutorial on Tuesday 3 December)

1. This problem concerns the various states of angular momentum of a system made up of two spin-1 particles. Recall that $J_i = J_{1i} + J_{2i}$, where $i = x, y, z$ and 1 and 2 label the individual particles. As in appendix B, our goal will be to expand states $|J; m\rangle$ of the composite system in terms of superpositions of two-particle states, i.e., $\Sigma c_{m_1,m_2}|J_1 = 1; m_1\rangle|J_2 = 1; m_2\rangle$. **Remember to show all your work.**

   (a) What are the possible values of J (the total angular momentum quantum number) for the composite system (we follow the conventional definition of the quantum number $\mathbf{J}^2|\Psi\rangle = J(J+1)\hbar^2|\Psi\rangle$)?

   (b) If the composite system has $m = 2$ (defined, as usual, by $J_z|\Psi\rangle = m\hbar|\Psi\rangle$), what can you say about J for the composite system? What about m for the individual spin-1 particles?

   (c) Write the state $|J = 2, m = 2\rangle$ in terms of the single-particle states.
   Use the lowering operator $J_- = J_{1-} + J_{2-}$ (see equation 3.122 and the surrounding discussion) to find $|J = 2, m = 1\rangle$.

   (d) Since $|J = 1, m = 1\rangle$ can be distinguished from $|J = 2, m = 1\rangle$ by measuring J, they must be orthogonal. Given your knowledge of which states of the individual particles could yield $m = 1$, and this orthogonality relation, find an expression for $|J = 1, m = 1\rangle$.

   (e) Use the lowering operator on the results of 1c and 1d to find expressions for $|J = 2, m = 0\rangle$ and $|J = 1, m = 0\rangle$.

   (f) Given the three states of the pair of spin-1's which can combine to form $m = 0$, and the knowledge that $|J = 0, m = 0\rangle$ must be orthogonal to the two states you found in 1e, determine what the expansion must be for $|J = 0, m = 0\rangle$.

   (g) Using the spin-1 operators defined in equation 3.28, calculate $J_x|J = 0, m = 0\rangle$ explicitly. Does this agree with what you expect given your knowledge of $\mathbf{J}^2$? Explain.

2. Problem 10.5

NOTE: To review for the final, you should be sure to go over all of the homework solutions from the term and be sure you understand how to approach all the problems we have done. You should also practice other problems from the textbook. For the current material, I recommend in particular looking at problems 9.8, 9.12, and 9.16; and 10.2, 10.3, 10.7, 10.13.