Math 140

Lecture 1

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with modifications by T. Milev

University of Massachusetts Boston

September 4-6, 2013

Outline

1. Ways to Represent a Function
   - The Definition of a Function
   - The Vertical Line Test
   - Piecewise Defined Functions
   - Symmetry
   - Increasing and Decreasing Functions
   - A Note on Domains of Functions

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   - The Definition of a Function
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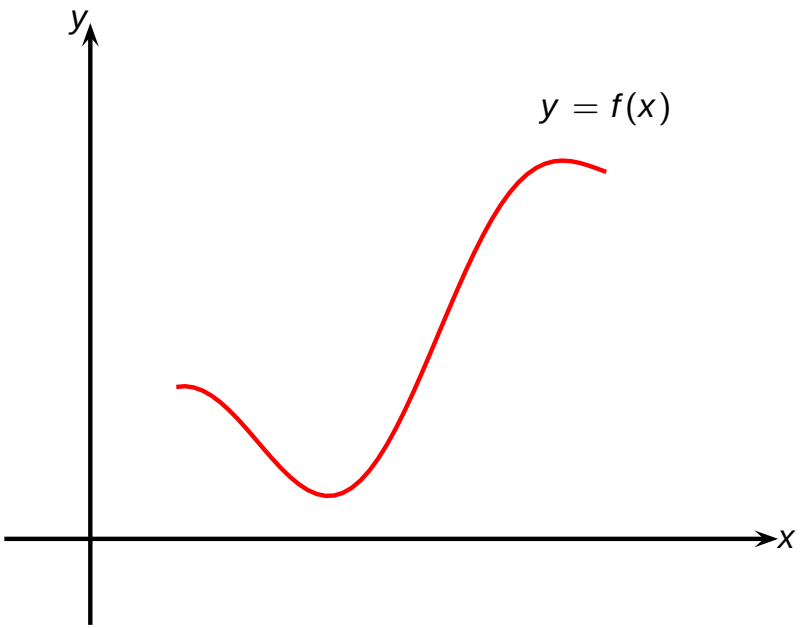
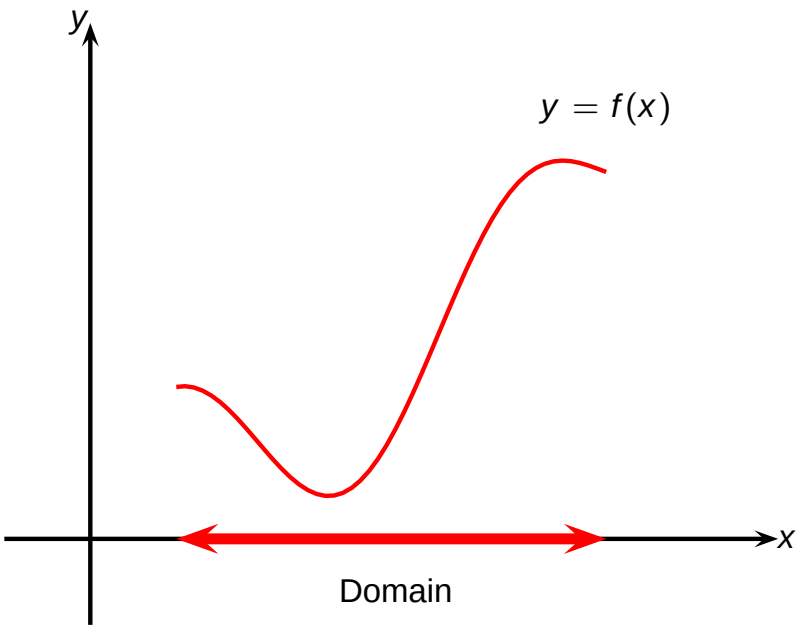
   - A Note on Domains of Functions
2. A Catalog of Essential Functions
   - Linear Functions



Definition (Function)

A function f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.



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Definition (Domain)

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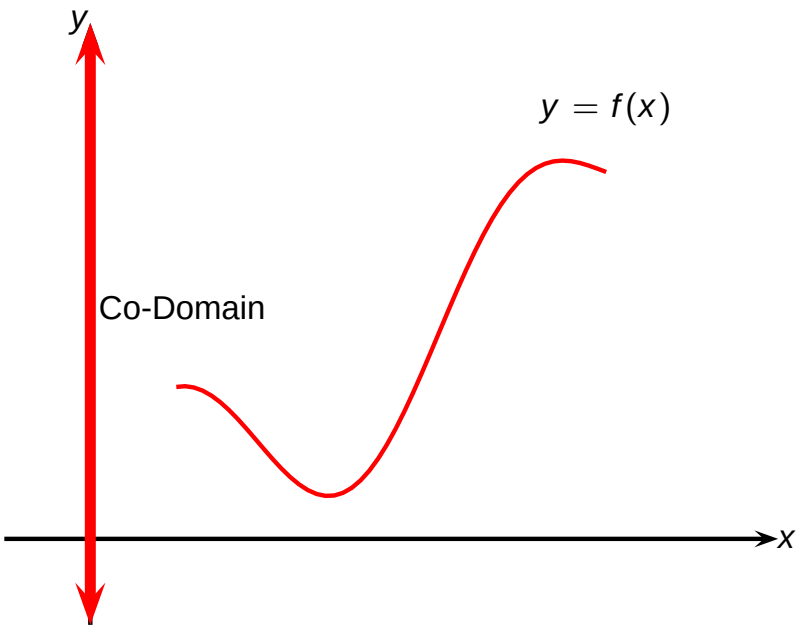

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Definition (Co-domain)

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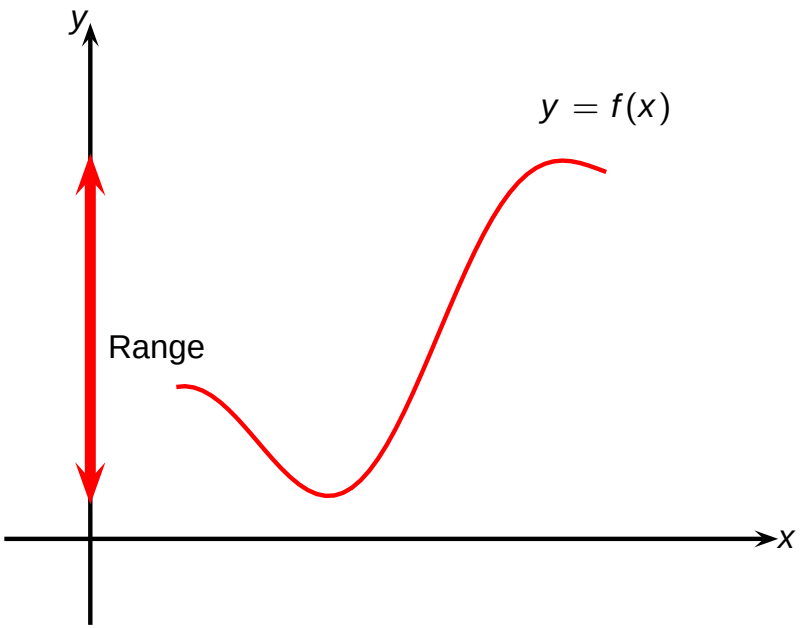

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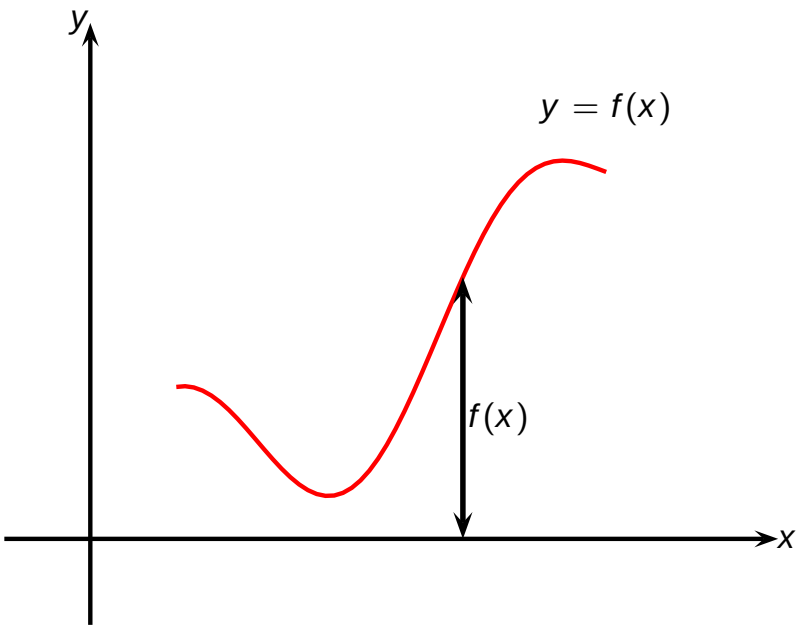
Definition (Range)

The set of all possible values taken by $f(x)$ as the element x runs over elements of D is called the range of f.



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Definition (Value of f at x)

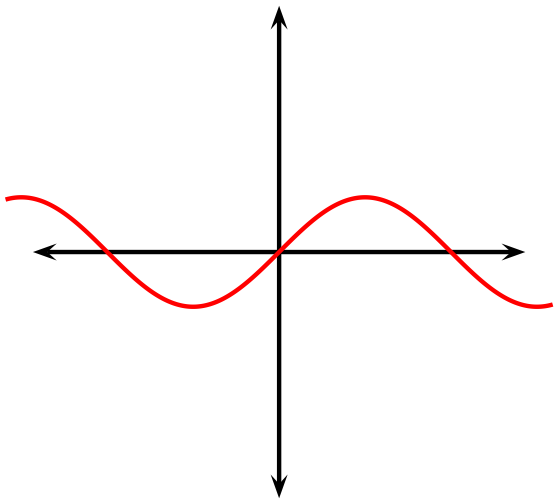
The number $f(x)$ is called the value of f at x, and is read "f of x."

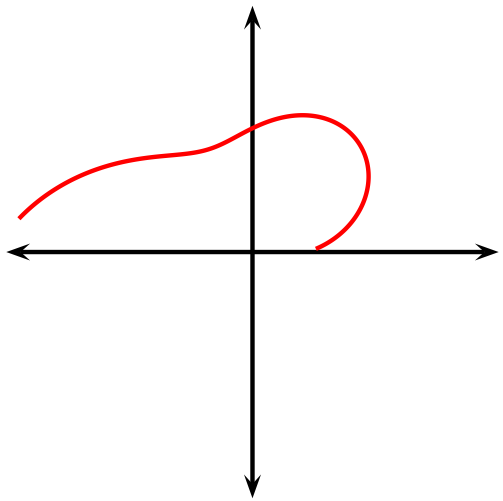
The Vertical Line Test

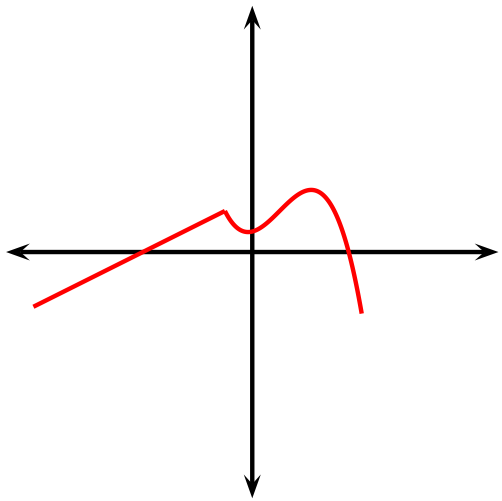
Question: How can we tell if a curve is the graph of a function or not?
Answer: Use the vertical line test.

The Vertical Line Test.

A curve is the graph of a function if and only if no vertical line intersects it more than once. □







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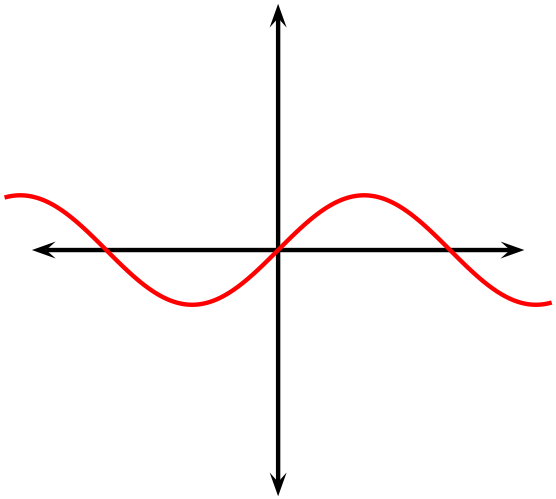

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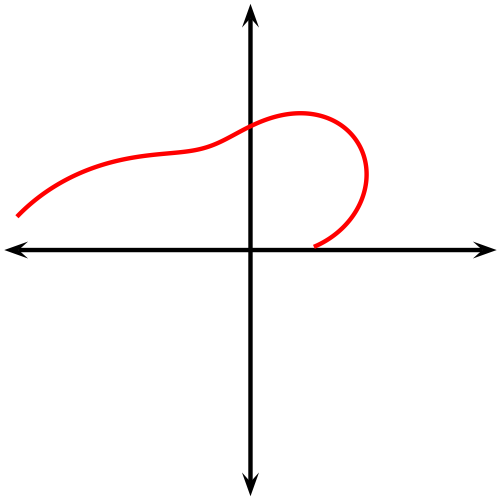

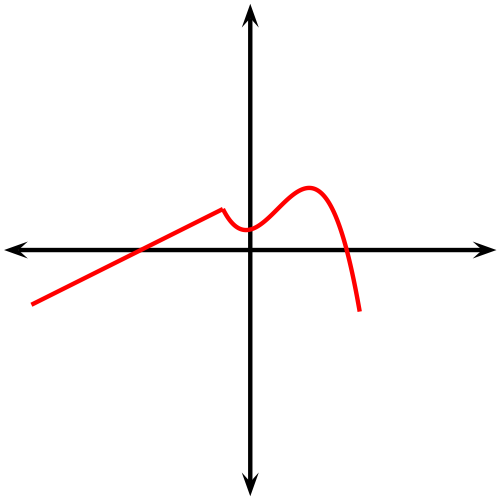

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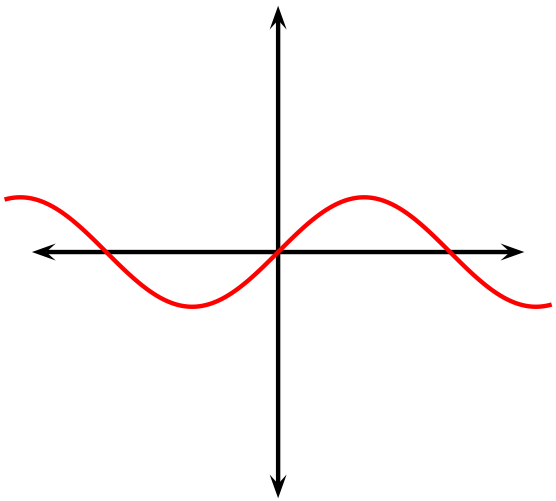

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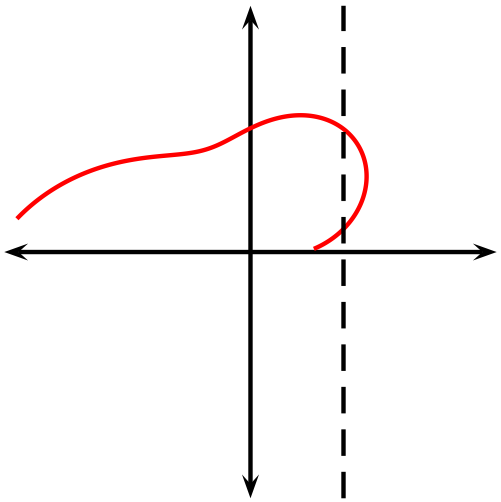

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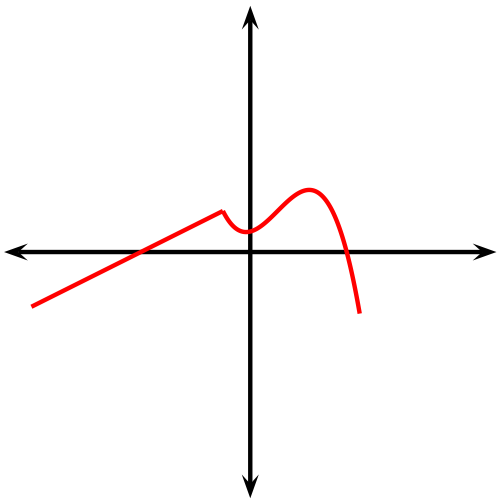

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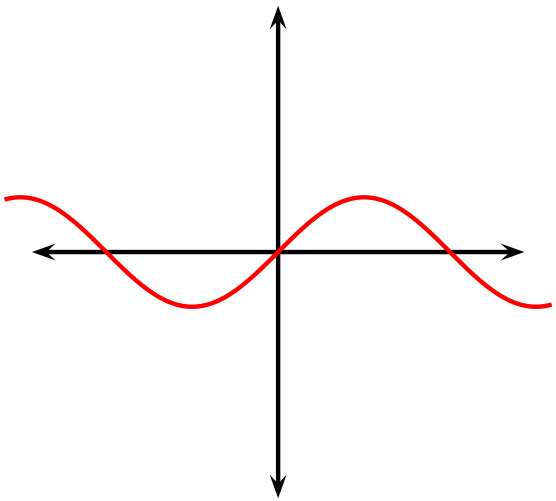

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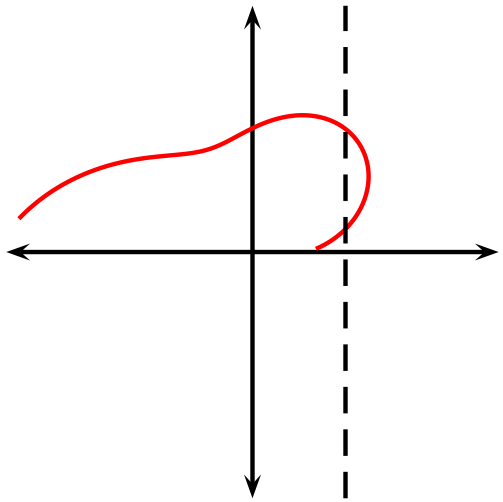

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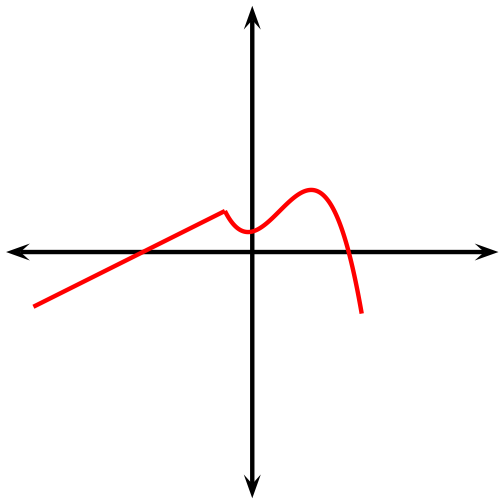

Function

Piecewise Defined Functions

Definition (Piecewise Defined Function)

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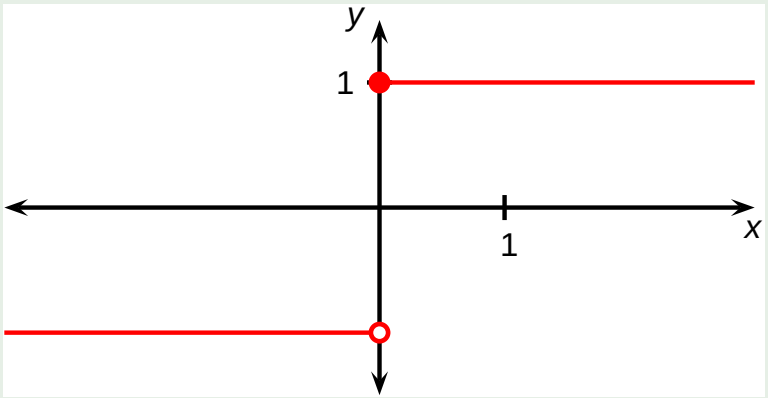
Example



$$f(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ -1 & \text{if} \quad x < 0 \end{cases}$$

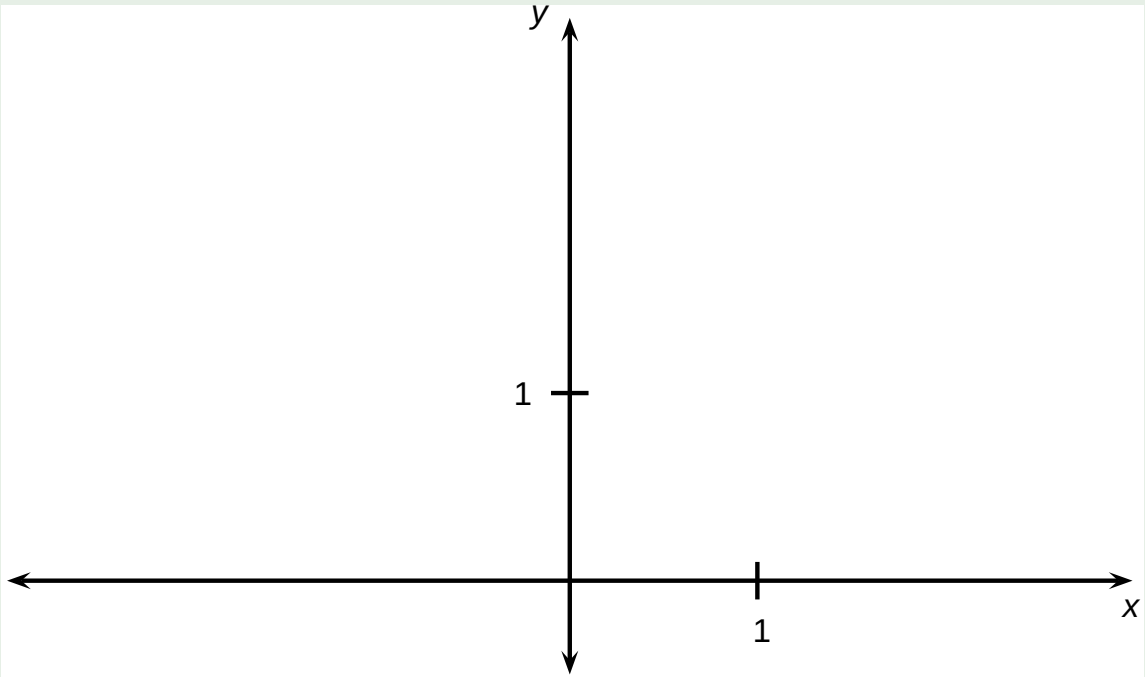
The filled red circle means $(0, 1)$ is on the curve.
The hollow circle means $(0, -1)$ is not on the curve.

Example

The absolute value $|x|$ of a number x is defined to be

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

Sketch a graph of the function $f(x) = |x|$.



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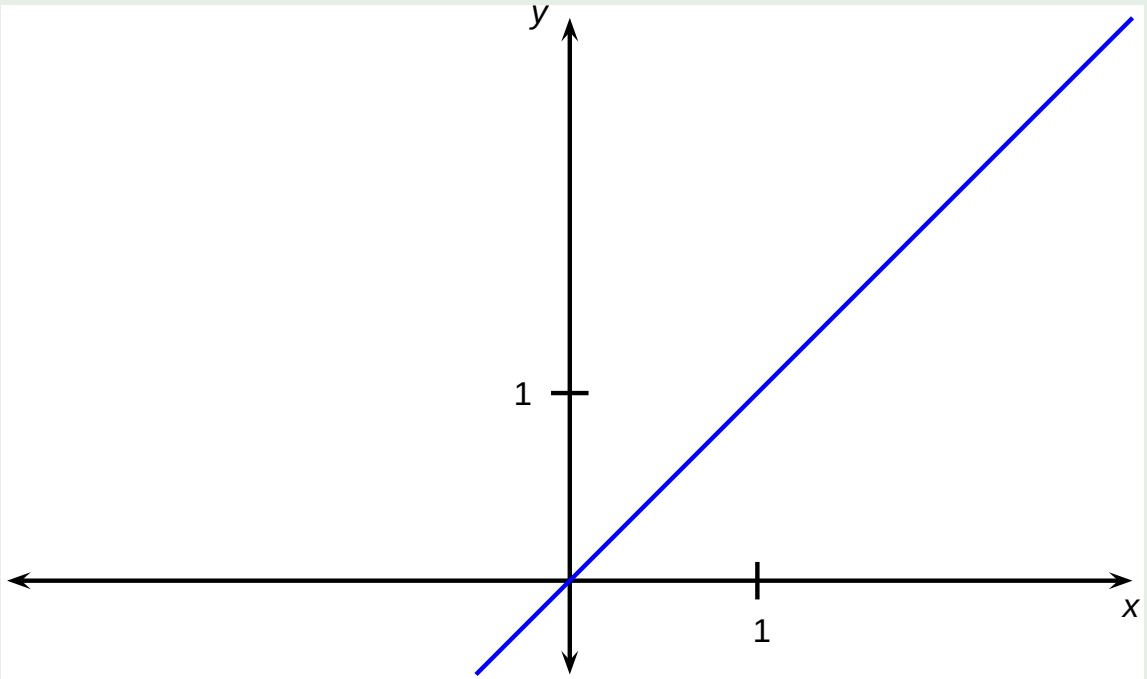

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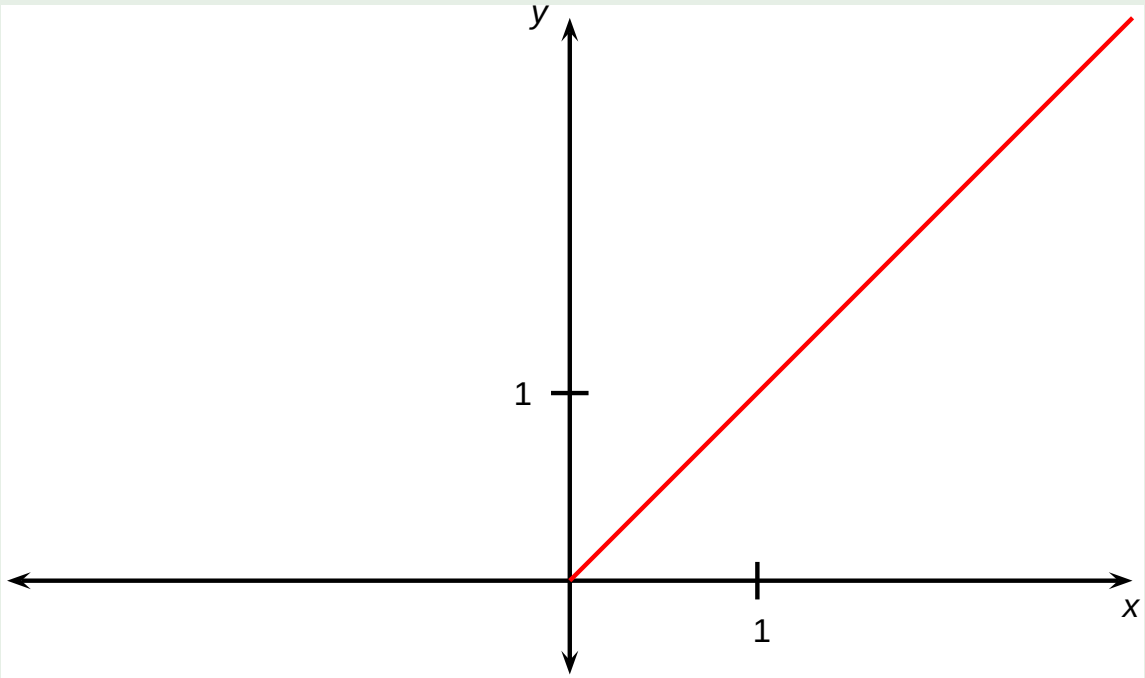

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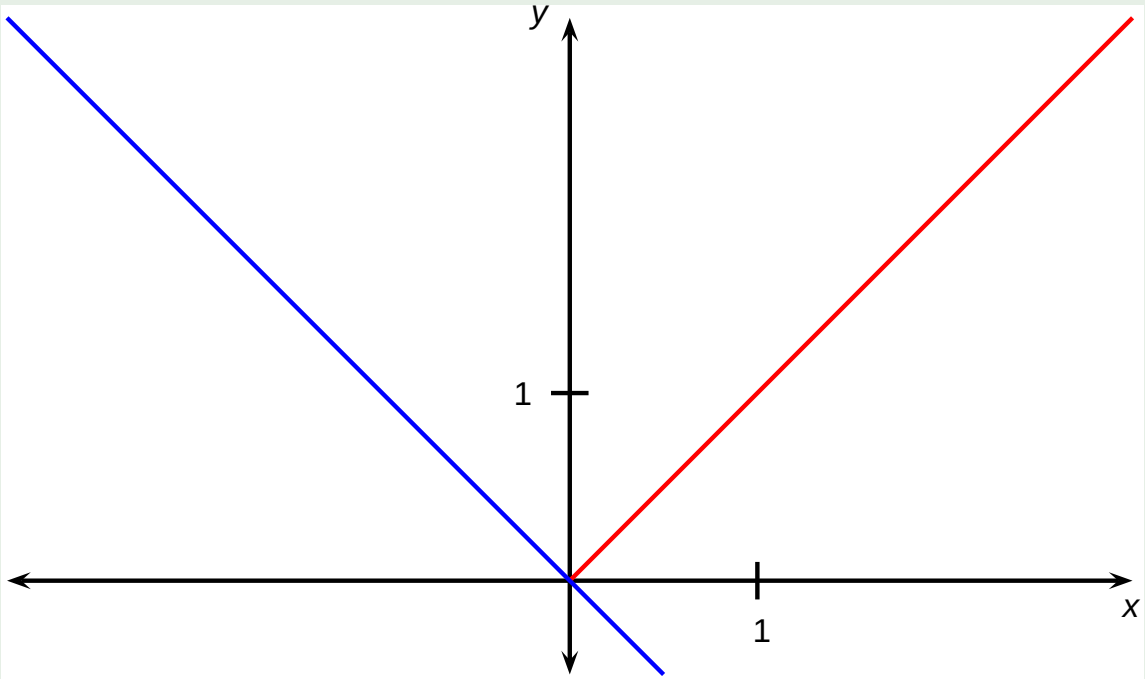

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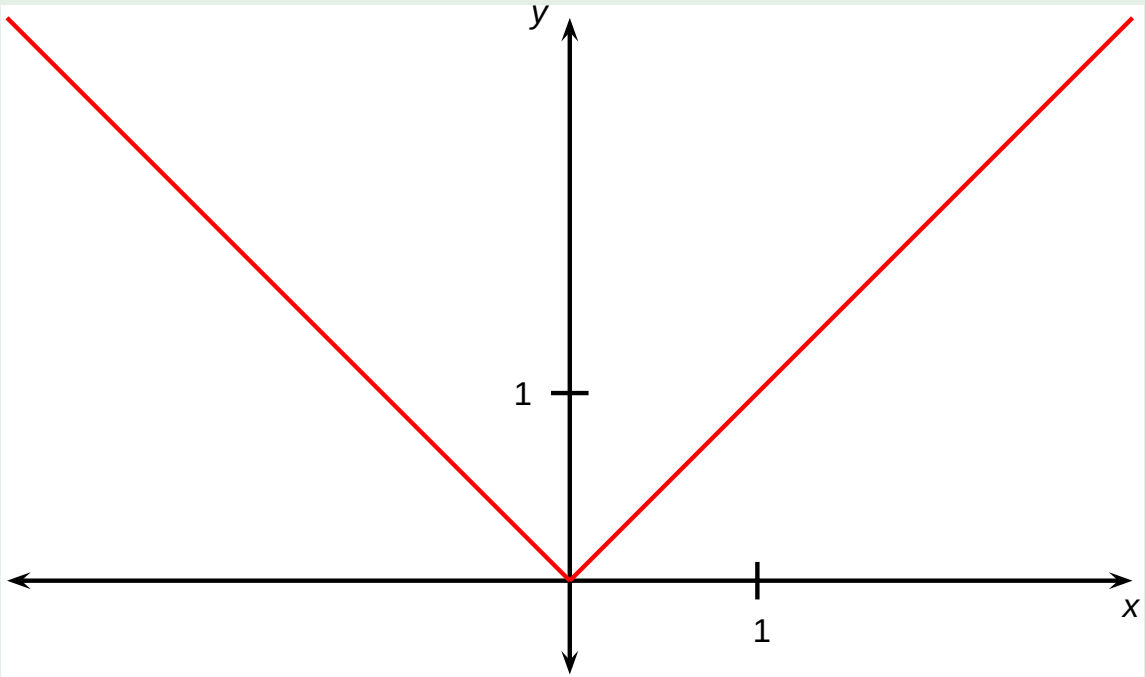
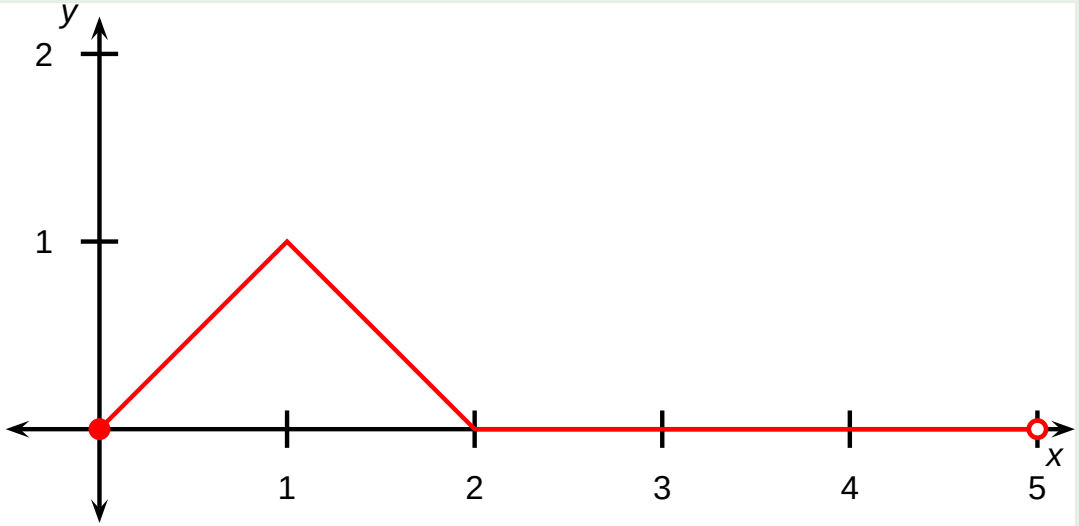


Example

Find a formula for the function f in the graph.



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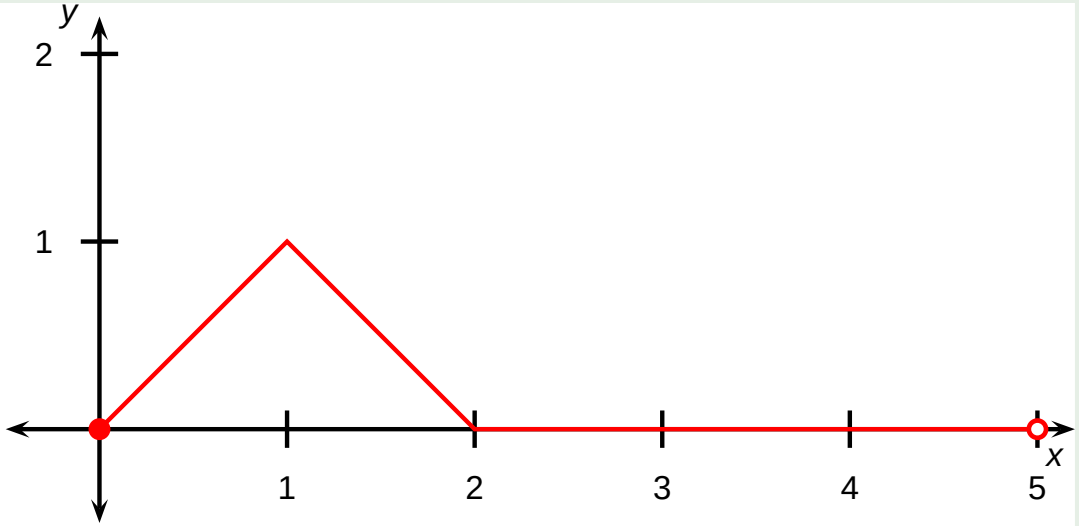

Different formulas on $[0, 1)$, $[1, 2)$, and $[2, 5)$.

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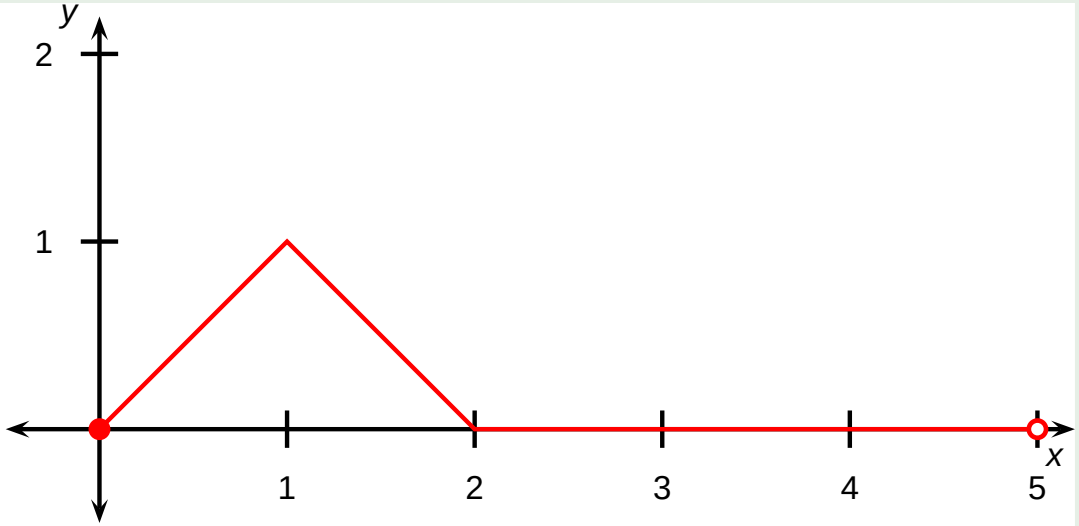

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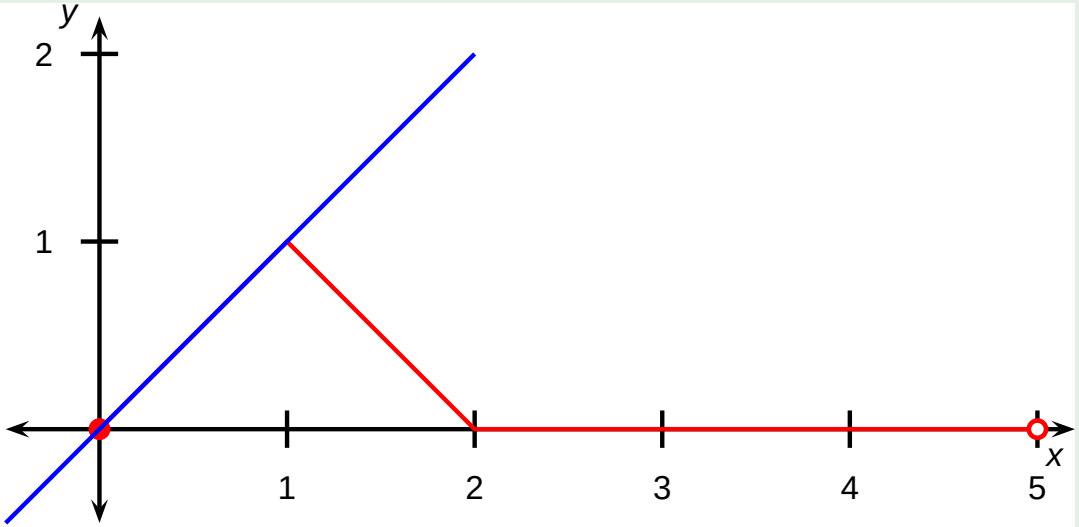

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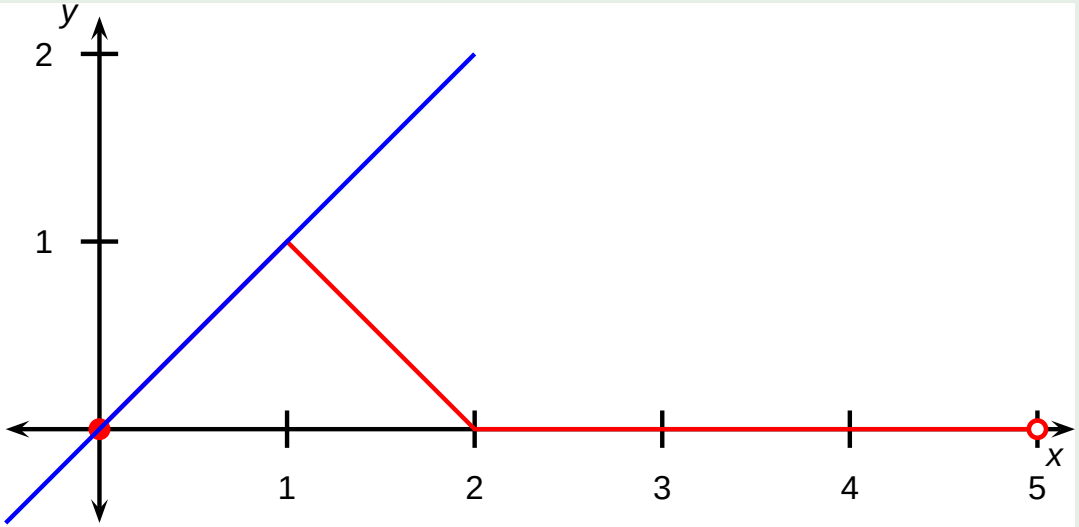

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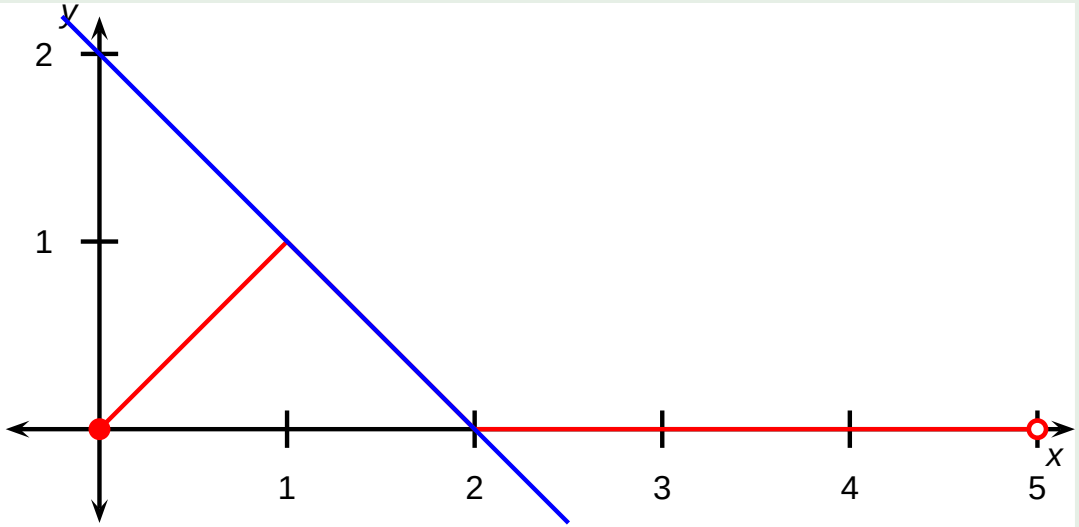

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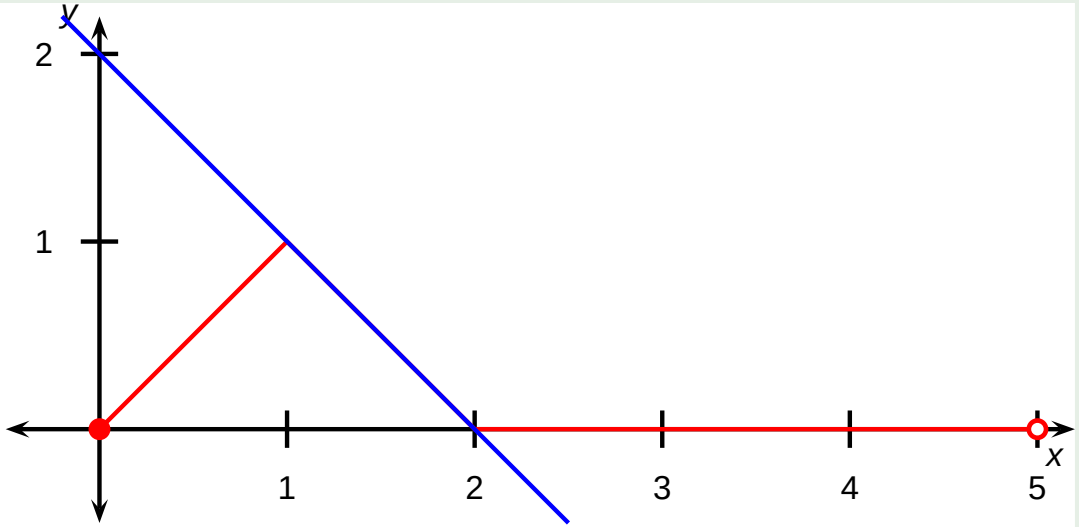

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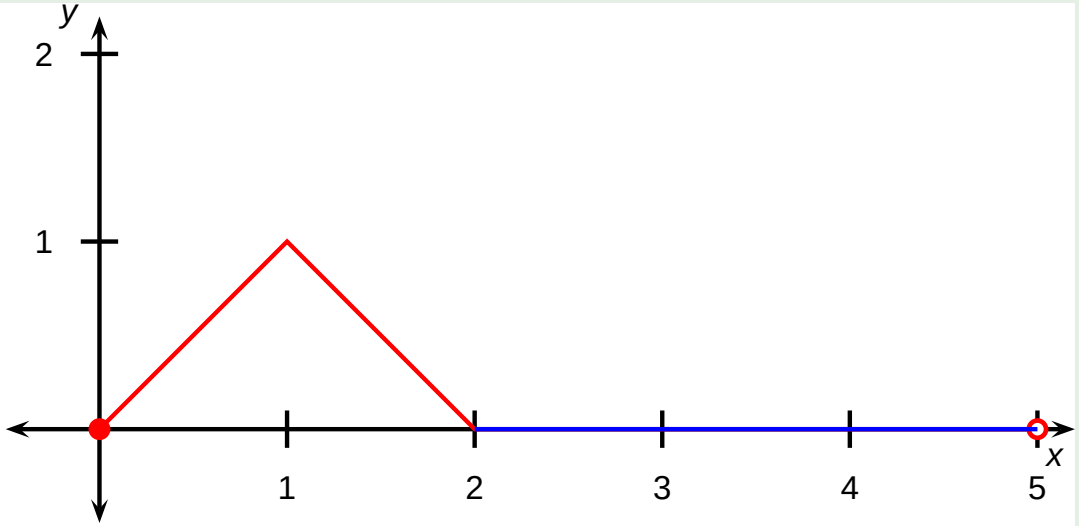

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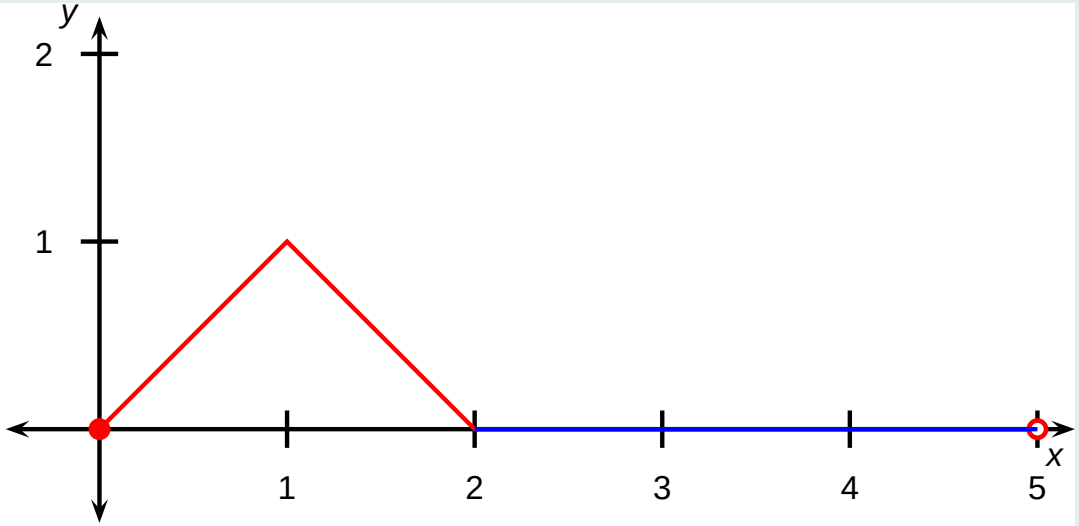

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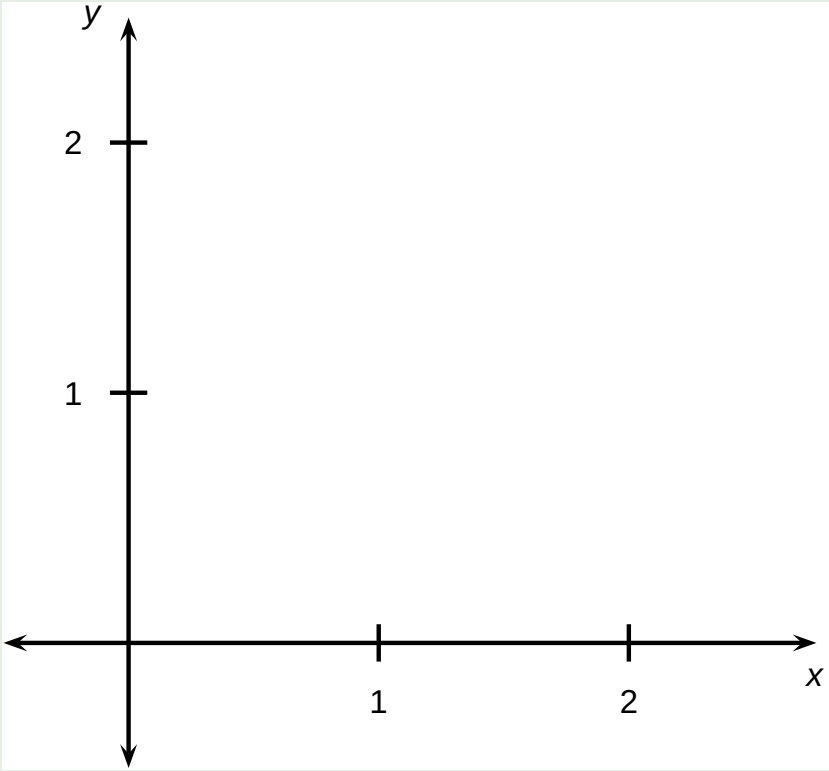
Example

Sketch the function $f(x) = |2x - 3|$.



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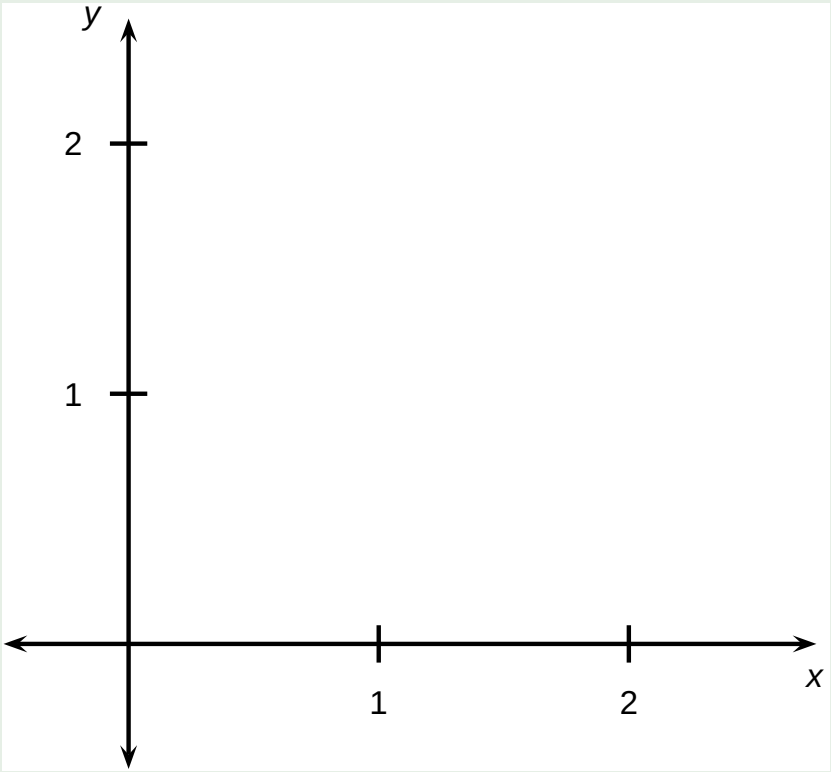

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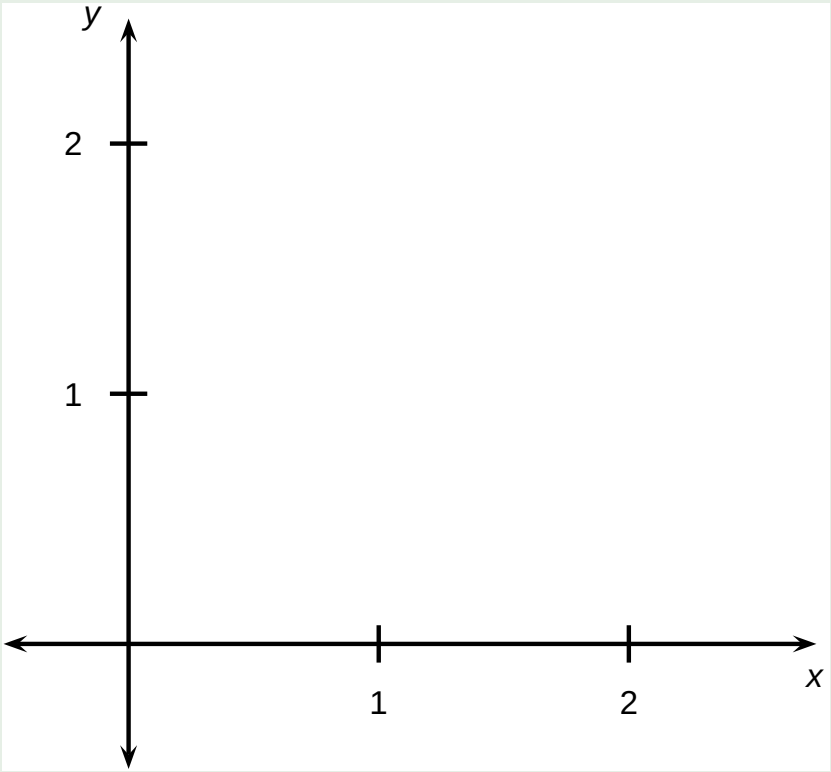

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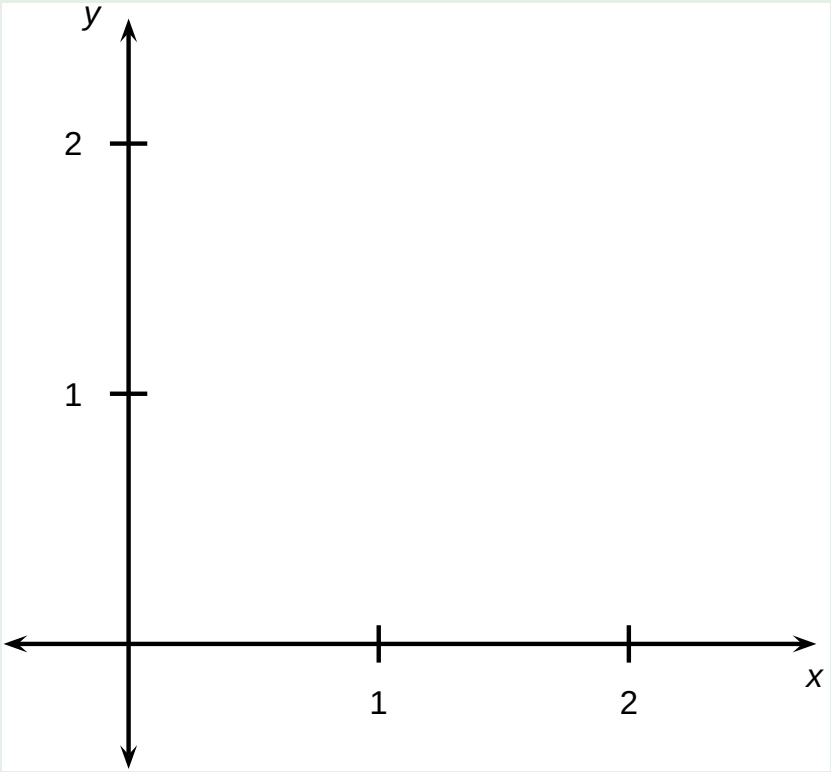

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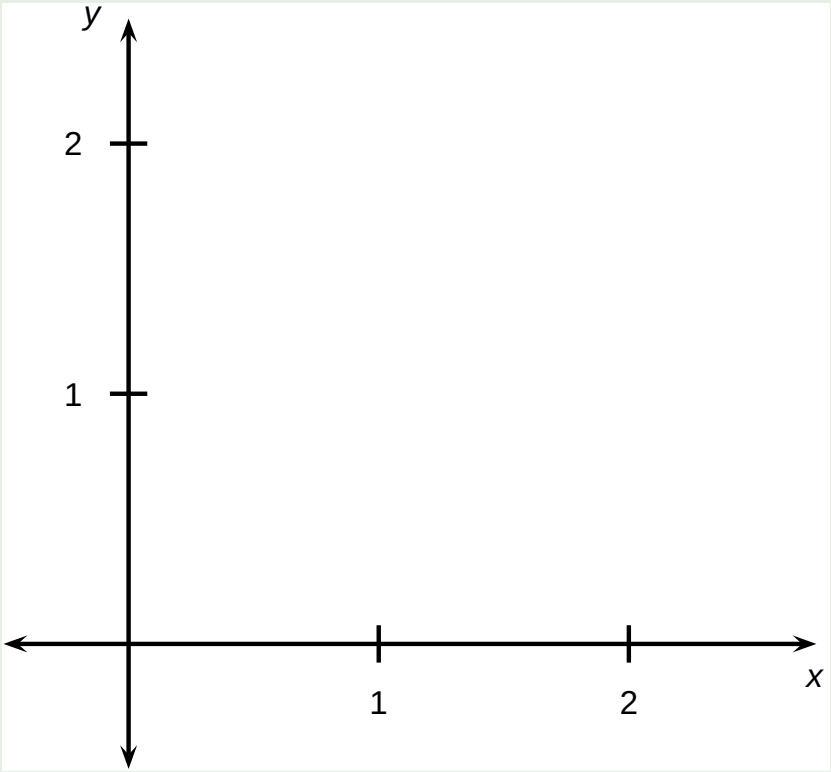

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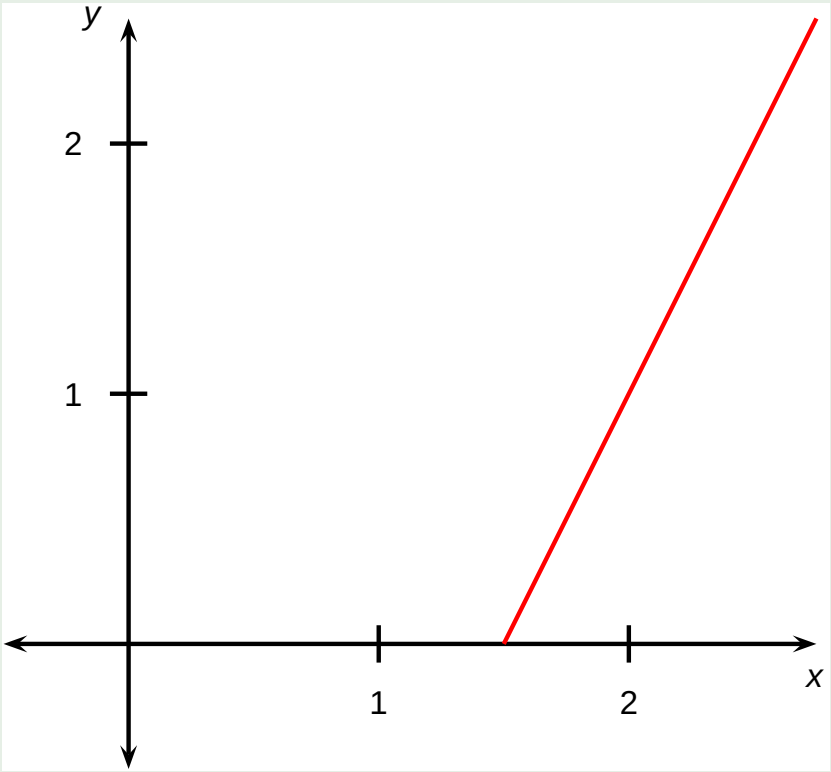

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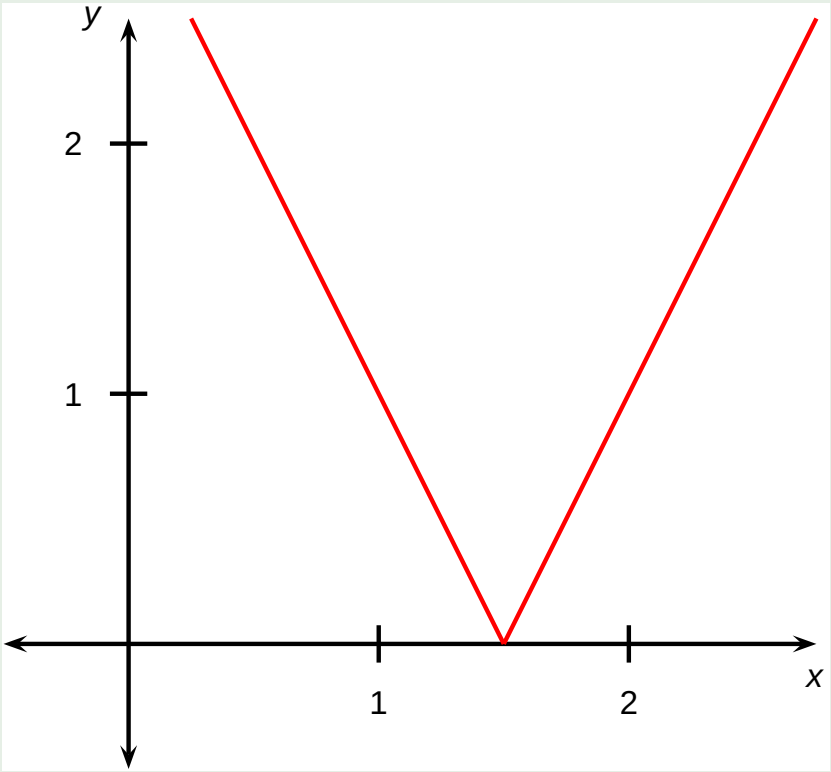

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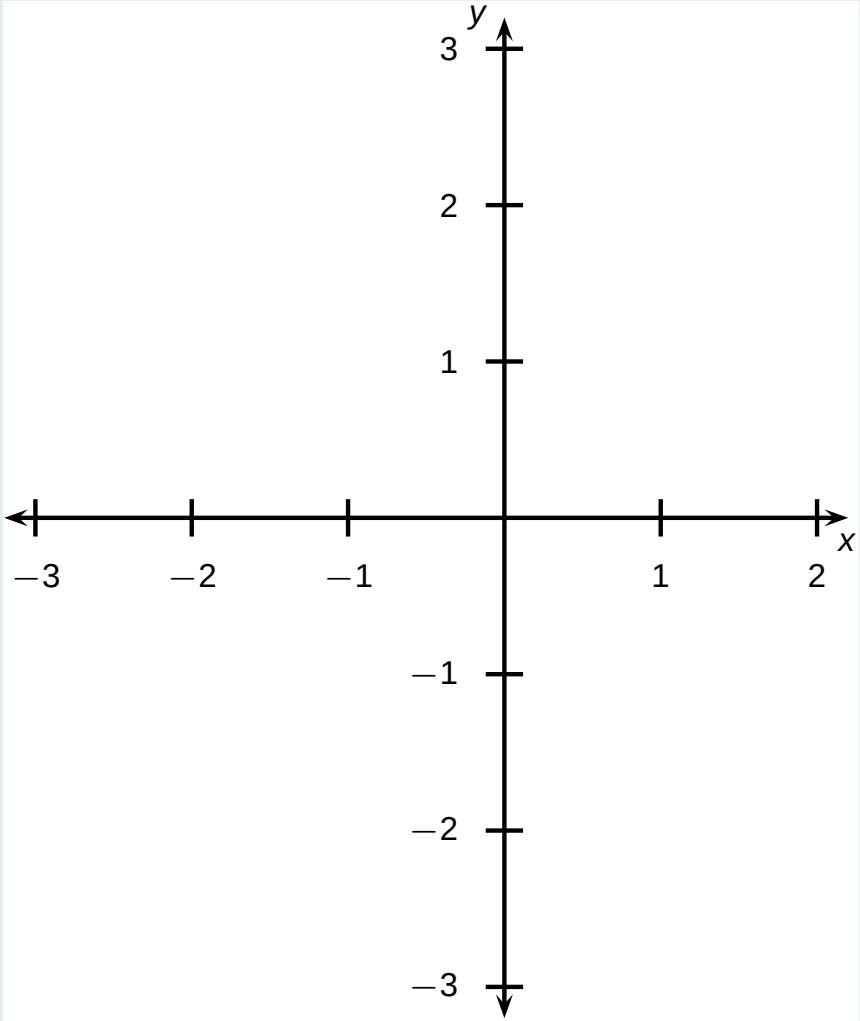
Example

Sketch the function $f(x) = \dfrac{|4x+2|}{2x+1}$.



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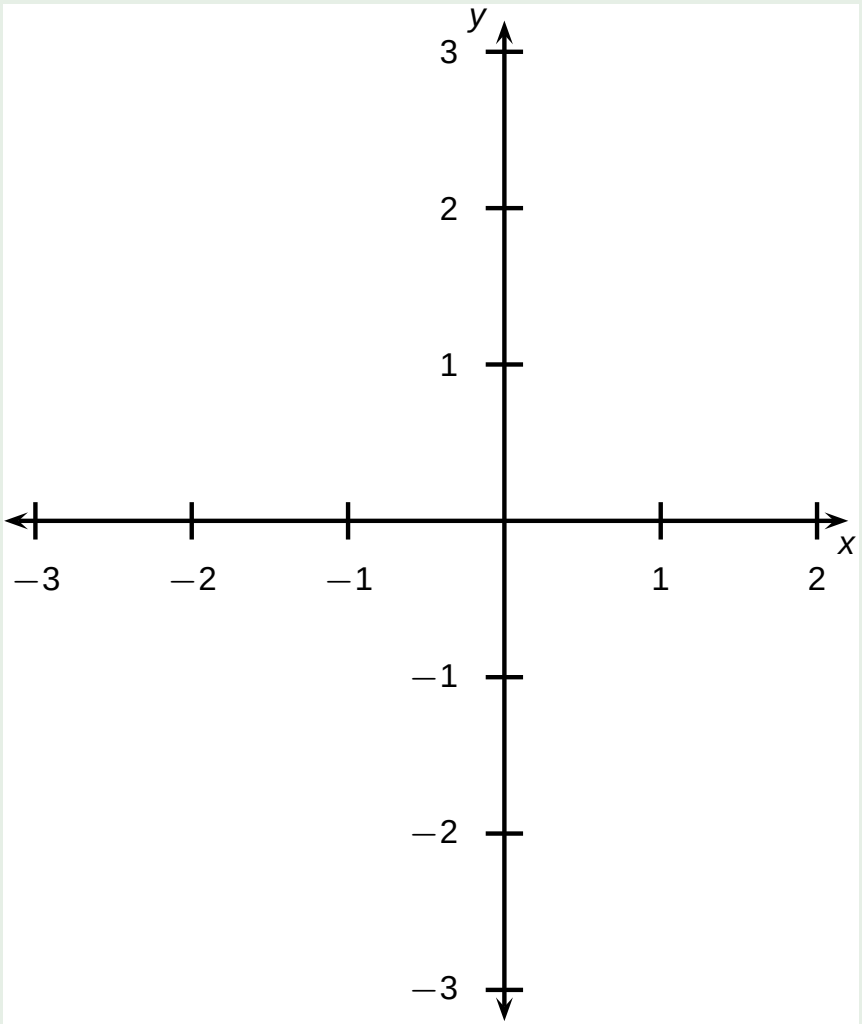

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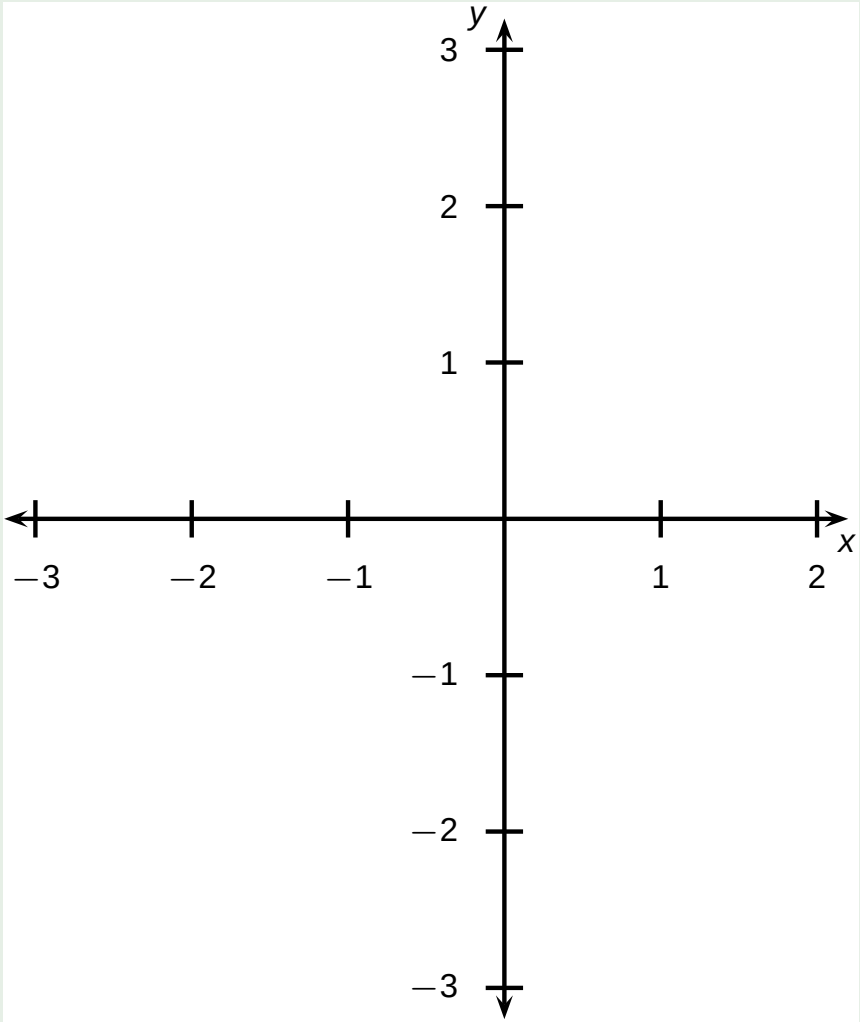

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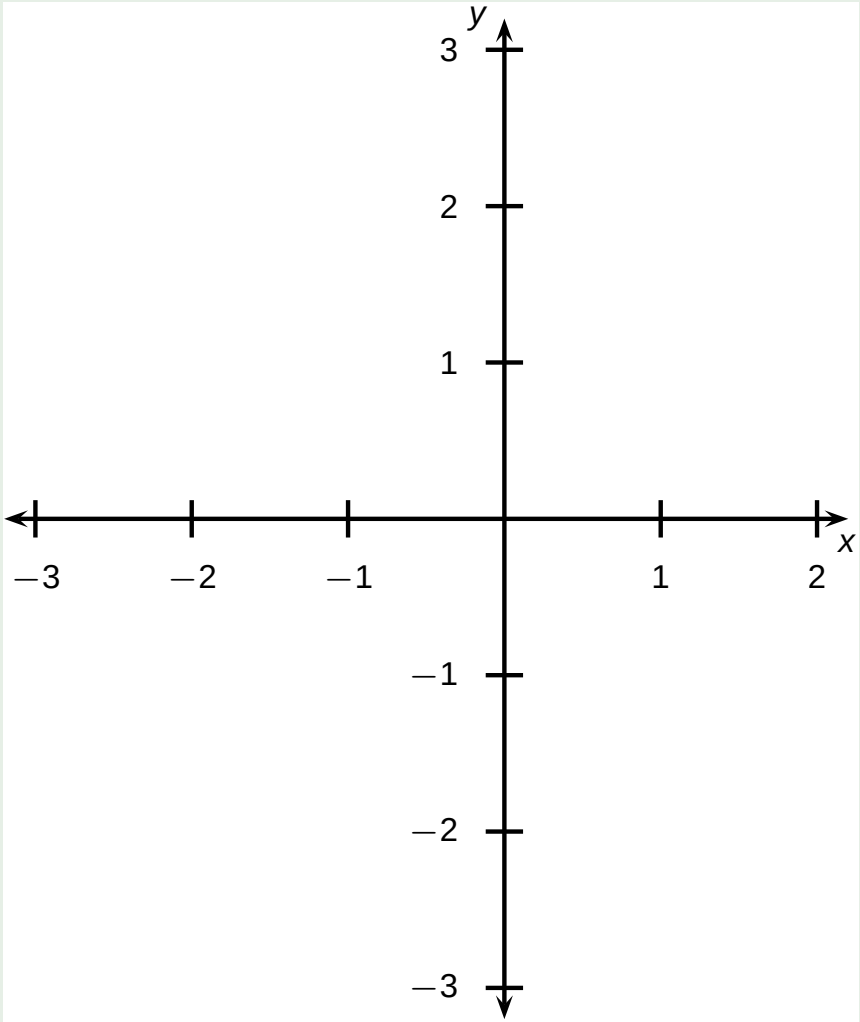

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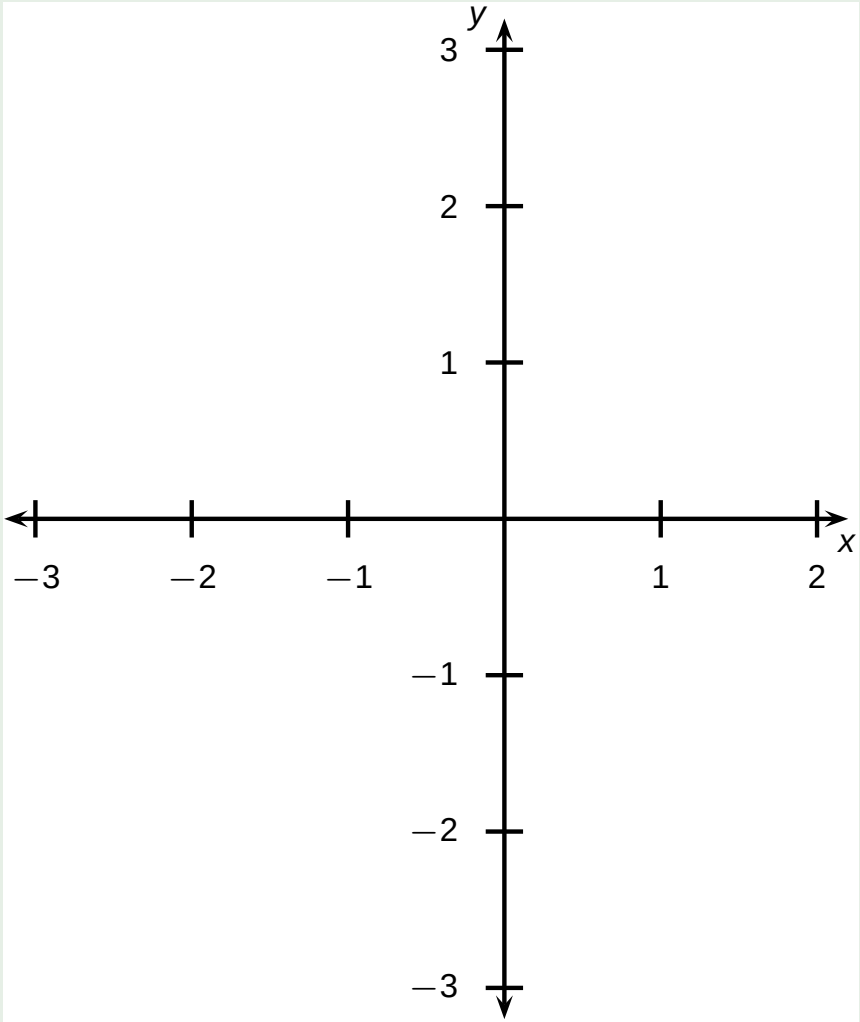

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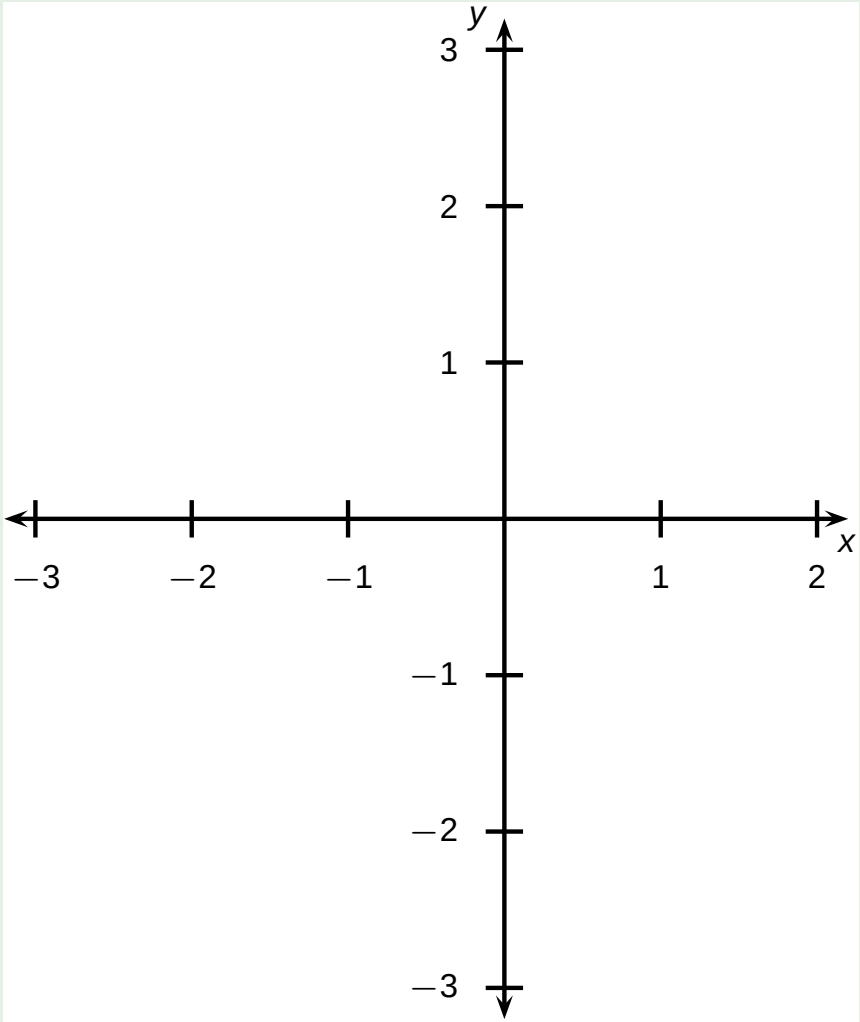

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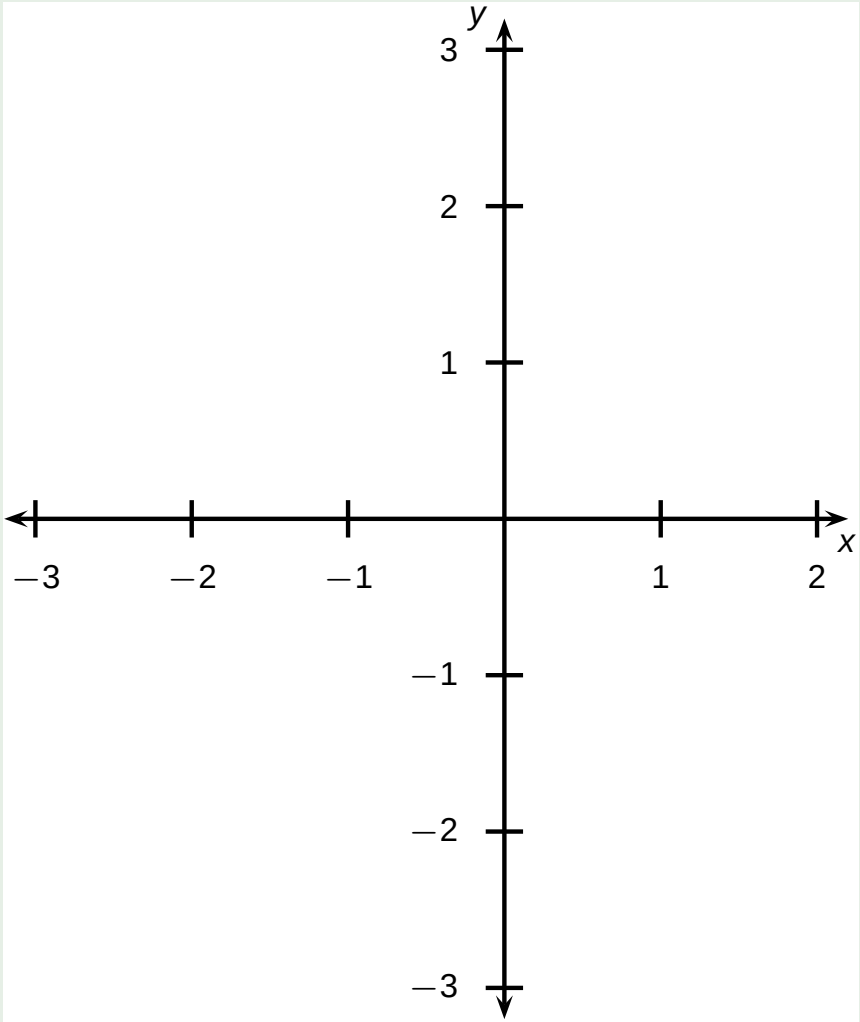

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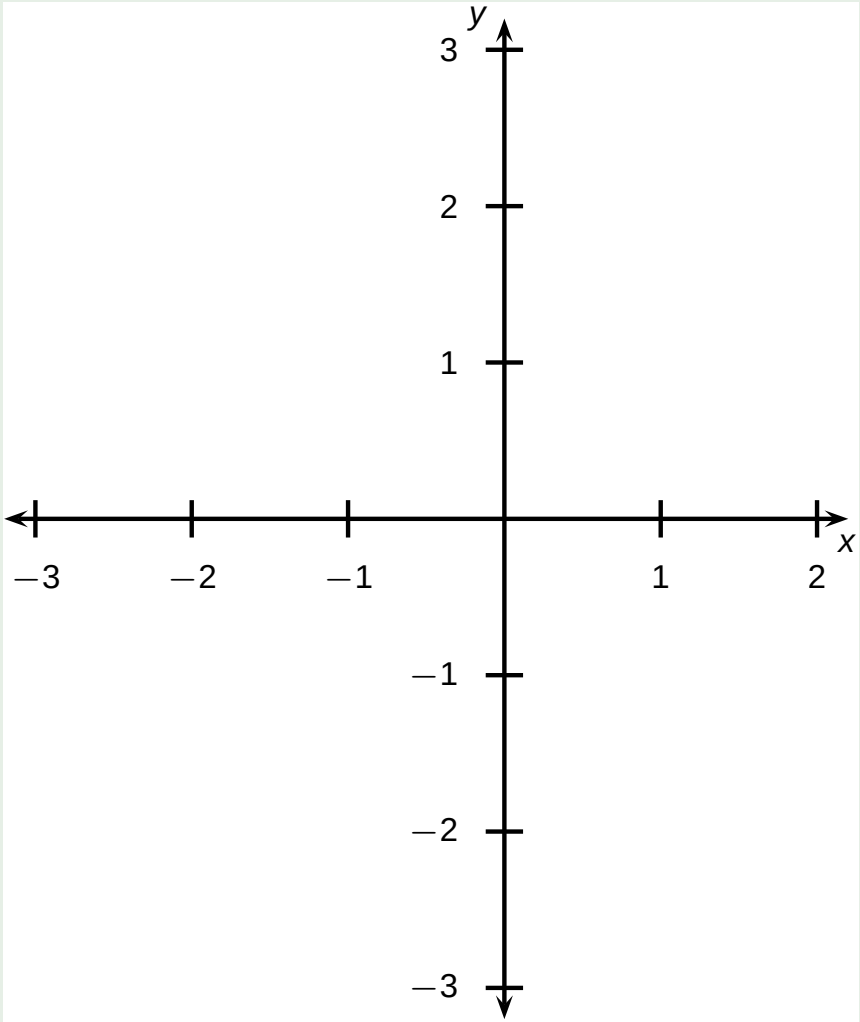

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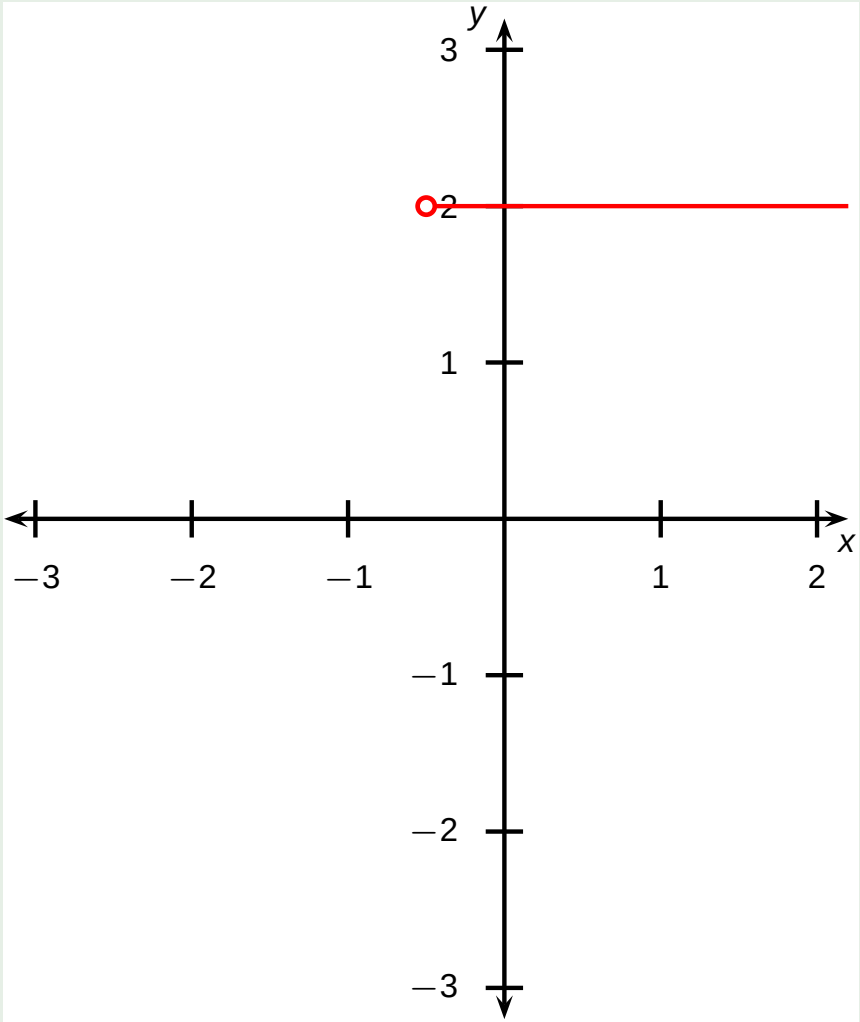

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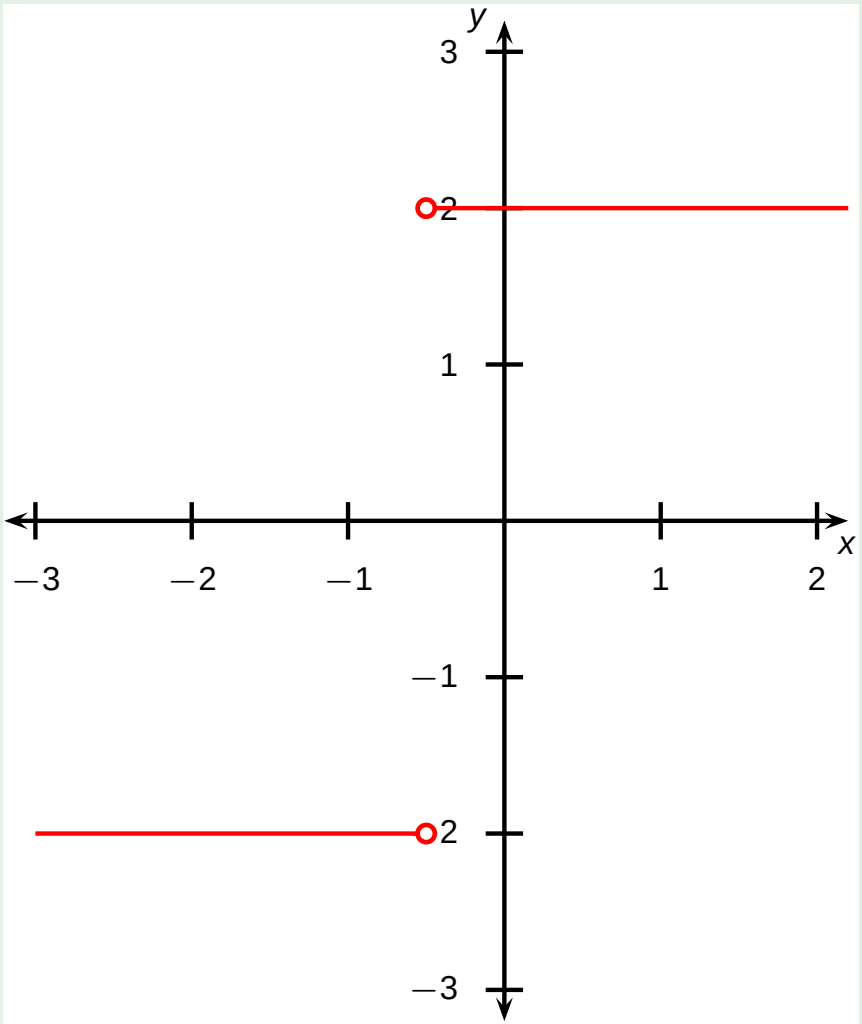

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Therefore f is odd.

$$g(x) = 1 - x^4$$

$$\begin{aligned} g(-x) &= 1 - (-x)^4 \\ &= 1 - x^4 \\ &= g(x) \end{aligned}$$

Therefore g is even.

$$h(x) = 2x - 1$$

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Therefore h is neither even nor odd.

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A function f is called increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
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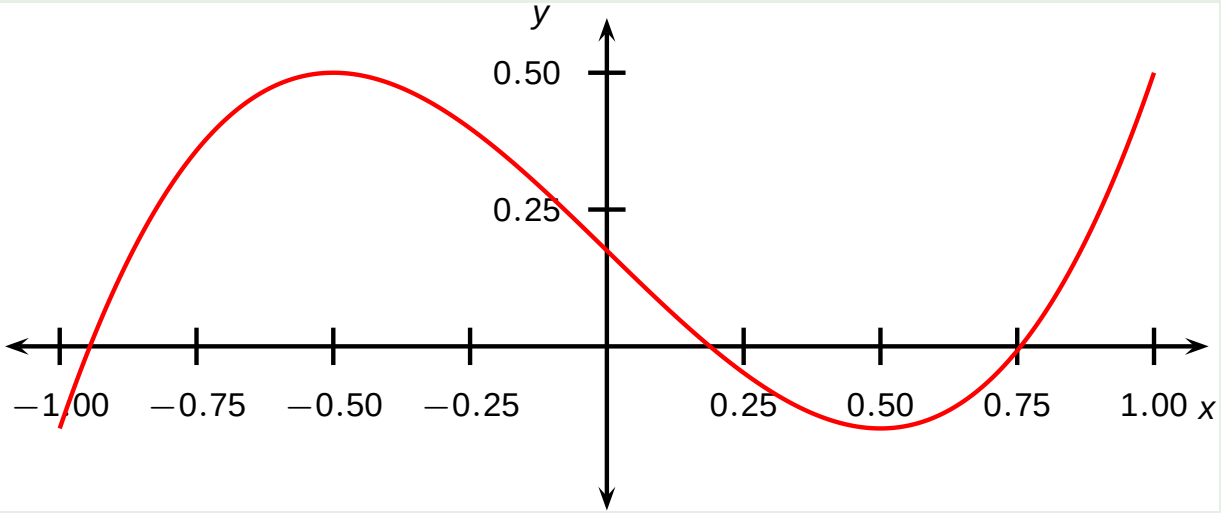

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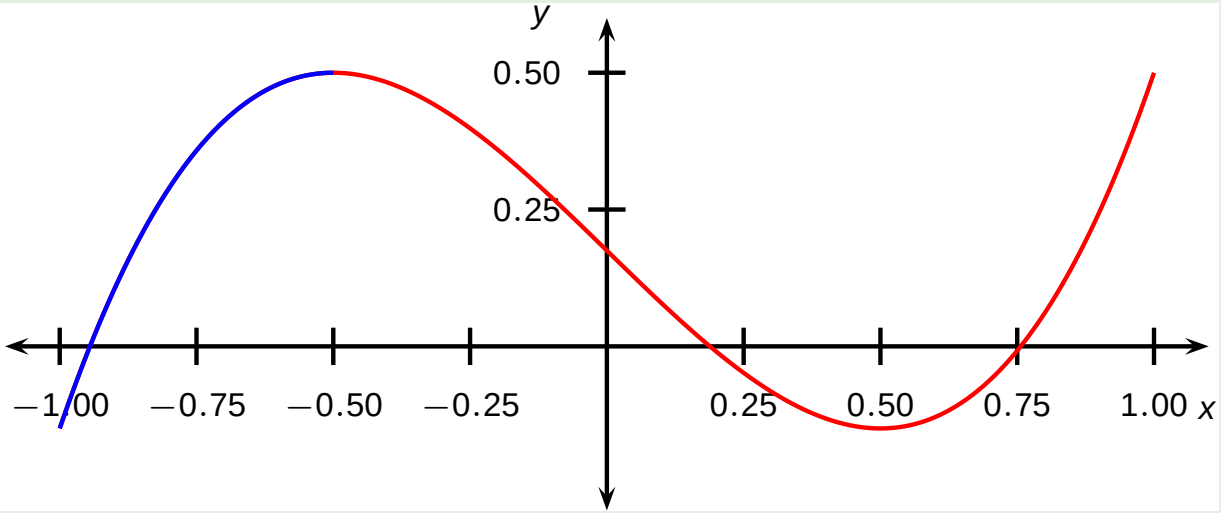

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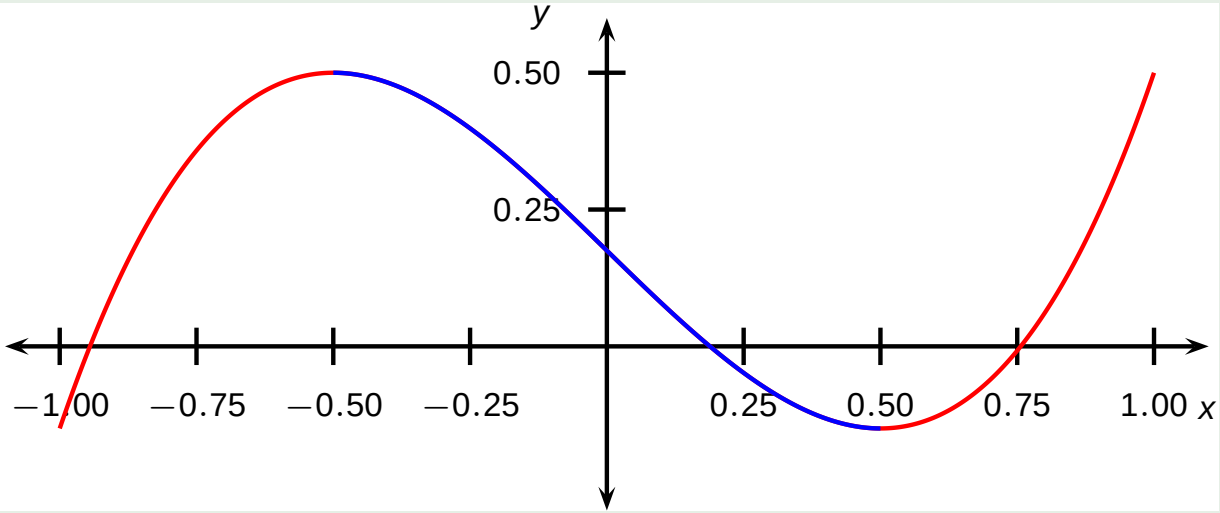

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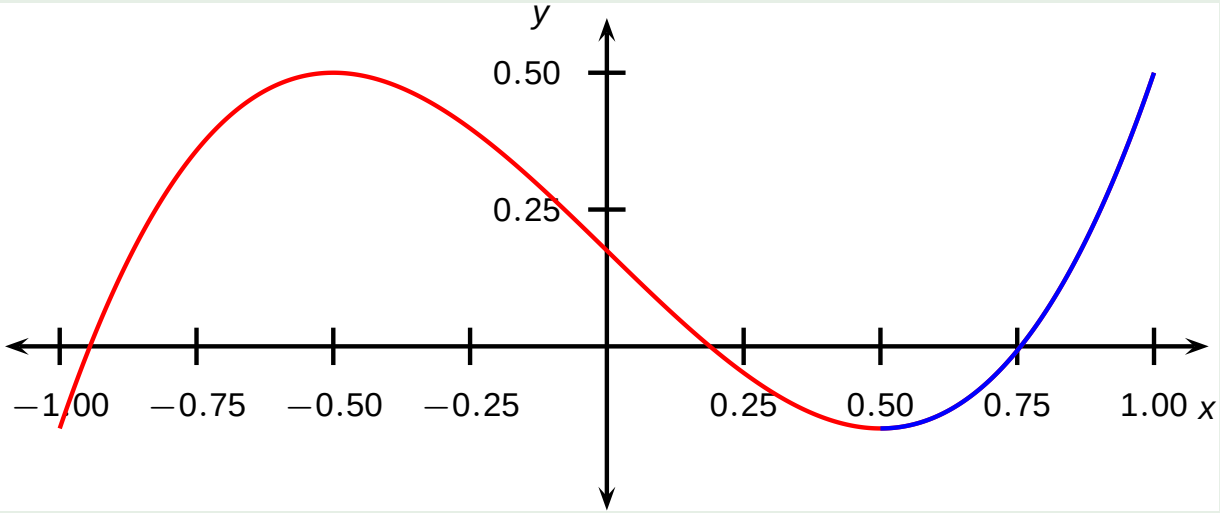

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- Even roots of a negative number are not defined in this course ($\sqrt{-1}$, $\sqrt[4]{-2053}$, $\sqrt[6]{-15}$ ... not allowed).

A Note on Domains of Functions

If the domain of a function isn't specified, it is implied to be all numbers x for which the formula $f(x)$ is defined. There are some restrictions to consider:

- Can't divide by 0.
- Even roots of a negative number are not defined in this course ($\sqrt{-1}$, $\sqrt[4]{-2053}$, $\sqrt[6]{-15}$ ... not allowed).
- Taking $\log x$ if $x \leq 0$ is not allowed in this course; taking $\log 0$ is not allowed in any course.

Example (Two Functions and Their Domains)

Find the implied domains of the following two functions:

$$f(x) = \sqrt[4]{x-2} + \sqrt[3]{6-x}$$

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$$x^2 - x - 6 \neq 0$$

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Domain is all real numbers except 3 and -2; that is, $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$.

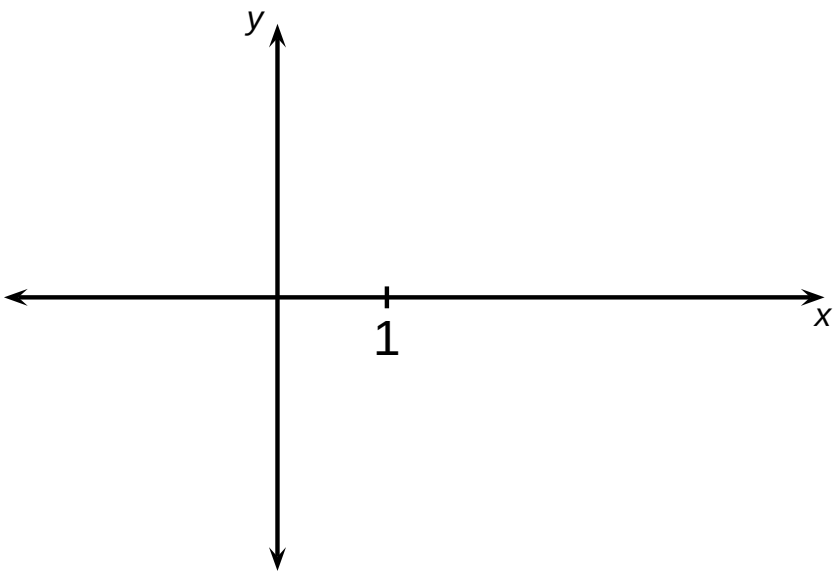
Linear Functions

Definition (Linear Function)

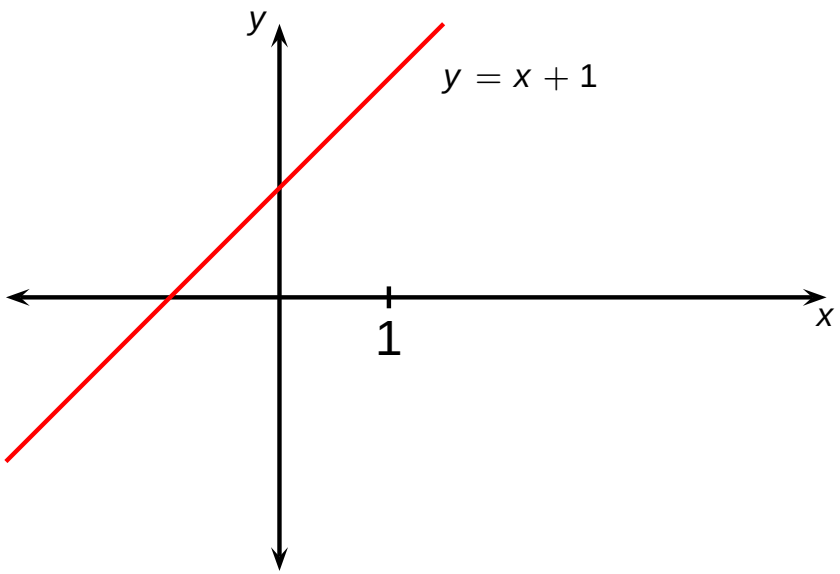
A linear function is a function the graph of which is a line. We can write any linear function in slope-intercept form:

$$f(x) = mx + b.$$

m is called the slope, and b is called the y-intercept.

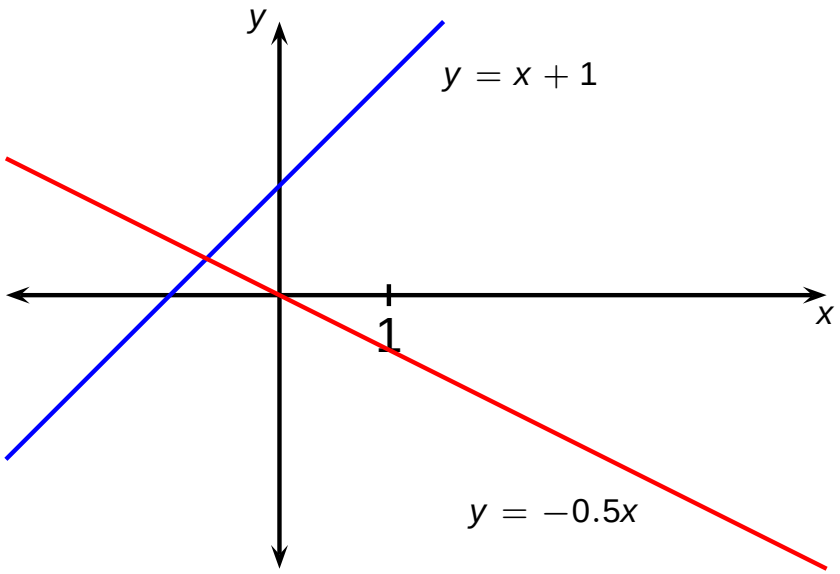


$f(x)$	Direction	y-intercept
$x+1$		
$-0.5x$		
-1		



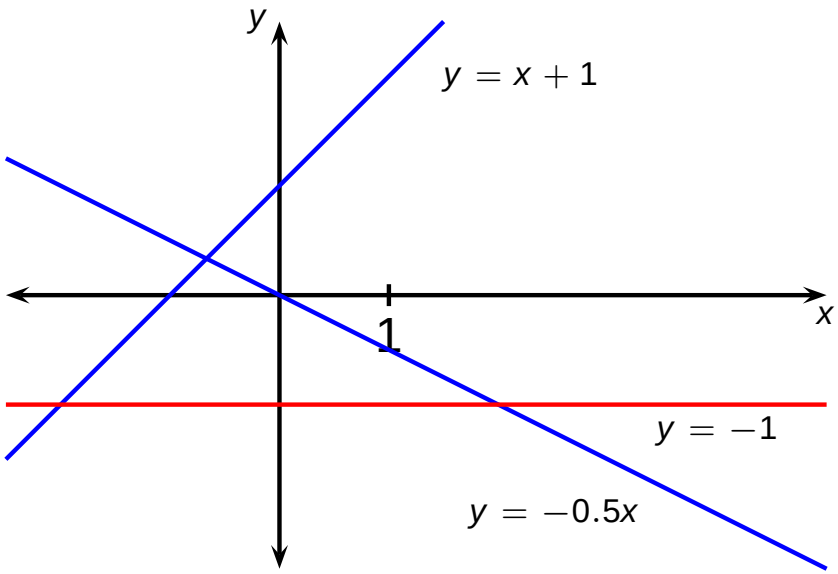
$f(x)$	Direction	y-intercept
$x + 1$	↗	
$-0.5x$		
-1		

- $m > 0$ means the graph of f points up (↗).



$f(x)$	Direction	y-intercept
$x + 1$	↗	
$-0.5x$	↘	
-1		

- $m > 0$ means the graph of f points up (↗).
- $m < 0$ means the graph of f points down (↘).



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$x + 1$	↗	
$-0.5x$	↘	
-1	→	

- $m > 0$ means the graph of f points up (↗).
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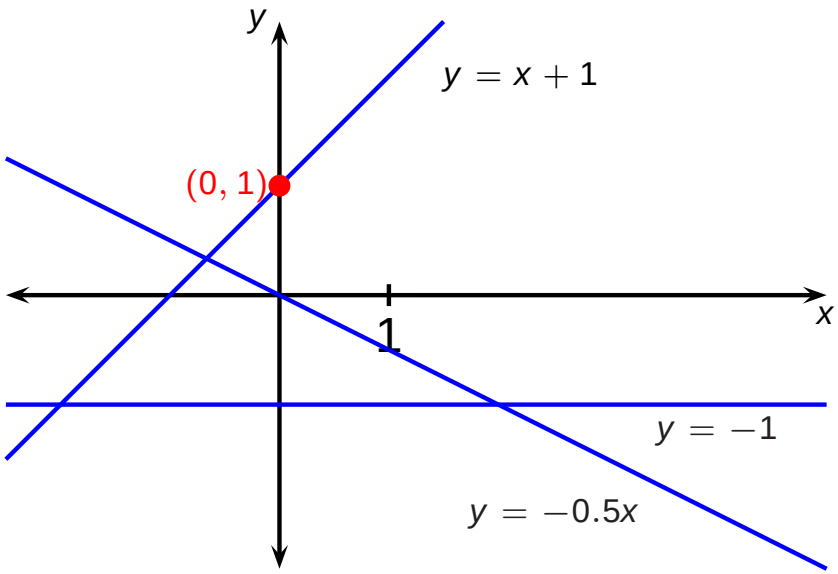
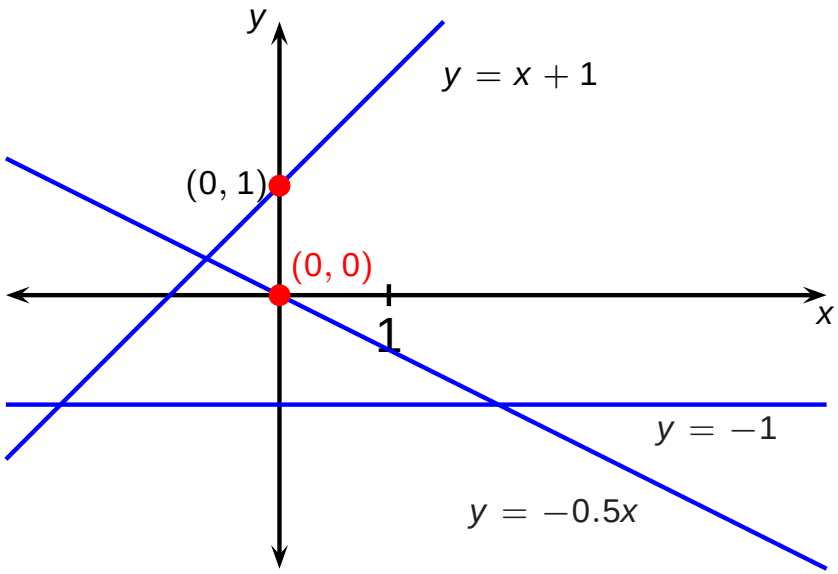
- $m = 0$ means the graph of f is horizontal (→).



$f(x)$	Direction	y-intercept
$x + 1$	$\nearrow$	1
$-0.5x$	$\searrow$	
-1	$\rightarrow$	

- $m > 0$ means the graph of f points up ($\nearrow$).
- $m < 0$ means the graph of f points down ($\searrow$).
- $m = 0$ means the graph of f is horizontal ($\rightarrow$).
- b tells us the height of the point where the graph hits the y-axis.



$f(x)$	Direction	y-intercept
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$-0.5x + 0$	$\searrow$	0
-1	$\rightarrow$	

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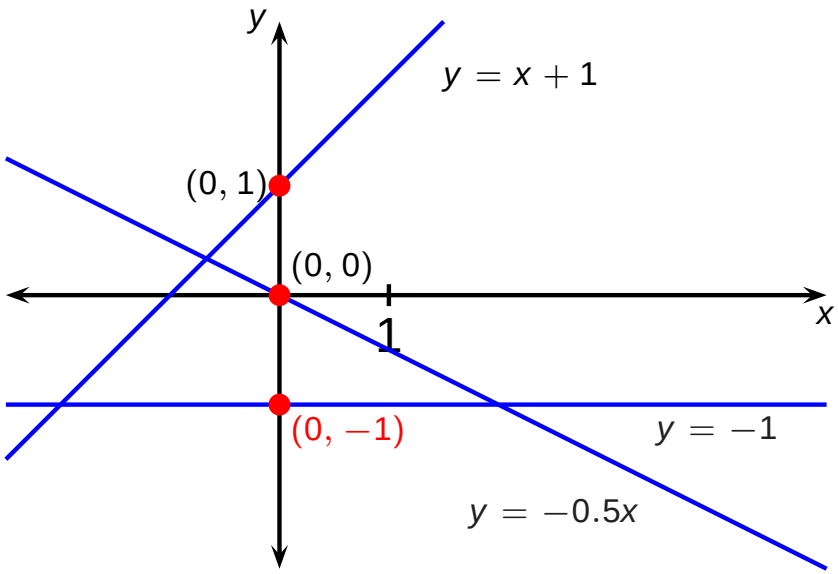

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