Math 140
Lecture 2

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Outline

1. A Catalog of Essential Functions
   - Polynomials
   - Power Functions
   - Rational Functions
   - Algebraic Functions
   - Transcendental Functions

Outline

Polynomials

Definition (Polynomial Function)

A polynomial function is a function f of the form

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1} + a_nx^n,$$

where n is a non-negative integer and $a_0, \ldots, a_n$ are real numbers, called the coefficients. If $a_n \neq 0$ the integer n is called the degree of f.

If we interpret x as an indeterminate formal expression, rather than a number, we say that $f(x)$ is a polynomial (rather than a polynomial function).

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$f(x)$	Polynomial?	Degree	a_0	a_1	a_2
$x^4 - x + 1$					
6					
$3x^2 - \frac{1}{2}x + \sqrt{x}$					
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6					
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$3x^2 - \frac{1}{2}x + \sqrt{x}$	No				
$3x^2 - \frac{1}{2}x + \sqrt{2}$	Yes	2	$\sqrt{2}$	$-\frac{1}{2}$	
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Polynomials

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A polynomial function is a function f of the form

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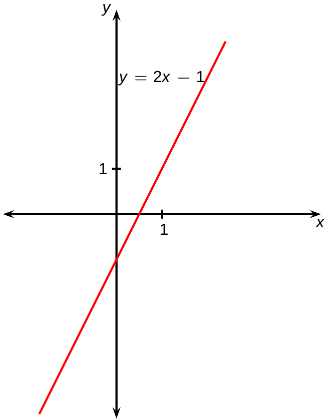

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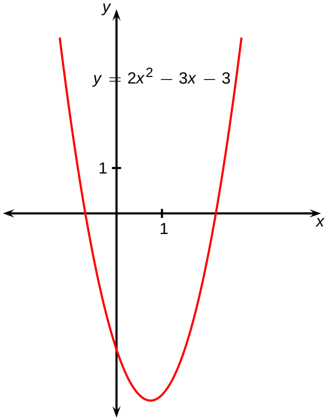

Quadratic

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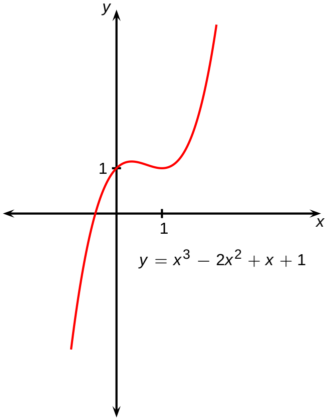

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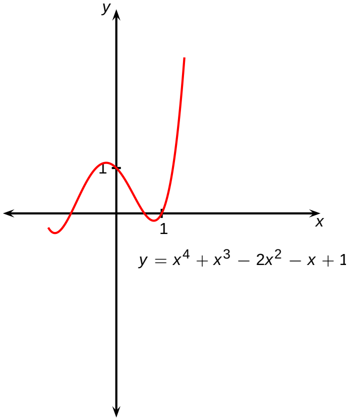

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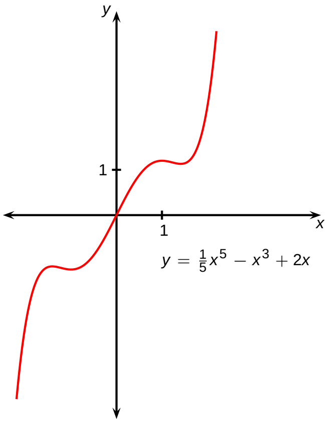

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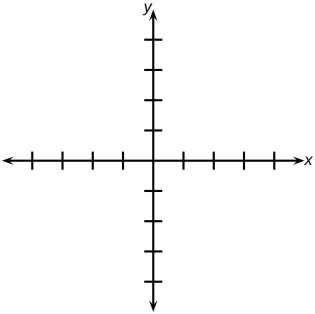

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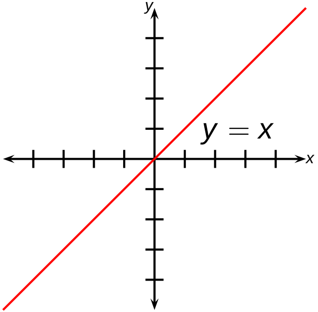

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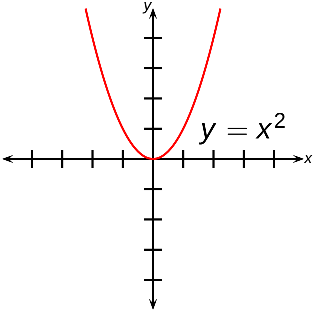

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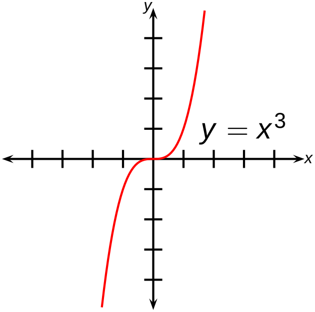

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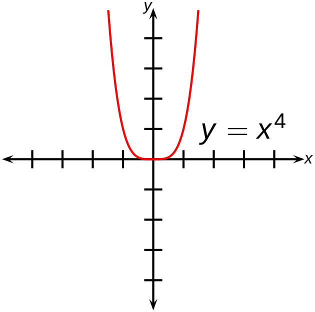

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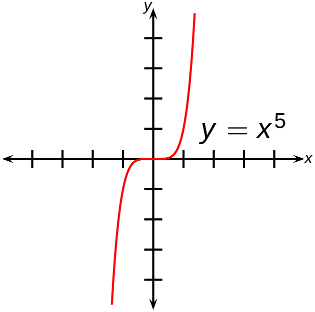

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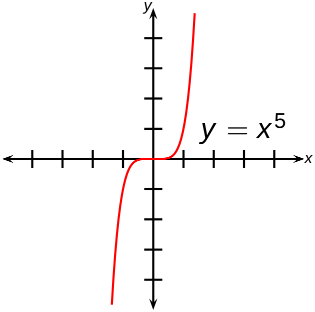

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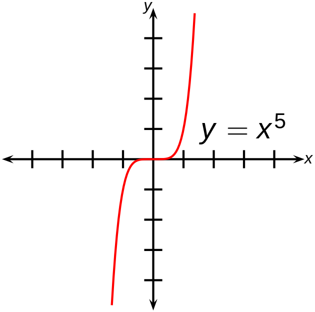

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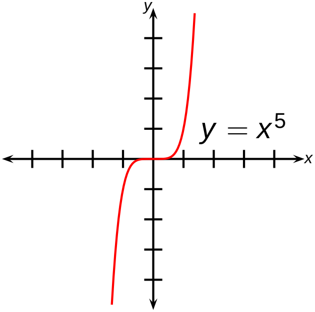

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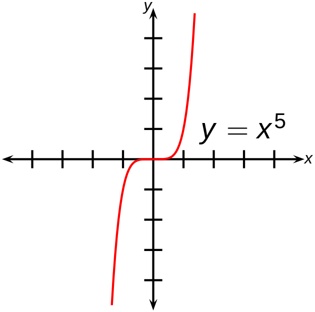

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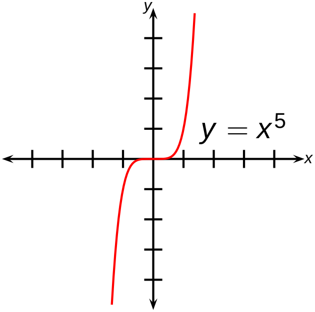

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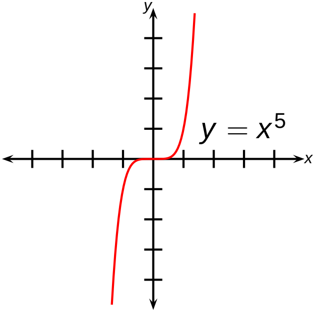

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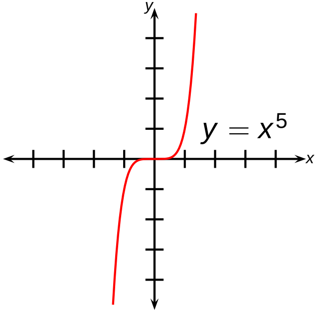

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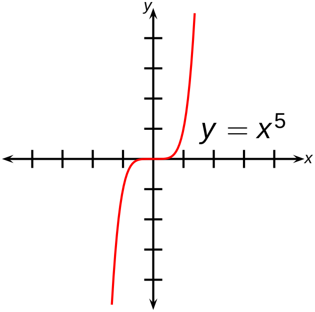

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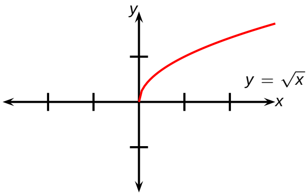

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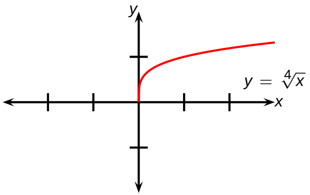

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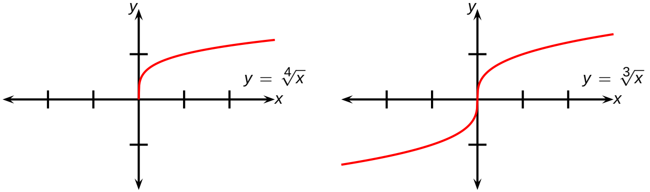

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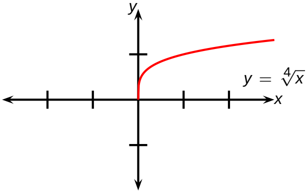
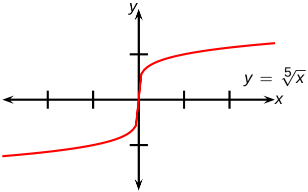
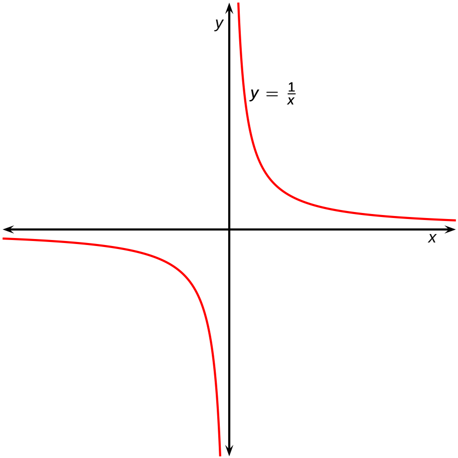




$f(x) = x^{-1} = \frac{1}{x}$ is called the reciprocal function. Its graph has equation $y = \frac{1}{x}$, or $xy = 1$, and is an hyperbola with the coordinate axes as its asymptotes.



Rational Functions

Definition (Rational Function)

A rational function is a quotient of two polynomials; that is, a function of the form

$$f(x) = \frac{g(x)}{h(x)},$$

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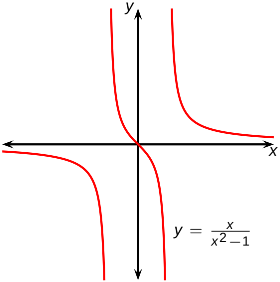

Example ($x/(x^2 - 1)$)

The function

$$f(x) = \frac{x}{x^2 - 1}$$

is a rational function.

Algebraic Functions

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A function in x that can be constructed using x, constants, and finitely many of the operations $+, -, *, /$, and $\sqrt[n]{\ }$ is an algebraic function.

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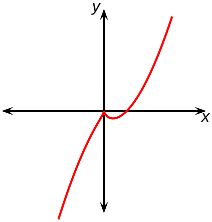
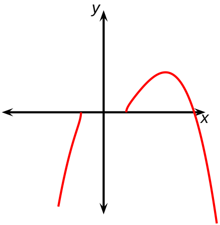
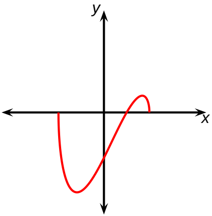
Examples.



$y = (x-1)\sqrt[3]{x^2}$



$y = \frac{1}{5}(4x - x^2)\sqrt{x^2 - 1}$



$y = (x-1)\sqrt{4 - x^2}$

Transcendental Functions

Transcendental functions include many classes of functions.

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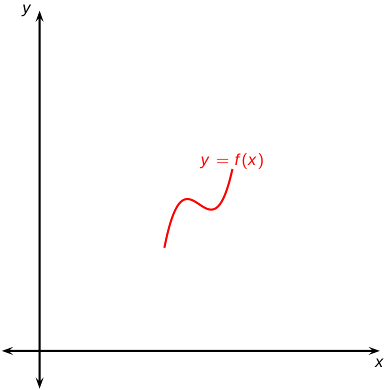
- And many more.
- Outside of Calculus I: by definition, a function is transcendental if it is not algebraic, i.e., if it satisfies no polynomial equation with polynomial coefficients.

Transformations of Functions



What happens if we add or subtract a positive constant c in the equation of a function f? What happens if we add or subtract c from x before applying the function f?

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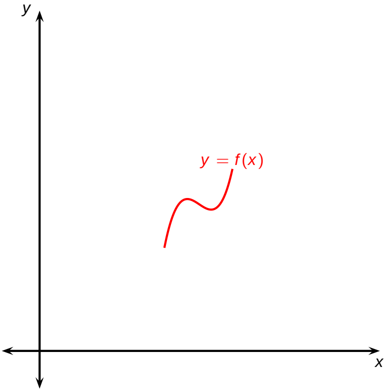

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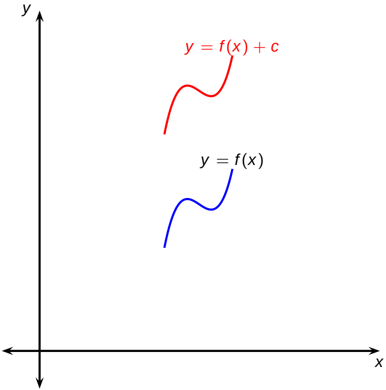

What happens if we add or subtract a positive constant c in the equation of a function f? What happens if we add or subtract c from x before applying the function f?

$f(x)+c$	Shift the graph of $f(x)$ c units up.
$f(x)-c$	
$f(x-c)$	
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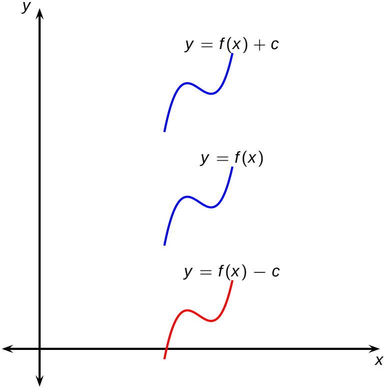

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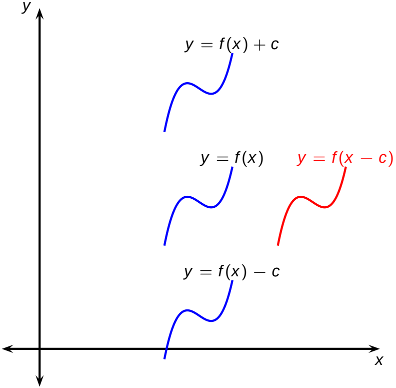

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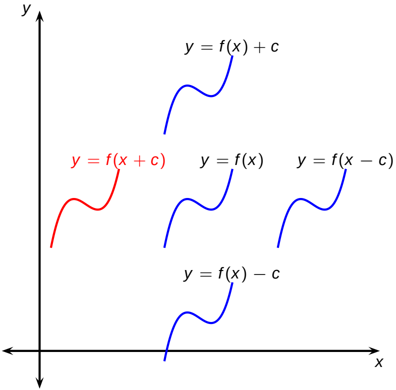

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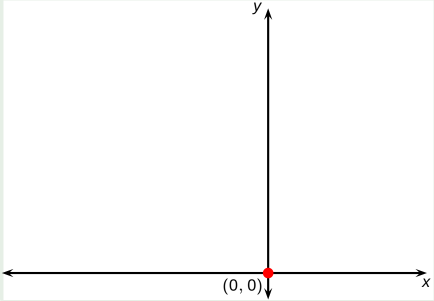
Example

Draw a graph of the function $f(x) = x^2 + 6x + 10$.



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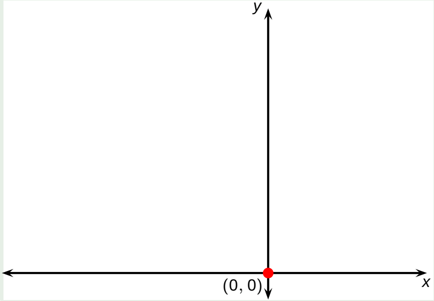

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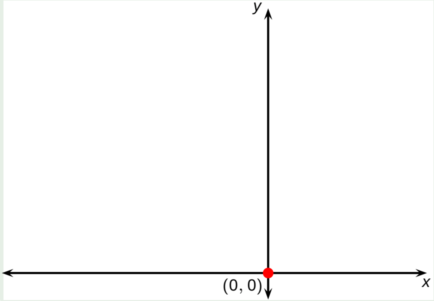

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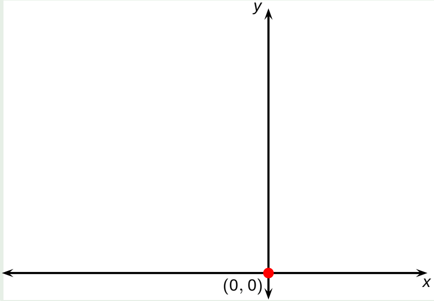

Complete the square:

$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= (x^2 + 6x \qquad) + 10 \end{aligned}$$

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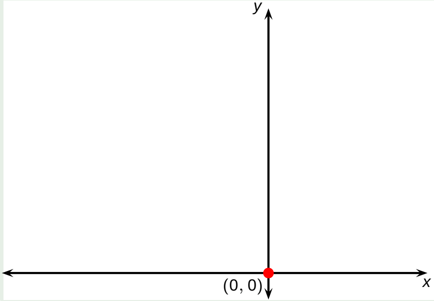

Complete the square:

$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= (x^2 + 6x + 9) + 10 - 9 \end{aligned}$$

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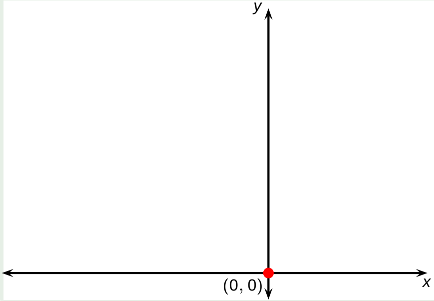

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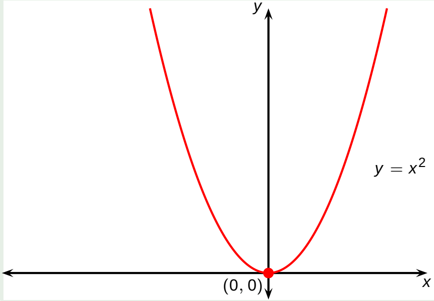

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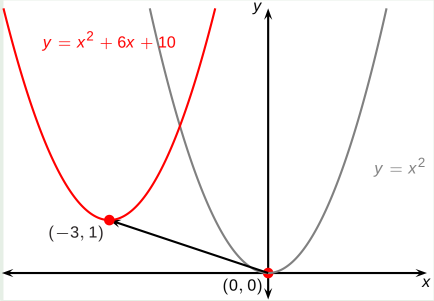

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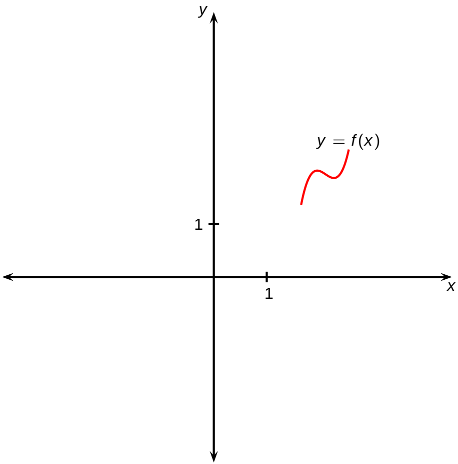
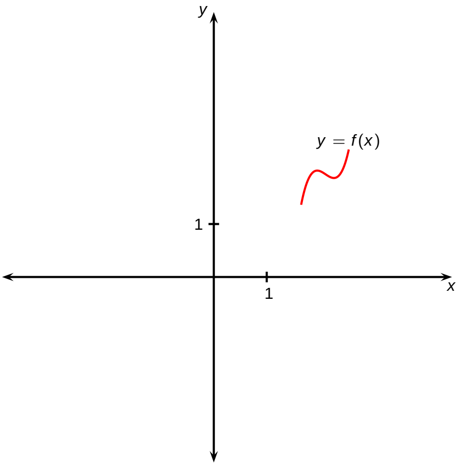


What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f? What happens if we multiply f by -1? What happens if we multiply x by -1 before applying f?

$cf(x)$	
$(1/c)f(x)$	
$-f(x)$	
$f(-x)$	



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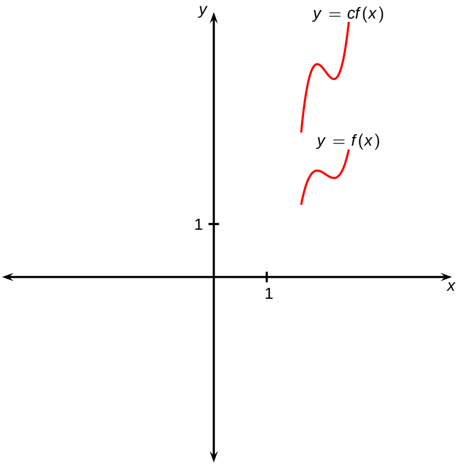

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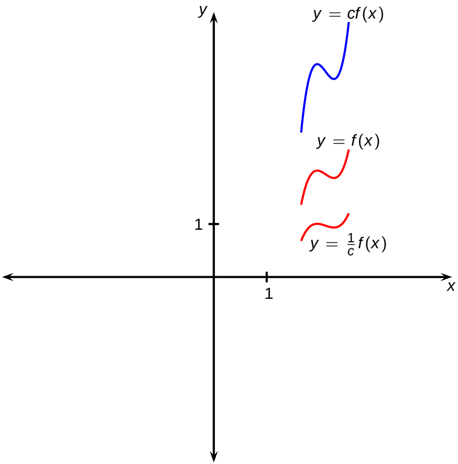

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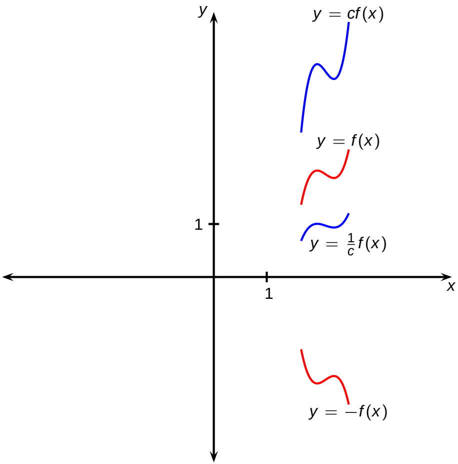

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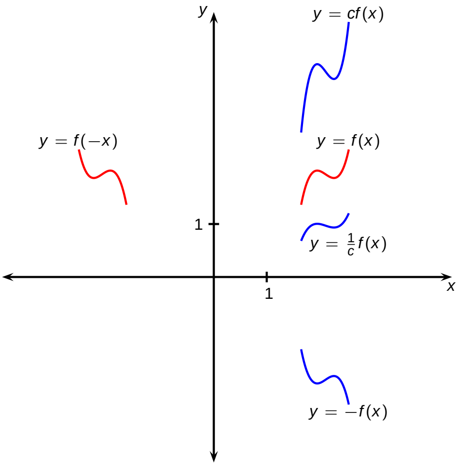

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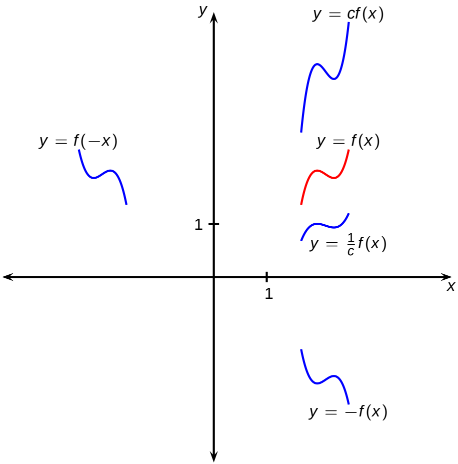

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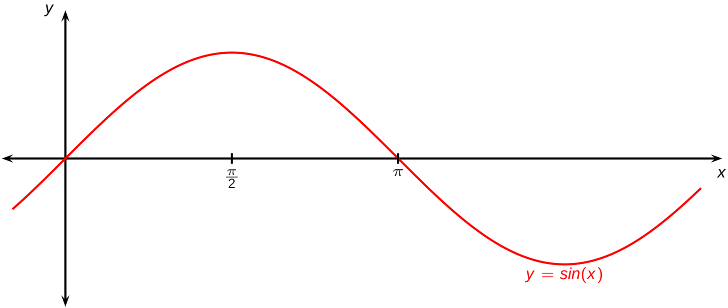

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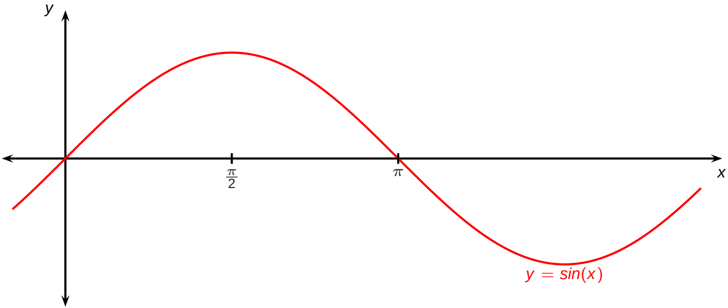

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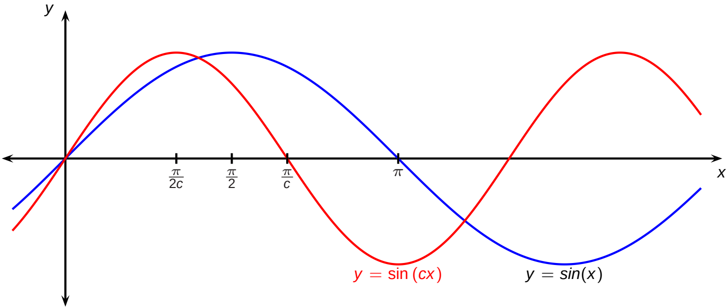
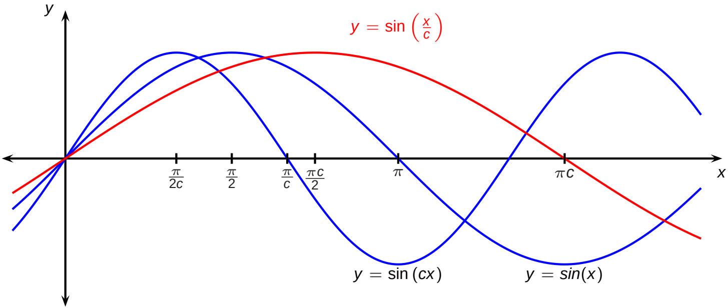
$f(cx)$ $f((1/c)x)$	



What happens if we multiply or divide x by a constant $c > 1$ before applying f?

$f(cx)$	Compress the graph of $f(x)$ horizontally by a factor of c.
$f((1/c)x)$	



What happens if we multiply or divide x by a constant $c > 1$ before applying f?

$f(cx)$	Compress the graph of $f(x)$ horizontally by a factor of c.
$f((1/c)x)$	Stretch the graph of $f(x)$ horizontally by a factor of c.

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Example

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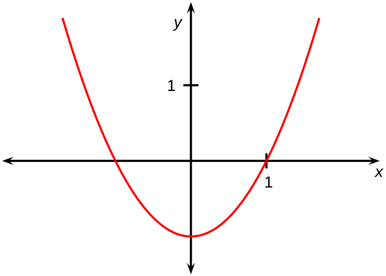

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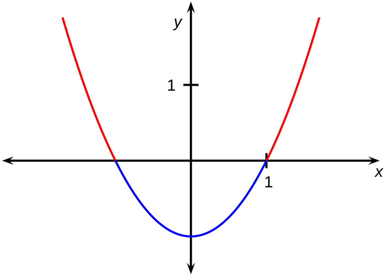

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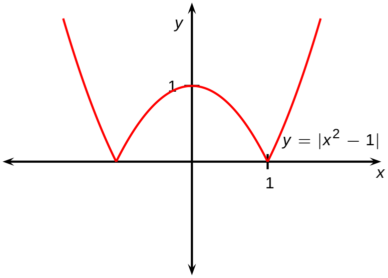

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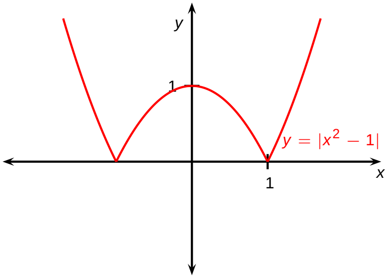

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Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, fg, and f/g. The sum and difference functions are defined by the formulas

$$(f+g)(x) = f(x) + g(x), \qquad (f-g)(x) = f(x) - g(x).$$

If A is the domain of f and B is the domain of g, then the domain of $f+g$ and $f-g$ is $A \cap B$, the intersection of A and B.
The product and quotient functions are defined by the formulas

$$(fg)(x) = f(x)g(x), \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}.$$

These functions also have the domain $A \cap B$, with one exception: in the quotient function, we aren't allowed to divide by 0, so we must exclude those values of x that make $g(x) = 0$. We write this domain as

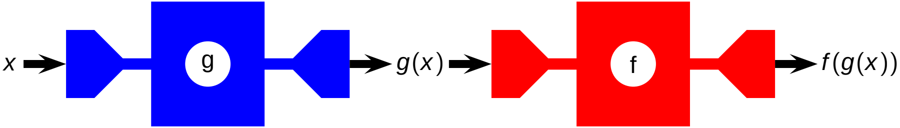
$$\{x \in A \cap B | \ g(x) \neq 0\}.$$

Definition (Composition of f and g)

If f and g are two functions, then the composition of f and g is written $f \circ g$ and is defined by the formula

$$(f \circ g)(x) = f(g(x)).$$

Imagine f and g as machines taking some input and producing some output. Then $f \circ g$ corresponds to attaching both machines end-to-end so that the output of g becomes the input of f.



Definition (Composition of f and g)

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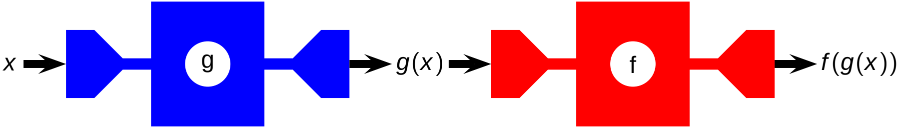

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f. If the domain of f is A and the domain of g is B, we write this as

$$\{x \in B | \; g(x) \in A\}.$$

Example

If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2 - x}$, find each function and its domain.

$f \circ g$ $\qquad$ $g \circ f$ $\qquad$ $g \circ g$

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Domain :

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Domain : $[0, 4]$.

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Domain : $[-2, 2]$.