

Review problems Test 1

Math 140 Calculus I

Instructor: Todor Milev

The exam is closed books, no calculators will be allowed. The time for work will be 50 minutes. The problems on the exam will be similar to the problems in the review sheet. You will be asked a theoretical question (see the last two problems).

Problem 1 Find an expression for the function $(f \circ g)(x)$ and $(g \circ f)(x)$. Simplify your answer to a single fraction.

$$1. f(x) = \frac{3x-5}{x-2}, g(y) = \frac{y-2}{y-4}.$$

$$\frac{x+1}{1-x} = (x)(f \circ g) \quad , \quad \frac{9+x}{41+x^2} = (x)(g \circ f) \quad \text{answer}$$

$$2. f(x) = \frac{x-3}{x+2}, g(y) = \frac{y+3}{y-4}.$$

$$\frac{11-x}{8+x^2} = (x)(f \circ g) \quad , \quad \frac{9-x}{51+(x)^2} = (x)(g \circ f) \quad \text{answer}$$

Problem 2 Find all solutions in the interval $[0, 2\pi]$ of the equation.

$$1. 2 \cos^2 x - (1 + \sqrt{2}) \cos x + \frac{\sqrt{2}}{2} = 0.$$

$$\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} = x \quad \text{answer}$$

$$2. \sqrt{3} \sin x = \sin(2x).$$

$$\pi, \frac{\pi}{2}, 0, \frac{9\pi}{11}, \frac{9}{\pi} = x \quad \text{answer}$$

Problem 3 Evaluate the limit if it exists.

$$1. \lim_{x \rightarrow 1} \frac{3x^2+4x-7}{x^3-x}$$

$$5 \quad \text{answer}$$

$$2. \lim_{x \rightarrow -1} \frac{2x^2-3x-5}{x^3+1}$$

$$\frac{3}{2} \quad \text{answer}$$

Problem 4 Evaluate the limit if it exists.

$$1. \lim_{x \rightarrow 3^+} \frac{\sqrt{\frac{x^2}{9}-1}}{2x^2-3x-9}.$$

$$\infty \quad \text{answer}$$

$$2. \lim_{x \rightarrow -2^-} \frac{\sqrt{\frac{x^2}{4}-1}}{2x^2+3x-2}.$$

$$\infty \quad \text{answer}$$

Problem 5

1. (a) Solve the equation $x^2+13x+41 = 1$. (b) Use the intermediate value theorem to prove that the equation $x^2+13x+41 = \sin x$ has at least two solutions, lying between the two numbers found in (a).

2. (a) Solve the equation $x^2-15x+55 = 1$. (b) Use the intermediate value theorem to prove that the equation $x^2-15x+55 = \cos x$ has at least two solutions, lying between the two numbers found in (a).

Problem 6 Evaluate the limit if it exists.

$$1. \lim_{x \rightarrow -\infty} \sqrt{x^2+x} - \sqrt{x^2-x}.$$

$$-1 \quad \text{answer}$$

$$2. \lim_{x \rightarrow \infty} \sqrt{x^2+2x} - \sqrt{x^2-2x}.$$

$$2 \quad \text{answer}$$

Problem 7 Find the horizontal and vertical asymptotes of the curve

$$1. y = \frac{2x}{\sqrt{x^2+x+3}-3}.$$

$$-2 = y, \text{ horizontal}; x = 2, x = -3, \text{ vertical} \quad \text{answer}$$

$$2. y = \frac{3x^2}{\sqrt{x^2+2x+10}-5}.$$

$$-5 = y, \text{ horizontal}; x = 3, x = -5, \text{ vertical} \quad \text{answer}$$

Problem 8

1. State the intermediate value theorem.

2. Give the ε, δ -definition of limit.