Review problems Test 2
Math 140 Calculus I

Instructor: Todor Milev

The exam is closed books, no calculators will be allowed. The material covered will be the material from Lecture 8 to Lecture 17 (including Lectures 8 and 17). The time for work will be 50 minutes. The problems on the exam will be similar to the problems in the review sheet. You will be asked a theoretical question (see the last problem).

Problem 1 *(Lecture 8) Solve the equation.*

1. $e^{4x} + 3e^{2x} - 4 = 0$.

answer: $x = 0$

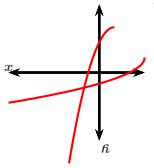
2. $4^{3x} - 2^{3x+2} - 5 = 0$.

answer: $x = \frac{\log_2 5}{3}$

Problem 2 *(Lecture 8) Find the inverse function f^{-1}. Plot roughly by hand $y = f(x)$. Using the plot of $y = f(x)$, plot roughly by hand $f^{-1}(x)$. Indicate the relationship between the graph of $f(x)$ and $f^{-1}(x)$.*

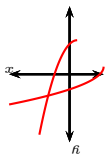
1. $f(x) = x^2 + 2x - 2, \qquad x \geq -1.$

answer: $f^{-1}(x) = \sqrt{x+3} - 1$



2. $f(x) = x^2 + x - 2, \qquad x \geq -\frac{1}{2}.$

answer: $f^{-1}(x) = \frac{\sqrt{4x+9}-1}{2}$



Problem 3 *(Lectures 10-16) Compute the derivative of the function.*

- $f(x) = \frac{1+x}{1+\frac{2}{x}}$.

answer: $\frac{x^2+4x+2}{(2+x)^2}$

- $f(x) = \frac{1+x}{1+\frac{3}{x}}$.

answer: $\frac{x^2+6x+3}{(3+x)^2}$

Problem 4 *(Lectures 10-16) Compute the derivative of the function.*

1. 2^{3^x}.

answer: $2^{3^x} 3^x (\ln 2)(\ln 3)$

2. 3^{2^x}.

answer: $3^{2^x} 2^x (\ln 2)(\ln 3)$

Problem 5 *Compute the derivative of the function.*

1. $\sec^2(3x^2)$.

answer: $12 \frac{x \sin(3x^2)}{(\cos(3x^2))^3}$

2. $\csc^2(3x^2)$.

answer: $-12 \frac{x \cos(3x^2)}{(\sin(3x^2))^3}$

Problem 6 *(Lecture 15) Use implicit differentiation to express $\frac{dy}{dx}$ via y and x, where x and y satisfy the following relation.*

1. $x^4(x + y) = y^2(3x - y)$.
2. $2x^3 + x^2y - xy^3 = 2$.

Problem 7 *(Lecture 15) Use implicit differentiation to find an equation of the tangent line to the curve at the given point.*

- $x^{2/3} + y^{2/3} = 4$ *at* $(-3\sqrt{3}, 1)$.

answer: $y = \frac{\sqrt{3}}{3}x + 4$

- $x^2y^3 + x^2 - y^2 = 1$ *at* $(1, 1)$.

answer: $y = -4x + 5$

Problem 8 *(Lecture 17) A wedge of radius 2 (depicted below) is folded into a cone cup. The volume varies depending on the angle of the wedge. Find the maximal possible volume of the cone cup and the angle of the wedge for which this maximal volume is achieved.*

Problem 9 *(Lecture 14)*

- *State the quotient rule for computing the derivative of* $\left(\frac{f}{g}\right)'$*. Derive the quotient rule using the chain rule, the negative power rule and the product rule.*
- *State the power rule for computing the derivative* $(x^r)'$ *for an arbitrary real number* r *and* $x > 0$*. Derive the power rule using the chain rule, the rule* $(e^x)' = e^x$*, the constant multiple derivative rule and the logarithm derivative rule* $(\ln x)' = \frac{1}{x}$*.*