

Problem Set 2:

1. Solve following differential equations:

① $\frac{dy}{dx} = y \cdot (10 - y)$

② $x^2 \cdot \frac{dy}{dx} = y + 1$

③ $\sin x \cdot \frac{dy}{dx} = y^2 + 1$

④ $\frac{dy}{dx} = -y + \sin x$

⑤ $y'' = y' \cdot \tan x + 1$

⑥ $\frac{dy}{dx} + (1 + 3x^2) \cdot y = 0$

⑦ $y' = \sqrt{y}$

*⑧ $\frac{dy}{dx} = (y + x)^2$

Check which of the following could be applied to integrating factor:

1) $y' = y \cdot (y + x)$

5) $(y')^2 + y = x$

2) $\sin x \cdot y' = y + 1$

6) $x^2(y') = y^2 - 1$

3) $y \cdot y' = \sin(y + x)$

7) $y' + e^y = e^x$

4) $y' + \sin y = x$

8) $y' + \sin^2 y = \cos^2 y$

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2. Application:

17. A storage tank contains 2000 gal of gasoline that initially has 100 lb of an additive dissolved in it. In preparation for winter weather, gasoline containing 2 lb of additive per gallon is pumped into the tank at a rate of 40 gal/min. The well-mixed solution is pumped out at a rate of 45 gal/min. How much of the additive is in the tank 20 min after the pumping process begins?

2) A 66-kg cyclist on a 7-kg bicycle starts coasting on level ground at 9 m/sec. The k is 3.9 kg/sec. The resistance is proportional to its velocity.

- How far will the cyclist coast before reaching a complete stop?
- How long will it take the cyclist's speed to drop to 1 m/sec.

3). A population P_0 of fish is living in a pool with maximum carrying capacity M_0 . Suddenly, a catastrophe came and decreased the carrying capacity to M (considerably less than M_0), actually less than P_0 . How will the population become after time t ?

Hint: $\frac{dP}{dt} = r \cdot P \cdot (M - P)$

4) Social diffusion: to spread a piece of news, divide people into two parts: those who know and those who don't. In a fixed population, say N , assume the rate of diffusion is proportional to the number who know times the number yet to receive it. Describe X : number who know by differential equation.

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3. Taylor Formula: (general n-expression). at $x=0$.

1) $\ln \frac{1+t}{1-t}$;

8) $\arcsin x$;

2) $\sin x \cdot \cos x$;

9) $\sqrt[3]{1+4t+4t^2}$

3) $\frac{1}{2-3x+x^2}$;

10) $\frac{1}{\sqrt{1-t}}$

4) e^{t^2} ;

11) $\frac{1}{\sqrt{1-t^2}}$

5) $\frac{\sin x}{x}$;

6) $\arctan x$;

7) $\sin(x + \frac{\pi}{4})$;

4. Find k satisfies:

1) $o(x^2) - o(x^2)$

2) $o(x^2) \cdot (x^2 - 1)$

3) $o(x) - o(x^3)$

4) $\sin x^2 \cdot o(x)$

5) $(e^x - 1) \cdot o(x^2)$

6) $\sin x \cdot \cos x$

7) $3o(x)$.

8) $\sqrt{1+x} - \sqrt{1-x}$

5. Estimate Error:

1) $f(x) = \sqrt{1+x}$, $R_n f(x)$, $0 \leq x \leq 1$

2) $f(x) = e^x$, $R_n f(x)$, $0 \leq x \leq 1$

3) $f(x) = \sin x^2$, $R_n f(x)$, $0 \leq x \leq 1$.