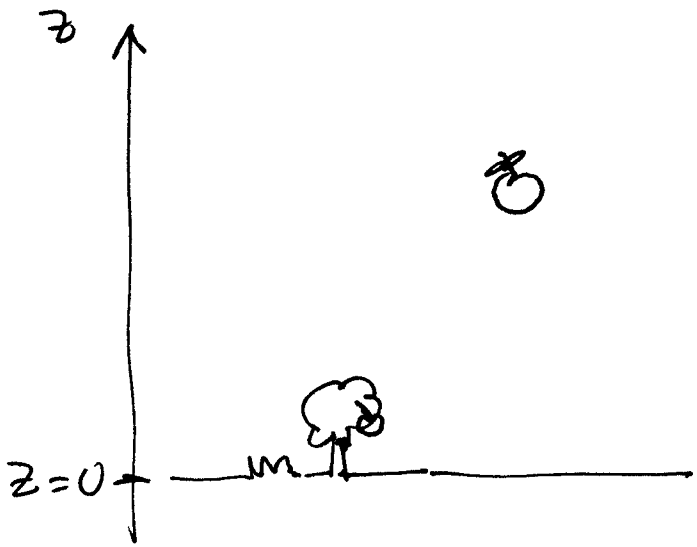
N2 : Free fall in the Flat-Earth approximation.



$$F = m\ddot{z}$$

$$F = -mg$$

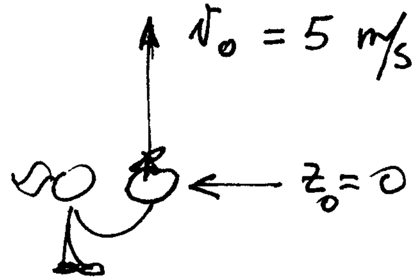
$$-\cancel{m}g = \cancel{m}\ddot{z} \rightarrow \boxed{\ddot{z} = -g}$$

$$\ddot{z} = -g \rightarrow \begin{cases} v = \dot{z} & (1) \\ \dot{v} = -g & (2) \end{cases}$$

$$(2) \rightarrow \quad v = -gt + A$$

$$(1) \rightarrow \quad z = -\frac{1}{2}gt^2 + At + B$$

At $t=0$:



$v_0 = A = 5$ m/s

$z_0 = B = 0$

$$\rightarrow \quad z = -\frac{1}{2}gt^2 + (5\ \text{m/s})\,t$$

Slightly different approach:

$v = \dot{z}$ (1)

$\dot{v} = -g$ (2)

Integrate (2): $\dfrac{dv}{dt} = -g$

$$\int_{v_0}^{v(t)} dv = -g\int_0^t dt \qquad \text{(same } t\text{)}$$

$$v(t) - v_0 = -gt \quad \rightarrow \quad \boxed{v(t) = v_0 - gt}$$

Integrate 1 : $\dfrac{dz}{dt} = v = v_0 - gt$

$$\int_{z_0}^{z(t)} dz = \int_0^t (v_0 - gt)\,dt$$

$$z(t) - z_0 = v_0 t - \frac{1}{2} g t^2$$

$$z(t) = z_0 + v_0 t - \frac{1}{2} g t^2$$

Integrating the EOM when F is a function of position only.

The EOM is $$m \frac{dv}{dt} = F(x)$$

$$m\, dv = F(x)\, dt$$

?

Use $v = \frac{dx}{dt} \rightarrow dt = \frac{dx}{v}$

Then

$$m\, dv = F(x) \frac{dx}{v}$$

$$\int_{v_0}^{v(x)} m v\, dv = \int_{x_0}^{x} F(x)\, dx$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \underbrace{\int_{x_0}^{x} F(x)\,dx}_{W(x_0 \to x)}$$

$$T - T_0 = W(x_0 \to x)$$

This is the <u>work-KE theorem</u>.

If $F(x)$ can be written as $F(x) = -\frac{dV(x)}{dx}$

$$\cancel{T(x)} - T_0 = -\int_{x_0}^{x} \frac{dV}{dx}\cdot dx = -\int_{V(x_0)}^{V(x)} dV = V_0 - V(x)$$

$$T(x) + V(x) - \left[T_0 + V_0\right] = 0$$

$V(x)$ is the potential energy associated with F.

A handy tool is the energy diagram: