Drag forces:

and $|\vec{F}|$ is increasing function of v, so

non-conservative.

In general, $F = -c_1 v - c_2 v|v|$

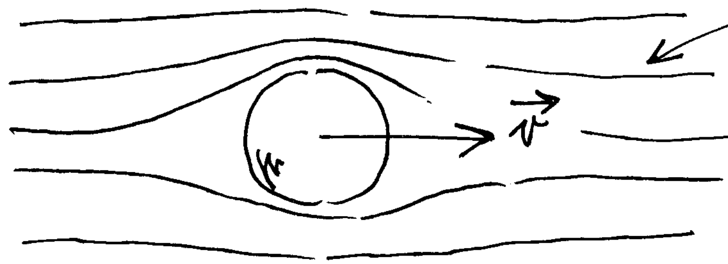
For air drag, $c_1 = 1.55 \times 10^{-4} D$ in SI units

$c_2 = 0.22 D^2$

Two scenarios :

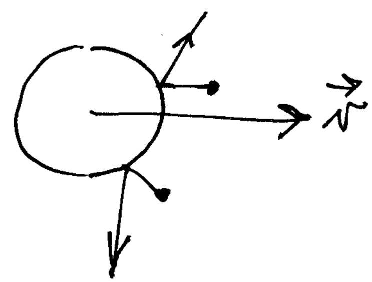
"Slow" motion :

$|\vec{F}| \propto |\vec{v}|$



friction between layers : viscosity of fluid

"Fast" motion :



$|\vec{F}| \propto |\vec{v}|^2$

The critical speed v_c at which $\frac{\text{linear term}}{\text{quadratic}} = 1$

is given by

$$c_1 |v_c| = c_2 |v_c|^2 \rightarrow |v_c| = \frac{c_1}{c_2}$$

so $$|v_c| = \frac{1.55 \times 10^{-4} D}{0.22 D^2} = \frac{7.05 \times 10^{-4}}{D}$$

Let's solve the linear drag problem:

1. Horizontal motion:

$$m \frac{dv}{dt} = -c_1 v$$

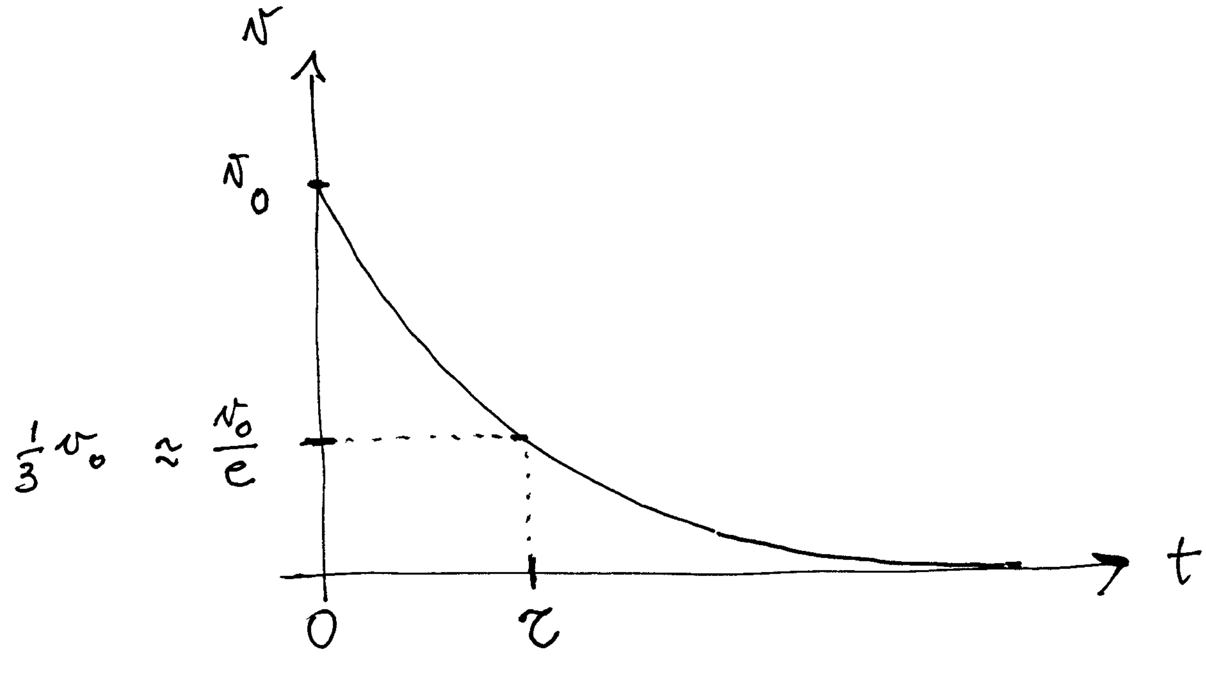
$$\frac{dv}{dt} = -\frac{c_1}{m} v$$

has dimensions of $(\text{time})^{-1}$ call it $\frac{1}{\tau}$

$$\frac{dv}{dt} = -\frac{v}{\tau}$$

$$\int_{v_0}^{v(t)} \frac{dv}{v} = -\int_0^t \frac{dt}{\tau} \rightarrow \ln\left(\frac{v}{v_0}\right) = -\frac{t}{\tau}$$

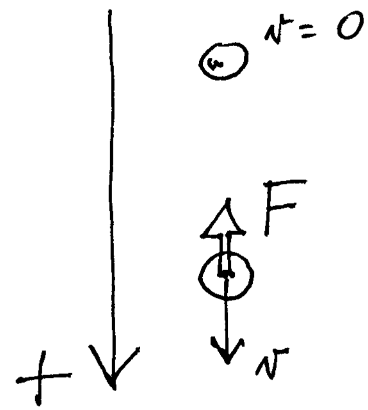
$$\rightarrow \boxed{v = v_0 e^{-t/\tau}}$$



See textbook for the quadratic case.

2. Object in free fall (starting from rest)
Linear drag force:

$$m\frac{dv}{dt} = -c_1 v + mg$$



It reaches "terminal velocity":

$$0 = -c_1 v_t + mg$$

$$v_t = \frac{m}{c_1} g$$