Recap: $$\frac{d^2 f}{dt^2} - \omega^2 f = 0$$

In terms of D, $$[D^2 - \omega^2] f = 0$$

ω has dimensions of t^{-1}, or frequency

$$[D + \omega][D - \omega] f = 0$$

Either $[D - \omega] f = 0 \longrightarrow f_1 = e^{\omega t}$

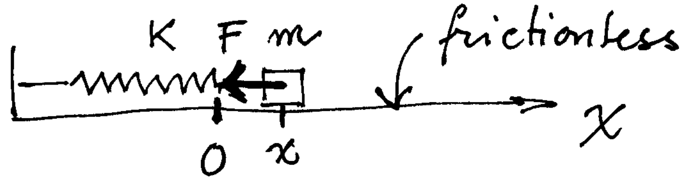
or

$[D + \omega] f = 0 \longrightarrow f_2 = e^{-\omega t}$

General solution: $f = Ae^{\omega t} + Be^{-\omega t}$

SIMPLE HARMONIC OSCILLATIONS (SHO)

Prototype:



$$F = -kx \qquad \text{Hooke's law}$$

EOM:

$$m\ddot{x} = -kx$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$(\text{frequency})^2$ call it ω^2

In terms of D,

$$\left[D^2 + \omega^2\right]x = 0$$

$$[D + i\omega][D - i\omega]x = 0$$

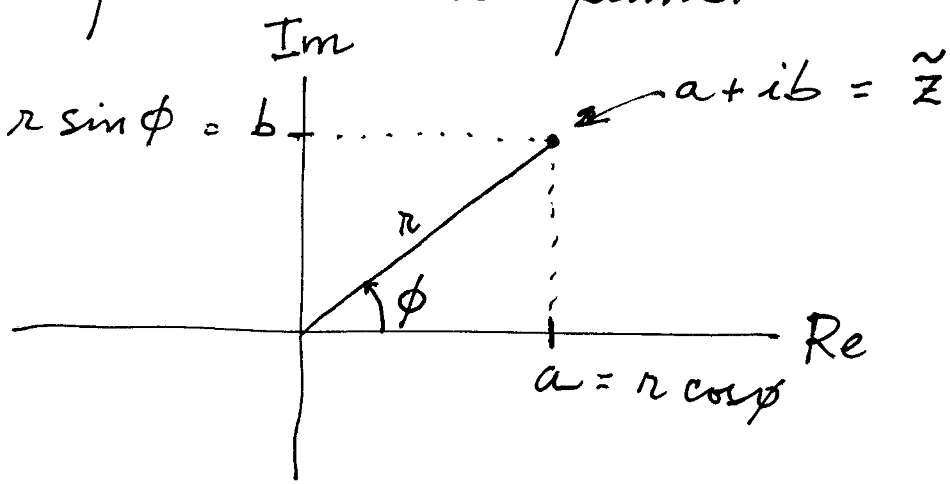
where $i^2 = -1$.

2 solutions, $e^{i\omega t}$ and $e^{-i\omega t}$

General solution: $Ae^{i\omega t} + Be^{-i\omega t}$

Digression: COMPLEX NUMBERS

Represents point on a plane:



Complex conjugation : $\tilde{z} = a + ib$

everywhere do the sub $i \rightarrow -i$

$$\tilde{z}^* = a - ib$$

$$\tilde{z} \cdot \tilde{z}^* = (a+ib)(a-ib) = a^2 + b^2 = |\tilde{z}|^2$$

$$\tilde{z} + \tilde{z}^* = a + ib + a - ib = 2a = 2\,\mathrm{Re}[\tilde{z}]$$

$$\tilde{z} - \tilde{z}^* = a + ib - a + ib = 2ib = 2i\,\mathrm{Im}[\tilde{z}]$$

Euler's identity : $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$

$f(\theta)$ is a function that satisfies $\frac{df}{d\theta} = if$

$e^{i\theta}$ is such a function.

but so is $\cos\theta + i\sin\theta$!