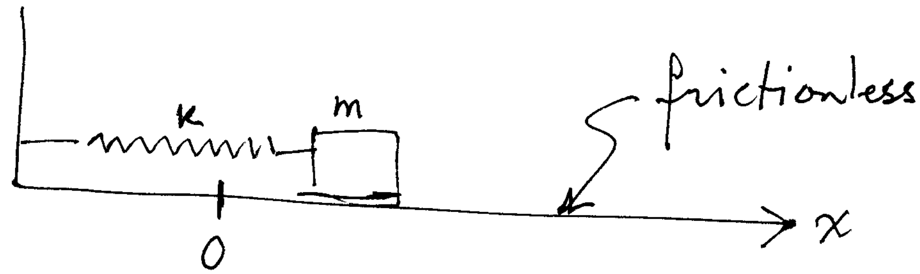
SHO:



$$F = -kx = m\frac{d^2x}{dt^2}, \quad \text{or} \quad \ddot{x} + \omega^2 x = 0$$

$$\frac{K}{m}$$

Two independent solutions: $e^{i\omega t}$, $e^{-i\omega t}$

The general solution is $x(t) = \tilde{A}e^{i\omega t} + \tilde{B}e^{-i\omega t}$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad //$

Because $x(t)$ is real, $x^*(t) = \tilde{A}^* e^{-i\omega t} + \tilde{B}^* e^{+i\omega t}$

$$\Rightarrow \quad \tilde{A}^* = \tilde{B}$$

$$x(t) = A\cos\omega t + B\sin\omega t$$

$$v(t) = -\omega A\sin\omega t + \omega B\cos\omega t$$

Initial conditions: At $t=0$ $x = x_0$ $v = v_0$

$$x_0 = A \quad \rightarrow \quad A = x_0$$

$$v_0 = \omega B \qquad B = \frac{v_0}{\omega}$$

If we wrote $x(t) = A\cos(\omega t + \phi)$

$$v(t) = -\omega A \sin(\omega t + \phi)$$

$$x_0 = A\cos\phi$$

$$v_0 = -\omega A\sin\phi$$

Write $\tilde{A} = a + ib$, $\tilde{B} = a - ib$

then $$x(t) = (a+ib)e^{i\omega t} + (a-ib)e^{-i\omega t}$$

$$= a\underbrace{\left(e^{i\omega t} + e^{-i\omega t}\right)}_{2\cos\omega t} + ib\underbrace{\left(e^{i\omega t} - e^{-i\omega t}\right)}_{2i\sin\omega t}$$

$$= \underbrace{2a}_{C}\cos\omega t \; \underbrace{-\,2b}_{D}\sin\omega t$$

$$\boxed{x(t) = C\cos\omega t + D\sin\omega t}$$

If instead we write $\tilde{A} = Ae^{i\phi}$, $\tilde{B} = Ae^{-i\phi}$

Then $$x(t) = \underbrace{Ae^{i(\omega t+\phi)} + Ae^{-i(\omega t+\phi)}}_{2A\,\mathrm{Re}\left[e^{i(\omega t+\phi)}\right]}$$

$$= \underbrace{2A}_{\equiv\, C} \cos(\omega t+\phi)$$

$$\boxed{x(t) = C \cos(\omega t+\phi)}$$

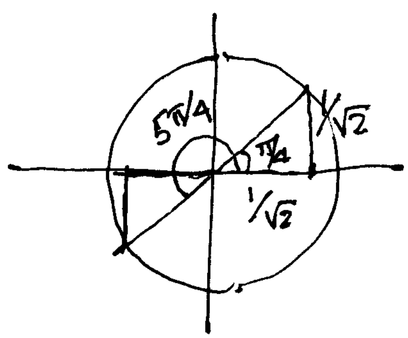
$$\tan\phi = -\frac{v_0}{\omega x_0}$$

Beware! 2 solutions.

F. ex.: $\tan\phi = 1 \rightarrow \phi = \frac{\pi}{4}$

Check $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \rightarrow \tan\frac{\pi}{4} = 1$

But $\phi = \frac{5\pi}{4}$ also works: $\left.\begin{array}{l} \sin\frac{5\pi}{4} = -\frac{1}{\sqrt{2}} \\ \cos\frac{5\pi}{4} = -\frac{1}{\sqrt{2}} \end{array}\right\} \rightarrow \tan\frac{5\pi}{4} = 1$



$$x_0^2 + \left(\frac{v_0}{\omega}\right)^2 = A^2 \cos^2\phi + A^2 \sin^2\phi = A^2$$