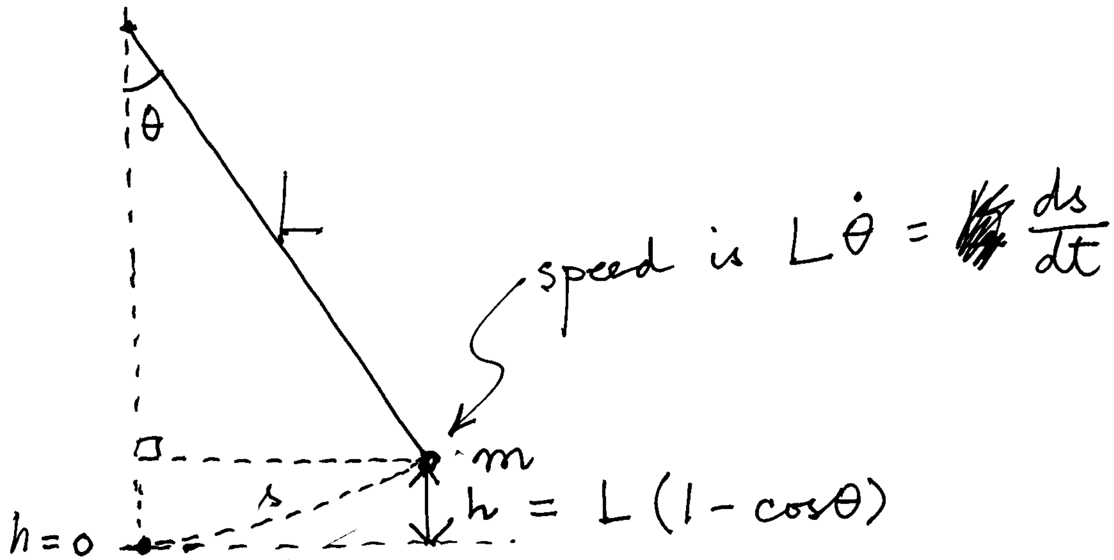
The pendulum:



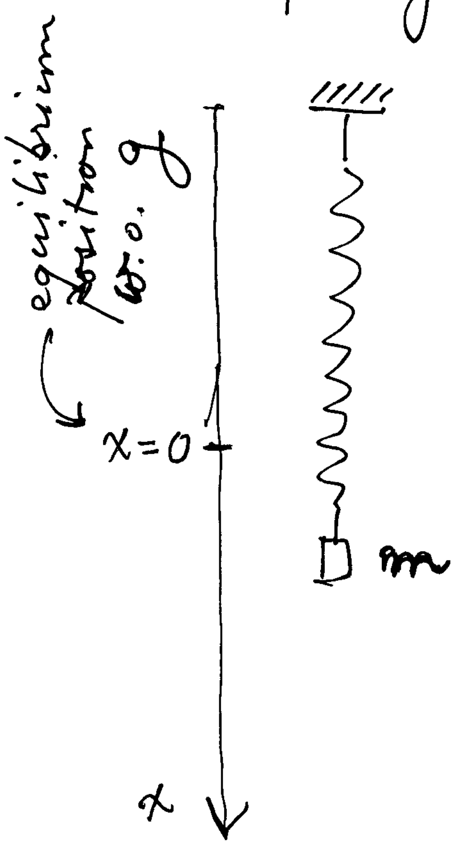
$$E = \frac{1}{2} m L^2 \dot\theta^2 + mgL(1-\cos\theta)$$

$$\frac{dE}{dt} = 0 = mL^2 \dot\theta \ddot\theta + mgL \sin\theta \cdot \dot\theta = mL\dot\theta \underbrace{\left(L\ddot\theta + g \sin\theta\right)}_{=0}$$

$$\boxed{\ddot\theta + \frac{g}{L}\sin\theta = 0}$$

For small amplitudes, $\sin\theta \approx \theta$, and it's a SHO.

Spring, with gravity:



$$\text{N2:} \quad m\ddot{x} = -kx + mg = 0$$

or

$$m\ddot{x} + \underbrace{kx - mg}_{k\underbrace{\left(x - \frac{mg}{k}\right)}_{\text{call this } z}} = 0$$

so $\dot{z} = \dot{x}$, $\ddot{z} = \ddot{x}$

$$m\ddot{z} + kz = 0$$

$$\longrightarrow z = A\cos(\omega t + \phi_0), \quad \omega = \sqrt{\frac{k}{m}}$$

Then $\quad x = \frac{mg}{k} + A\cos(\omega t + \phi_0)$

GENERIC SHO: $\quad \ddot{x} + \omega^2 x = 0$

given by the parameters of the oscillator

deviation from equilibrium

Mass, spring: $\quad \omega^2 = \frac{k}{m}$

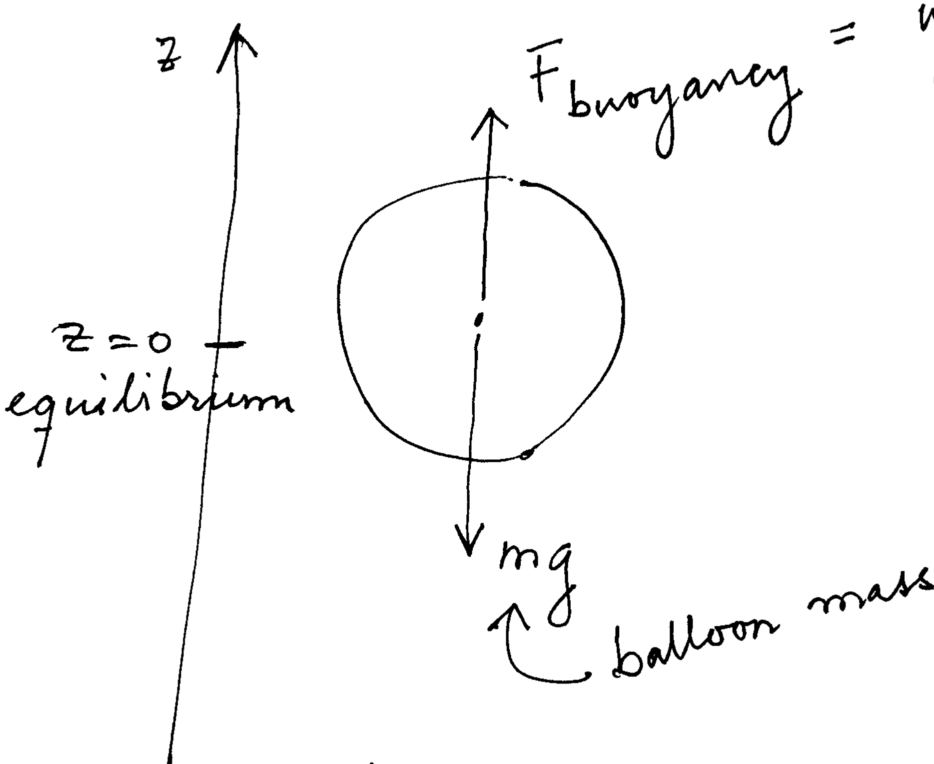
Pendulum: $\quad \omega^2 = \frac{g}{L}$

Solution: $\quad x = A\cos(\omega t + \phi_0)$

given by the IC

Example:

$$F_{\text{buoyancy}} = \text{weight of displaced air} = m_{\text{air}}\, g$$
$$= \rho_{\text{air}} V g$$



At $z = 0$, $mg = m_{\text{air}}\, g$

A model of the atmosphere: $\rho_{\text{air}}(z) = \underset{\text{at } z=0}{\rho_0}\, e^{-z/H}$

The net force on the balloon:

$$F(z) = (m_{air} - m)g = mg\left(e^{-z/H} - 1\right)$$

$$m_{air} = \underbrace{\rho_0 e^{-z/h} \cdot V}_{\substack{\times \\ \shortparallel \\ m}}$$

We will have SHO if $F(z) = -kz$