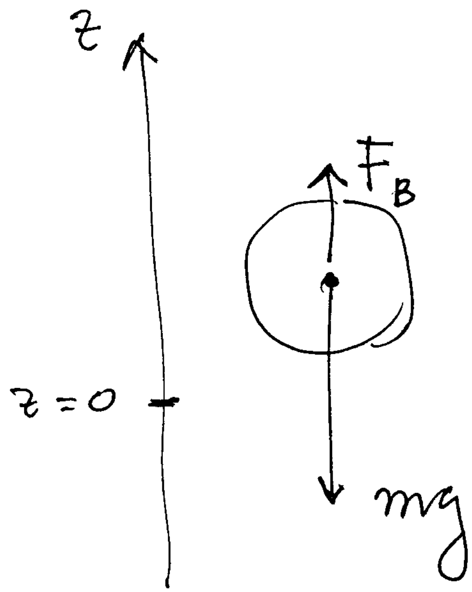


scale height

$$F = mg\left(e^{-z/H} - 1\right)$$

Q1: For $z \ll H$, show F is an elastic restoring force, i.e. $F \propto -z$

Find the 1st order approximation to $e^{-z/H}$ around $z=0$:

$$e^{-z/H} \approx 1 - \frac{1}{H} z \quad \rightarrow \quad F \approx -\frac{mg}{H} z$$

Q2: Write the EOM. Is it a SHO?

$$\ddot{z} + \frac{g}{H} z = 0 \quad \rightarrow \quad \omega = \sqrt{\frac{g}{H}}$$

Q3 : $H = 10$ km. What is the period?

(A) 1 s

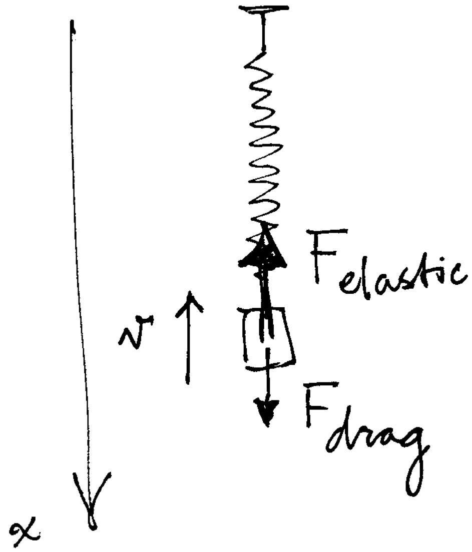
(B) 1 min

(C) 1 hr

(D) 1 day

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{H}{g}} \approx 3 \text{ min}$$

DAMPED OSCILLATIONS:



N2: $\quad m\ddot{x} = -kx - c\dot{x}$

linear because $|\dot{x}| \ll v_t$

$$\ddot{x} + \underbrace{\frac{k}{m}}_{\omega_0^2} x + \underbrace{\frac{c}{m}}_{2\gamma}\dot{x} = 0$$

$$\left[D^2 + 2\gamma D + \omega_0^2\right] x = 0$$

Pretend we have some number λ: $\quad \lambda^2 + 2\gamma\lambda + \omega_0^2 = 0$

$$\lambda = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2} = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

$$\left(\lambda + \gamma - \sqrt{\gamma^2 - \omega_0^2}\right)\left(\lambda + \gamma + \sqrt{\gamma^2 - \omega_0^2}\right) = 0$$

$$\left[\left(D + \gamma - \sqrt{\gamma^2 - \omega_0^2}\right)\left(D + \gamma + \sqrt{\gamma^2 - \omega_0^2}\right)\right] x = 0$$

Case 1 : $\gamma < \omega_0$. Call $\sqrt{\gamma^2 - \omega_0^2} = iq$

$$\left[(D + \gamma - iq)(D + \gamma + iq)\right] x = 0$$

Either $(D + \gamma - iq)\, x = 0 \rightarrow x = \tilde{A}\, e^{-(\gamma - iq)t}$

or $(D + \gamma + iq)\, x = 0 \longrightarrow x = \tilde{B}\, e^{-(\gamma + iq)t}$

General solution is

$$x = \tilde{A}\, e^{-(\gamma - iq)t} + \tilde{B}\, e^{-(\gamma + iq)t}$$

$=$

$$x^* = \tilde{A}^*\, e^{-(\gamma + iq)t} + \tilde{B}^*\, e^{-(\gamma - iq)t}$$

$$\therefore \quad \tilde{A}^* = \tilde{B}$$

$$x = e^{-\gamma t} \cdot 2\,\mathrm{Re}\left[\tilde{A} e^{iqt}\right]$$

Write $\tilde{A} = A e^{i\phi}$

$$x = 2 e^{-\gamma t}\,\mathrm{Re}\left[A e^{i(qt+\phi)}\right]$$

$$= \underbrace{2A}_{C} e^{-\gamma t} \cos(qt + \phi)$$

$$x = C e^{-\gamma t} \cos(qt + \phi)$$