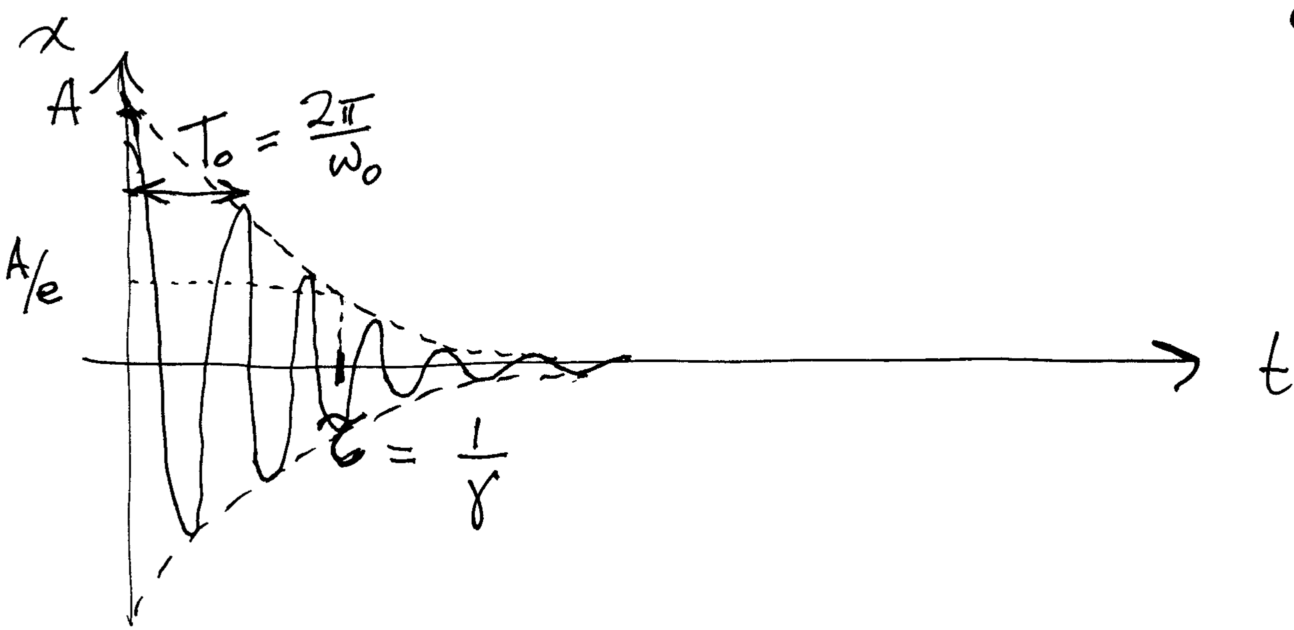
Quality Factor:

We begin with a slightly underdamped osc.

$$x(t) = A e^{-\gamma t} \cdot \cos(\omega_d t + \phi_0)$$

$$\omega_d = \sqrt{\omega_0^2 - \gamma^2}$$

If $\gamma \ll \omega_0$, $\omega_d \approx \omega_0$. If we write $\gamma = \frac{1}{\tau}$



Define quality factor $Q = \dfrac{\omega_0}{\gamma}$

Another interpretation of Q:

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \quad \text{in SHM}$$

For a damped oscillator, $E \propto (e^{-\gamma t} \cdot A)^2$

$$E(t) = E(t=0) \cdot e^{-2\gamma t}$$

Let's try to estimate ΔE in one cycle.

beginning of cycle

$$\Delta E = \underbrace{E(t+T)}_{E(t) \cdot e^{-2\gamma T}} - E(t) = E(t)\left(e^{-2\gamma T} - 1\right)$$

Taylor approximation of 1st order of $e^{-2\gamma T}$ around $T=0$ is ... $1 - 2\gamma T$

$$\Delta E \simeq -2\gamma T_0 \cdot E(t)$$

$$\frac{|\Delta E|}{E} = 2\gamma T_0 = 2 \times 2\pi \times \frac{\gamma}{\omega_0} = \frac{4\pi}{Q}$$

$$Q = 4\pi \Big/ {|\Delta E|}/{E}$$

fractional change in energy over one cycle.