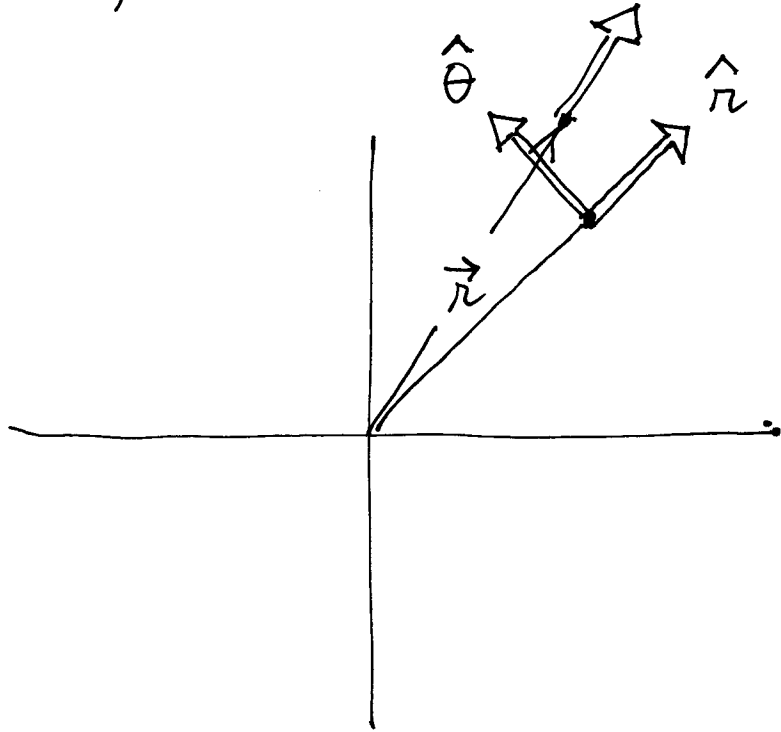
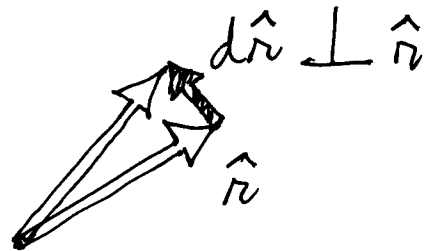


Position, velocity and acceleration in 2D polar coordinates:

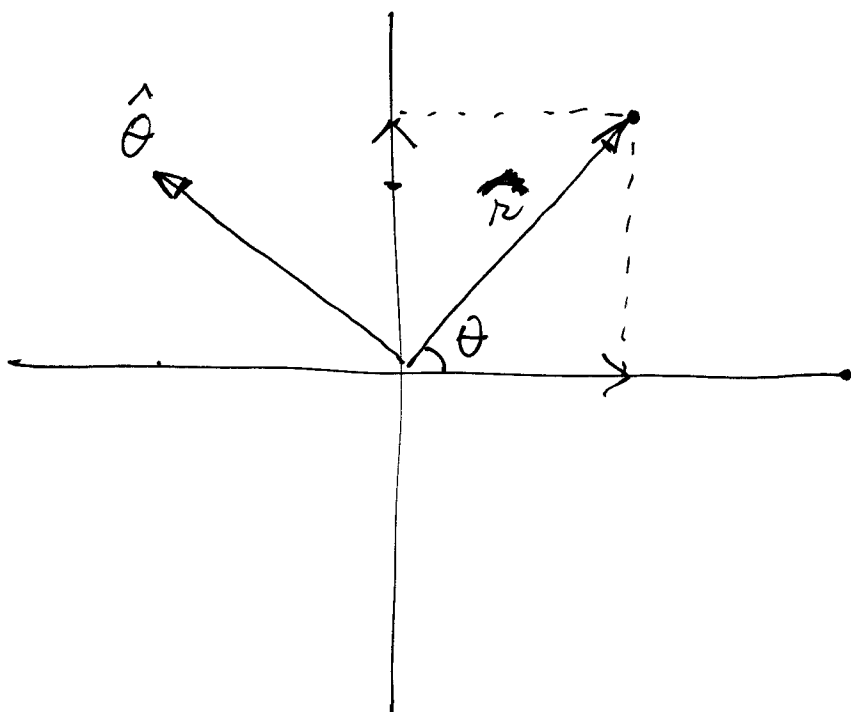


$$\vec{r} = r \hat{r}$$

$$\dot{\vec{r}} = \dot{r} \hat{r} + r \left(\dot{\hat{r}} \right) ?$$



Introduce $\hat{\theta} \perp \hat{r}$



$$\vec{r} = r \cos \theta \hat{i} + r \sin \theta \hat{j}$$

$$\hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\dot{\hat{r}} = -\sin \theta \cdot \dot{\theta} \hat{i} + \cos \theta \cdot \dot{\theta} \hat{j}$$

$$= \dot{\theta} \underbrace{(-\sin \theta \hat{i} + \cos \theta \hat{j})}_{\hat{\theta}}$$

$$\dot{\hat{r}} = \dot{\theta} \hat{\theta}$$

$\therefore$

$$\boxed{\dot{\vec{r}} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta}}$$

$$\ddot{\vec{r}} = \ddot{r} \hat{r} + 2\dot{r} \dot{\theta} \hat{\theta} + r \ddot{\theta} \hat{\theta} + r \dot{\theta} \dot{\hat{\theta}}$$

$$\begin{aligned} \hat{\theta} &= -\sin\theta \hat{i} + \cos\theta \hat{j} \\ \dot{\hat{\theta}} &= -\cos\theta \cdot \dot{\theta} \hat{i} - \sin\theta \cdot \dot{\theta} \hat{j} \\ &= -\dot{\theta} \hat{r} \end{aligned}$$

$$\boxed{\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2) \hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \hat{\theta}}$$

F. ex. circular motion, constant speed

$$\begin{aligned} \dot{r} &= 0 \\ \ddot{r} &= 0 \end{aligned}$$

$$\begin{aligned} \dot{\theta} &= \text{constant} = \omega \\ \ddot{\theta} &= 0 \end{aligned}$$

$$\ddot{\vec{r}} = -r\omega^2 \hat{r}$$

and $\dot{\vec{r}} = r\omega \hat{\theta}$