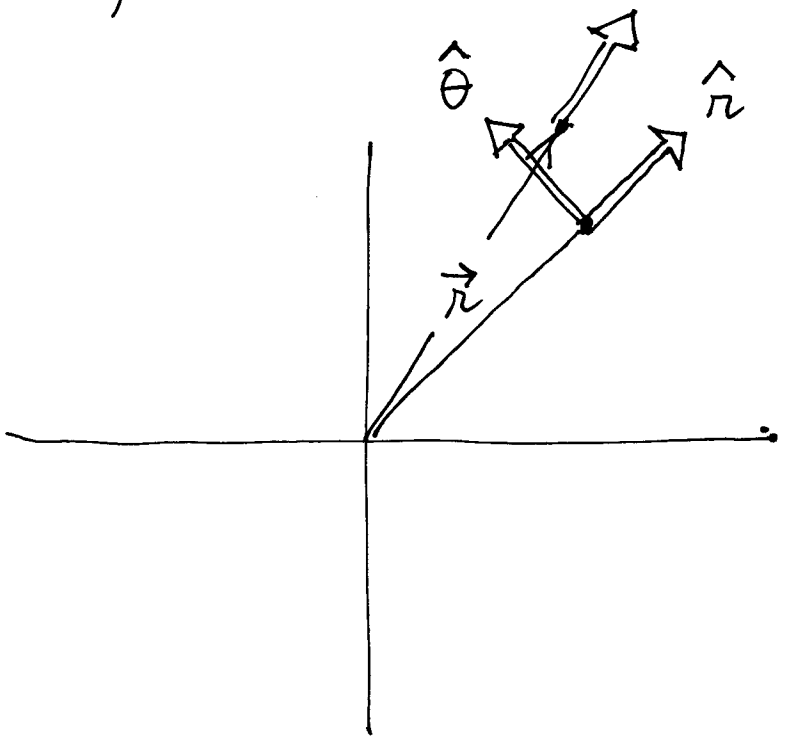
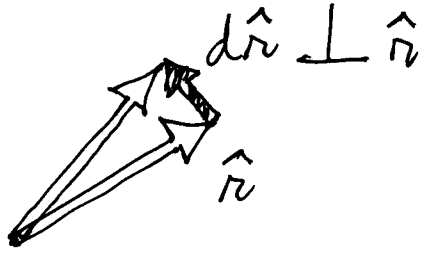
Position, velocity and acceleration in 2D polar coordinates:

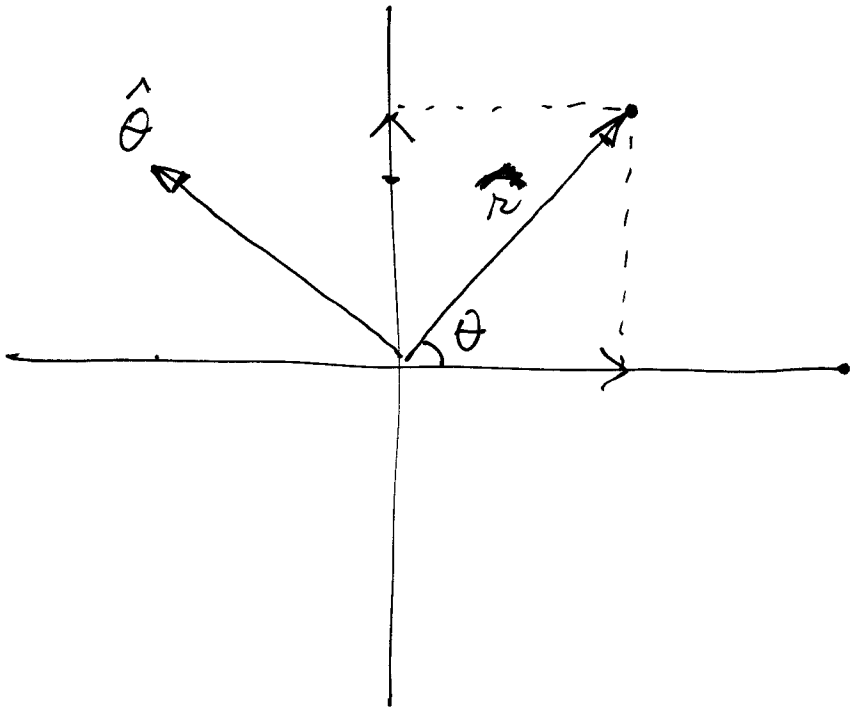


$$\vec{r} = r\hat{r}$$

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\hat{r}} \; ?$$



Introduce $\hat{\theta} \perp \hat{r}$



$$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$$

$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\dot{\hat{r}} = -\sin\theta\cdot\dot{\theta}\,\hat{i} + \cos\theta\cdot\dot{\theta}\,\hat{j}$$

$$= \dot{\theta}\underbrace{\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)}_{\hat{\theta}}$$

$$\dot{\hat{r}} = \dot{\theta}\,\hat{\theta}$$

$$\therefore \quad \boxed{\dot{\vec{r}} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta}}$$

$$\ddot{\vec{r}} = \ddot{r}\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}}$$

$$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$$

$$\dot{\hat{\theta}} = -\cos\theta\cdot\dot{\theta}\,\hat{\imath} - \sin\theta\cdot\dot{\theta}\,\hat{\jmath}$$

$$= -\dot{\theta}\hat{r}$$

$$\boxed{\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}}$$

F. ex. circular motion, constant speed

$$\dot{r} = 0 \qquad \dot{\theta} = \text{constant} = \omega$$

$$\ddot{r} = 0 \qquad \ddot{\theta} = 0$$

$$\ddot{\vec{r}} = -r\omega^2\hat{r}$$

and $\dot{\vec{r}} = r\omega\hat{\theta}$