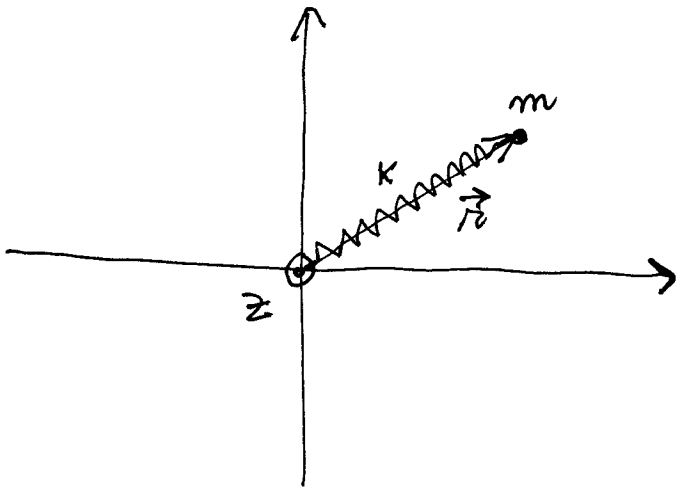
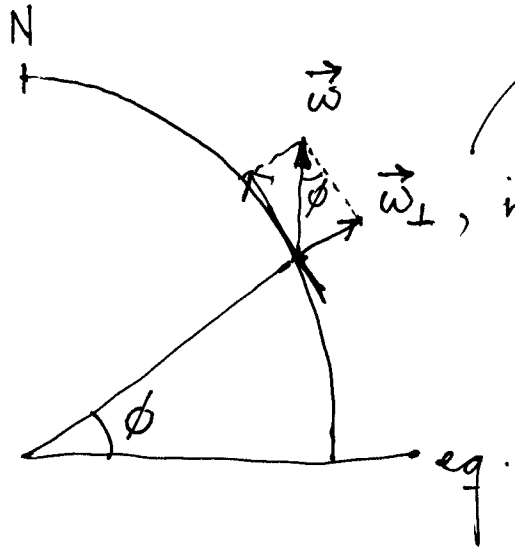
# FOUCAULT OSCILLATOR :




EOM: $m\ddot{\vec{r}} = -k\vec{r} - 2m\vec{\omega}_\perp \times \dot{\vec{r}} + m\omega_\perp^2 \vec{r}$

$\omega \sin\phi \cdot \hat{k}$

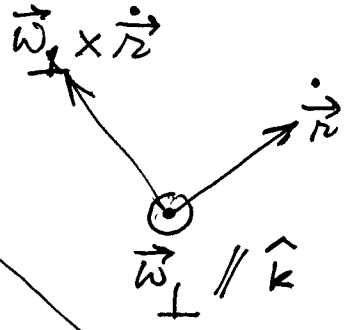



$\vec{\omega}_\perp$, in magnitude $\omega \sin\phi$

Use complex coordinates $\tilde{r} = x + iy$

$$\dot{\tilde{r}} = \dot{x} + i\dot{y}$$

$$\vec{\omega}_\perp \times \dot{\tilde{r}} = \omega \sin\phi \, \hat{k} \times \dot{\vec{r}}$$




$$m\ddot{\tilde{r}} = -(k + m\omega_\perp^2)\tilde{r} - 2mi\omega\sin\phi\,\dot{\tilde{r}}$$

$$\ddot{\tilde{r}} = -(\omega_0^2 - \omega_\perp^2)\tilde{r} - 2i\omega\sin\phi\,\dot{\tilde{r}}$$

$$\ddot{x} = -(\omega_0^2 - \omega_\perp^2)x - 2i\omega\sin\phi\,\dot{x}$$

$$\ddot{y} = -(\omega_0^2 - \omega_\perp^2)y - 2i\omega\sin\phi\,\dot{y}$$

$$\tilde{z} = |\tilde{z}|e^{i\theta}$$

$$i = e^{i\pi/2}$$

$$i\tilde{z} = |\tilde{z}|e^{i(\theta+\pi/2)}$$

$$\ddot{\tilde{r}} + (\omega_0^2 - \omega_\perp^2)\tilde{r} + 2i\omega_\perp \dot{\tilde{r}} = 0$$

Use diff. op. $D$:

$$\left[D^2 + 2i\omega_\perp D + (\omega_0^2 - \omega_\perp^2)\right]\tilde{r} = 0$$

Factor

Solve $\lambda^2 + 2i\omega_\perp \lambda + (\omega_0^2 - \omega_\perp^2) = 0$

$$\lambda = -i\omega_\perp \pm i\omega_0$$

$$\tilde{r} = e^{-i\omega_\perp t}\left(Ae^{+i\omega_0 t} + Be^{-i\omega_0 t}\right)$$

$$\tilde{r} = Ae^{i\omega_0 t} + 1$$

$$\ddot{\tilde{r}} = -\omega_0^2 Ae^{i\omega_0 t}$$

$$\lambda = \frac{-2i\omega_\perp \pm \sqrt{-4\omega_\perp^2 - 4(\omega_0^2 - \omega_\perp^2)}}{2}$$

$$= -i\omega_\perp \pm i\omega_0$$

$$\tilde{z} = e^{-i\omega_\perp t}\left(Ae^{i\omega_0 t} + Be^{-i\omega_0 t}\right)$$