NAME: _____

CSE 421
Introduction to Algorithms
Sample Midterm Exam Winter 2013

Anup Rao 30 October 2013

DIRECTIONS:

- Answer the problems on the exam paper.
- You are allowed one cheat sheet.
- Justify all answers with proofs, unless the facts you need have been proved in class or in the book.
- If you need extra space use the back of a page
- You have 50 minutes to complete the exam.
- Please do not turn the exam over until you are instructed to do so.
- Good Luck!

1	/20
2	/20
3	/25
4	/25
Total	/90

1. (20 points, 5 each) For each of the following problems answer **True** or **False** and BRIEFLY JUSTIFY you answer.

   (a) $n^{2.1} = O(n^2 \log n)$.

   (b) There is a polynomial time algorithm for deciding whether a graph is bipartite or not.

   (c) If the running time of an algorithm satisfies the recurrence $T(n) \leq T(n/10)+T(9n/10)+c(n)$, then $T(n) = O(n^2)$.

   (d) If an undirected connected graph G has a cycle with a unique heaviest weight edge e, and all edge weights in G are positive and distinct, then e cannot be part of any minimum spanning tree.

2. (25 points) A perfect matching of an undirected graph on $2n$ vertices is a set of n edges such that each vertex is part of exactly one edge. Give a polynomial time algorithm that takes a tree on $2n$ vertices as input and finds a perfect matching in the tree, if there is one.

3. Given a *sorted* array of n distinct integers $A[1, n]$, you want to find out whether there is an index i for which $A[i] = i$. Give an algorithm that runs in time $O(\log n)$ for this problem.

4. You are given k sorted arrays, each with n numbers in them. Give an algorithm for merging these arrays into a single sorted array of numbers that runs in time $O(nk \log k)$.