Chapter 14

THE IDEAL GAS LAW AND KINETIC THEORY

PREVIEW

Kinetic molecular theory involves the study of matter, particularly gases, as very small particles in constant motion. Because of the motion of the particles, an ideal gas has *internal energy* that can be transferred. We study gases by relating their *pressure, volume*, *number of moles*, and *temperature* in the *ideal gas law*.

The content contained in sections 1, 2, 3, and 5 of chapter 14 of the textbook is included on the AP Physics B exam.

QUICK REFERENCE

Important Terms

atomic mass unit
one-twelfth the mass of a carbon-12 atom
ideal gas law
the law which relates the pressure, volume, number of moles, and temperature of an ideal gas
internal energy
the sum of the potential and kinetic energy of the molecules of a substance
kinetic theory of gases
the description of matter as being made up of extremely small particles which are in constant motion
mole
one mole of a substance contains Avogadro's number (6.02×10^{23}) of molecules or atoms

Equations and Symbols

$PV = nRT$

$P_i V_i = P_f V_f$ (constant temperature)

$\frac{V_i}{T_i} = \frac{V_f}{T_f}$ (constant pressure)

$KE_{avg} = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} k_B T$

$v_{rms} = \sqrt{(v)_{avg}^2} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{\mu}}$

where

P = pressure
V = volume
n = number of moles
R = universal gas constant
= 8.31 J / (mol K)
KE_{avg} = average kinetic energy of molecules
v_{rms} = root-mean-square speed
k_B = Boltzmann constant
= 1.38 x 10^{-23} J/K
T = Kelvin temperature
M = molecular mass
μ = mass of molecule

Ten Homework Problems

Chapter 14 Problems 9, 10, 11, 14, 24, 26, 28, 29, 34, 52

DISCUSSION OF SELECTED SECTIONS

14.1 The Mole, Avogadro's Number, and Molecular Mass

When we are dealing with small particles like atoms and molecules, it is convenient to express their masses in atomic mass units (u) rather than kilograms. Atomic mass units and kilograms are related by the conversion 1 u = 1.6605 x 10^{-27} kg, which is approximately the size of a proton. When we buy 12 eggs we say we have a *dozen* eggs, but if we have 6.022 x 10^{23} atoms, we say we have a *mole* of those atoms. In other words, a *mole* of a substance is Avogadro's number (6.022 x 10^{23}) of atoms or molecules of that substance.

14.2 The Ideal Gas Law

All gases display similar behavior. When examining the behavior of gases under varying conditions of temperature and pressure, it is most convenient to treat them as *ideal gases*. An ideal gas represents a hypothetical gas whose molecules have no intermolecular forces, that is, they do not interact with each other, and occupy no volume. Although gases in reality deviate from this idealized behavior, at relatively low pressures and high temperatures many gases behave in nearly ideal fashion. Therefore, the assumptions used for ideal gases can be applied to real gases with reasonable accuracy.

The state of a gaseous sample is generally defined by four variables:
- pressure (p),
- volume (V),
- temperature (T), and
- number of moles (n),

though as we shall see, these are not all independent. The *pressure* of a gas is the force per unit area that the atoms or molecules exert on the walls of the container through collisions. The SI unit for pressure is the *pascal* (Pa), which is equal to one newton per meter squared. Sometimes gas pressures are expressed in *atmospheres* (atm). One atmosphere is equal to 10^5 Pa, and is approximately equal to the pressure the earth's atmosphere exerts on us each day. Volume can be expressed in liters (L) or cubic meters (m^3), and temperature is measured in Kelvins (K) for the purpose of the gas laws. Recall that we can find the temperature in K by adding 273 to the temperature in Celsius. Gases are often discussed in terms of standard temperature and pressure (STP), which refers to the conditions of a temperature of 273 K (0°C) and a pressure of 1 atm.

These four variables are related to each other in the ideal gas law:

$$PV = nRT$$

where R is a constant known as the *universal gas constant* = 8.31 J / (mol K). If the number of moles of a gas does not change during a process, then n and R are constants, and we can write the equation as the *combined gas law*:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

where a subscript of 1 indicates the state of the gas before something is changed, and the subscript 2 indicates the state of the gas after something is changed.

Example 1

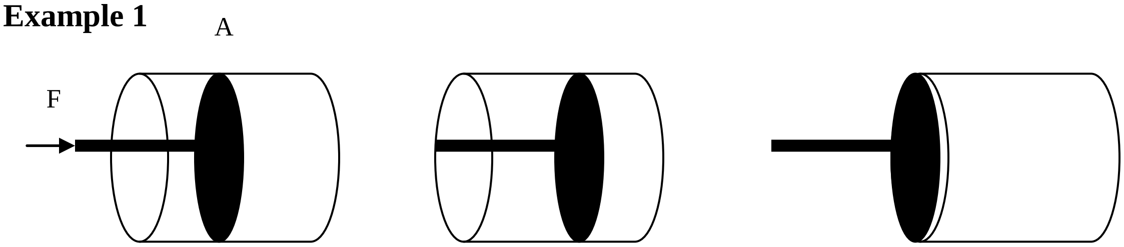


Fig. I Fig. II Fig. III

A cylinder is closed at one end with a piston which can slide to change the closed volume of the cylinder. When the piston is at the end of the cylinder, as in Figure III, the volume of the cylinder is 1.0 liter. The area of the piston is 0.01 m^2. The piston is positioned at half the length of the cylinder in Fig. I, and the cylinder is filled with an ideal gas. The force F necessary to hold the piston in this position is 10 N, and the temperature of the gas is 50˚ C.

(a) Determine the following for the gas in Fig. I:
i. the volume of the gas
ii. the pressure of the gas

A force is applied to the piston so that it is now positioned at one-third the length of the cylinder from its closed end, but the temperature of the gas remains at 50˚ C.

(b) Determine the following for the gas in Fig. II:
i. the volume of the gas
ii. the pressure of the gas

(c) The temperature of the gas is raised to 80˚ C between Fig. II and Fig. III. Determine the pressure of the gas in Fig. III.

Solution
(a) i. The volume of the gas is half of the full cylinder, or 0.5 liter.

ii. $P_I = \frac{F}{A} = \frac{10\,N}{0.1m^2} = 100\,Pa$

(b) i. The volume of the gas is one-third of the full cylinder, or 0.33 liter.
ii. At a constant temperature, the pressure and volume of the gas are inversely proportional according to Boyle's law:

$$P_I V_I = P_{II} V_{II}$$
$$(100\,Pa)(0.5l) = P_{II}(0.33l)$$
$$P_{II} = 151.5\,Pa$$

(c) Converting Celsius degrees to Kelvins:

$$T_{II} = 50°C + 273 = 323K$$
$$T_{III} = 80°C + 273 = 353K$$
$$\frac{P_{II} V_{II}}{T_{II}} = \frac{P_{III} V_{III}}{T_{III}}$$
$$\frac{(151.5\,Pa)(0.33l)}{323K} = \frac{P_{III}(1.0l)}{353K}$$
$$P_{III} = 54.7\,Pa$$

14.3 Kinetic Theory of Gases

As indicated by the gas laws, all gases show similar physical characteristics and behavior. A theoretical model to explain why gases behave the say they do was developed during the second half of the 19th century. The combined efforts of Boltzmann, Maxwell, and others led to the kinetic theory of gases, which gives us an understanding of gaseous behavior on a microscopic, molecular level. Like the gas laws, this theory was developed in reference to ideal gases, although it can be applied with reasonable accuracy to real gases as well.

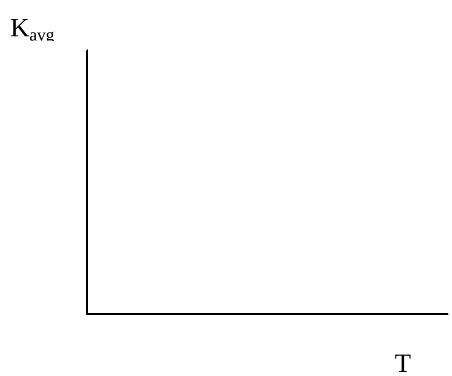
The assumptions of the kinetic theory of gases are as follows:

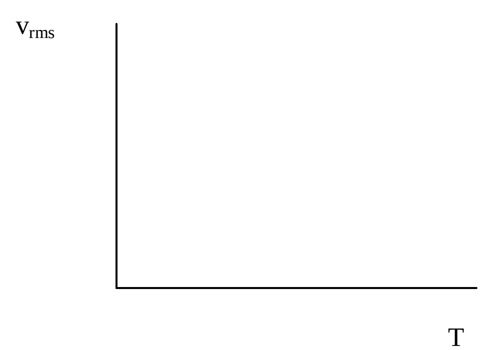
- Gases are made up of particles whose volumes are negligible compared to the container volume.
- Gas atoms or molecules exhibit no intermolecular attractions or repulsions.
- Gas particles are in continuous, random motion, undergoing collisions with other particles and the container walls.
- Collisions between any two gas particles are elastic, meaning that no energy is dissipated and kinetic energy is conserved.
- The average kinetic energy of gas particles is proportional to the absolute (Kelvin) temperature of the gas, and is the same for all gases at a given temperature. As listed in the list of equations, the average kinetic energy of each molecule is related to Kelvin temperature *T* by the equation

$K_{avg} = \frac{3}{2} k_B T$, where k_B is the Boltzmann constant, 1.38 x 10^{-23} J/K. The root-mean square speed of each molecule can be found by $v_{rms} = \sqrt{\frac{3k_B T}{\mu}}$, where μ is the mass of each molecule. This equation is very seldom used on the AP Physics B exam, and is provided on the exam if needed.

Example 2
The temperature of an ideal gas is 60˚ C.
(a) Find the average kinetic energy of the molecules of the gas.

(b) On the axes below, sketch a graph of
i. average kinetic energy K_{avg} vs Kelvin temperature T
ii. root-mean-square speed v_{rms} of each molecule in the gas vs. Kelvin temperature T.



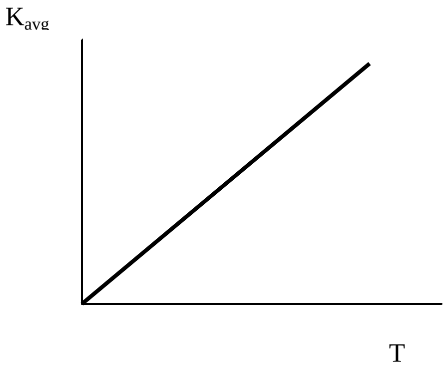


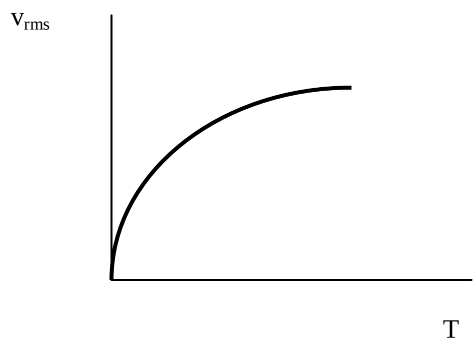
Solution

(a) $K_{avg} = \frac{3}{2}k_B T = \frac{3}{2}\left(1.38\,x10^{-23}\ J/K\right)\left(60°C + 273\right)$

$K_{avg} = 6.89\,x10^{-21}\ J$

(b) The average kinetic energy of each molecule is directly proportional to the Kelvin temperature of the gas, and v_{rms} is proportional to the square root of the Kelvin temperature of the gas:





CHAPTER 14 REVIEW QUESTIONS

For each of the multiple choice questions below, choose the best answer.

1. A *mole* is to *Avogadro's number* as
(A) kinetic energy is to temperature
(B) atomic mass unit is to kg
(C) gas is to liquid
(D) pressure is to volume
(E) decade is to ten years

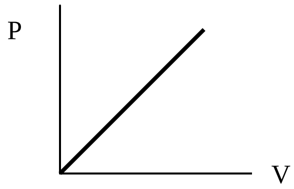
2. Which of the following is NOT true of an ideal gas?
(A) Gas molecules have no intermolecular forces.
(B) Gas particles are in random motion.
(C) Gas particles have no volume.
(D) The collisions between any two gas particles are elastic.
(E) The average kinetic energy of the gas molecules is proportional to the temperature in Celsius degrees.

3. A sample of argon occupies 50 liters at standard temperature. Assuming constant pressure, what volume will argon occupy if the temperature is doubled?
(A) 25 liters
(B) 50 liters
(C) 100 liters
(D) 200 liters
(E) 2500 liters

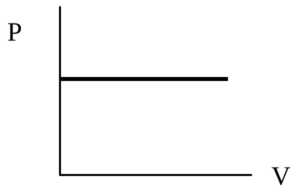
4. What is the final pressure of a gas that expands from 1 liter at 10°C to 10 liters at 100°C if the original pressure was 3 atmospheres?
(A) 0.3 atm
(B) 0.4 atm
(C) 3 atm
(D) 4 atm
(E) 30 atm

5. Which of the following pressure vs. volume graphs best represents how pressure and volume change when temperature remains constant?

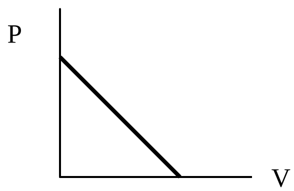
(A)



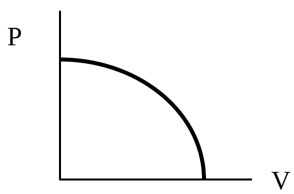
(B)



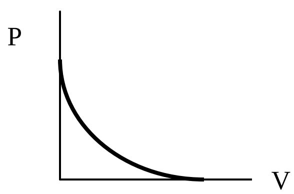
(C)



(D)

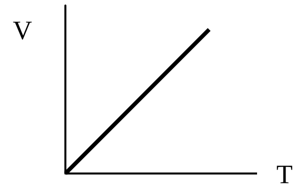


(E)

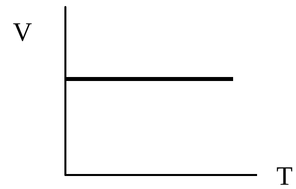


6. Which of the following volume vs. temperature graphs best represents how volume changes with Kelvin temperature if the pressure remains constant?

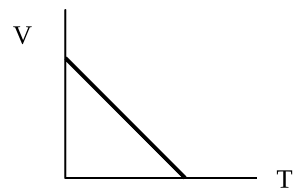
(A)



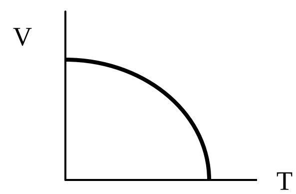
(B)



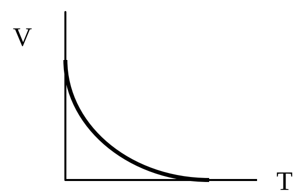
(C)



(D)



(E)



Free Response Question

<u>Directions:</u> *Show all work in working the following question. The question is worth 10 points, and the suggested time for answering the question is about 10 minutes. The parts within a question may not have equal weight.*

1. (10 points)
A special balloon contains 3 moles of an ideal gas and has an initial pressure of 3×10^5 Pa and an initial volume of 0.015 m^3.

(a) Determine the initial temperature of the gas.

(b) The pressure in the balloon is changed in such a way as to increase the volume of the balloon to 0.045 m^3 but the temperature is held constant. Determine the pressure of the gas in the balloon at this new volume.

(c) If the balloon contracts to one-third of its initial volume, and the pressure is increased to twice its initial value, describe the change in temperature that would have to take place to achieve this result.

ANSWERS AND EXPLANATIONS TO CHAPTER 14 REVIEW QUESTIONS

Multiple Choice

1. E
A *mole* is defined as a certain number of things (6×10^{23}), and a *decade* is a certain number of years (10).

2. E
All of the statements are true, except for "Celsius" should be replaced with "Kelvin".

3. C
With constant pressure, volume and temperature are proportional to each other, so twice the temperature would result in twice the volume.

4. B
First, we must convert Celsius to Kelvin:
$T_1 = 10^\circ$ C + 273 = 283 K
$T_2 = 100^\circ$ C + 273 = 373 K

$P_1 = 3$ atm
$V_1 = 1$ *l*
$V_2 = 10$ *l*

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Solving for P_2 and substituting, we get $P_2 = 0.4$ atm.

5. E
Pressure and volume are inversely proportional to each other at constant temperature.

6. A
Volume and temperature are proportional to each other at constant temperature.

<u>Free Response Question Solution</u>

(a) 4 points

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{(3x10^5\ Pa)(0.015\,m^3)}{(3\,moles)(8.31\ J/mol\,K)} = 180.5\,K$$

(b) 3 points
For a constant temperature,

$$P_1V_1 = P_2V_2$$

$$(3x10^5\,Pa)(0.015\,m^3) = P_2(0.045\,m^3)$$

$$P_2 = 1x10^5\,Pa$$

(c) 3 points

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

In order to keep the ratios constant on both sides,

$$\frac{P_1V_1}{T_1} = \frac{\left(\frac{1}{3}\right)P_1\left(\frac{1}{2}\right)V_1}{\left(\frac{1}{6}\right)T_1}$$

Thus, the temperature would have to decrease to 1/6 its initial value for these changes in the pressure and volume to take place.