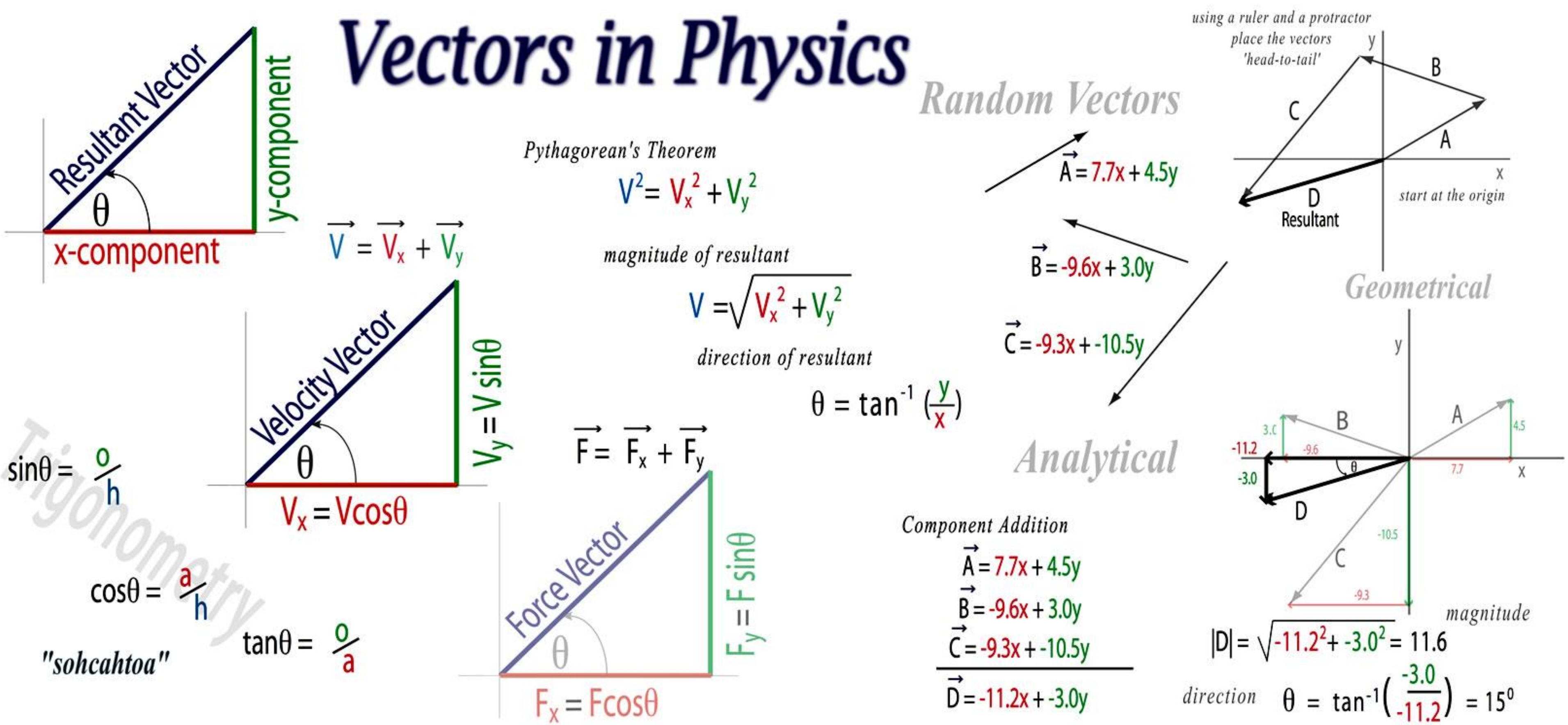
http://sdsu-physics.org/physics180/physics195/Topics/images_motion/3_vectors_ai.jpg



Class 7: Components of Motion; Vectors

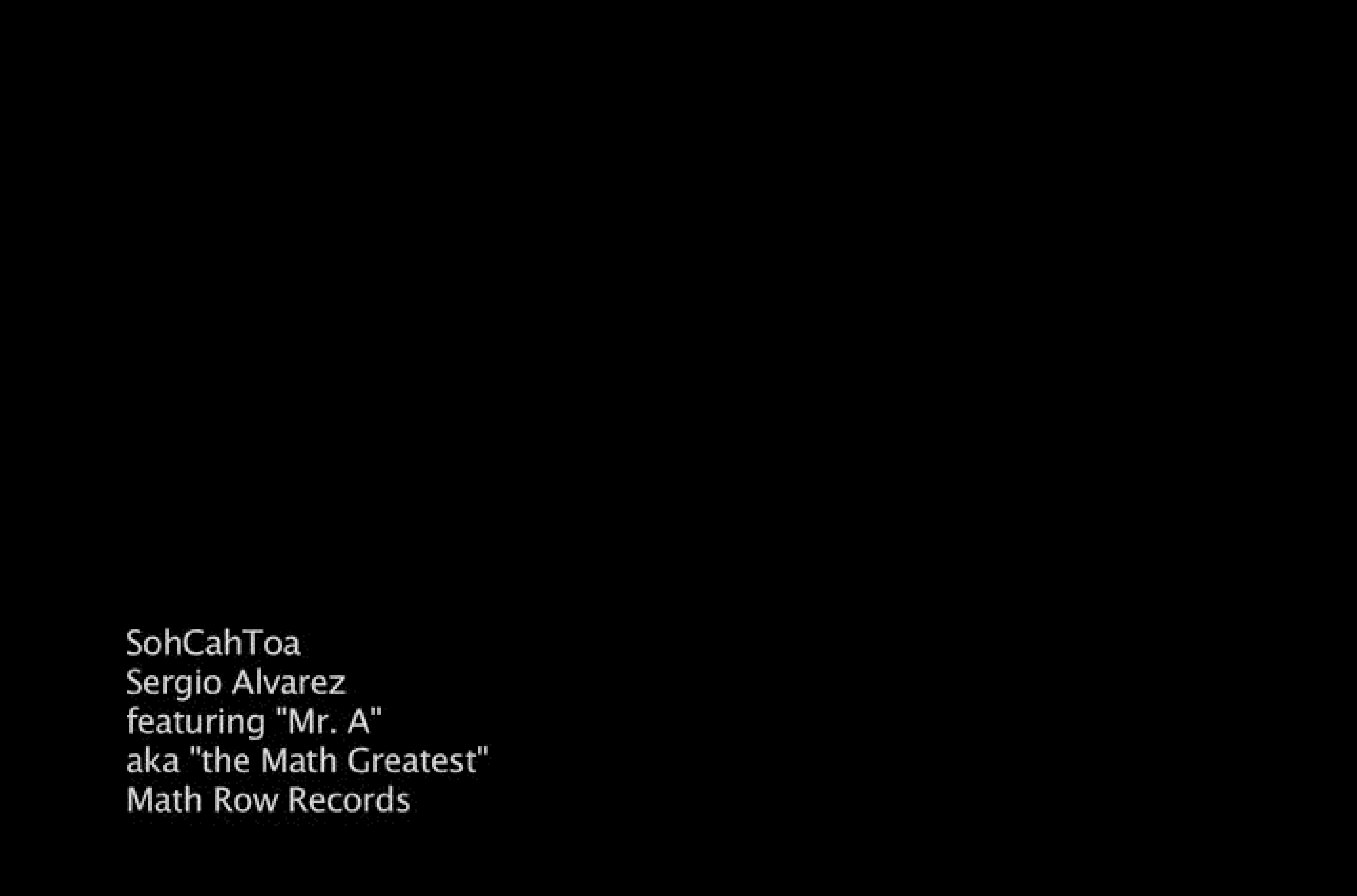
Refresher: Scalars vs. Vectors

- What is a **scalar**?
- A quantity that requires a single number to be completely specified
- Examples
- Temperature, volume, mass, distance, speed
- A quantity that requires both magnitude and direction to be fully specified is called a **vector**
- Examples
- Force, displacement, velocity

$1 + 1 \neq 2$

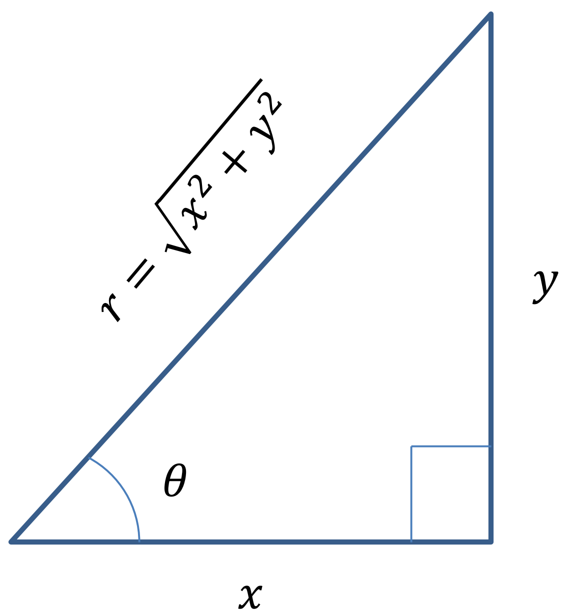
- Often, we wish to add vectors; adding forces, or displacements
- Vectors don't add or subtract like ordinary numbers; why?
- They depend on **direction** as well as **magnitude**!
- In more than 1 dimension, direction means angles, **trigonometry** is invaluable!

http://youtu.be/4iC-gjKvc7A



Review of Trigonometry

SOHCAHTOA



Sine:

- opposite over hypotenuse

$$\sin\theta = y/r$$

Cosine:

- adjacent over hypotenuse

$$\cos\theta = x/r$$

Tangent:

- opposite over adjacent

$$\tan\theta = y/x$$

Inverse functions, e.g.

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{y}{x}\right)$$

Quickuiz

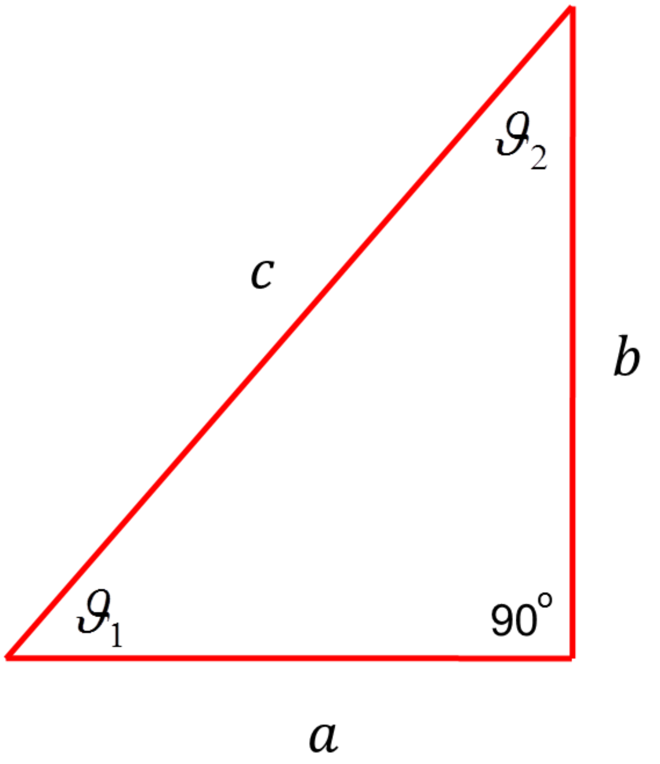
In the triangle below, which of the following is a correct expression for the length of side b?

$A.\ b = a \sin \vartheta_1$

$B.\ b = a \cos \vartheta_1$

$C.\ b = c \cos \vartheta_1$

$\boxed{D.\ b = c \sin \vartheta_1}$



Related, but not Related

- In more than one dimension, separate directions are treated independently
- But that's just a tool, directions are arbitrary and related through time

Example

- Cue ball rolls at speed $v_0 = 5.0\ \mathrm{m/s}$ for 2.0 s at an angle $\theta = 53°$.

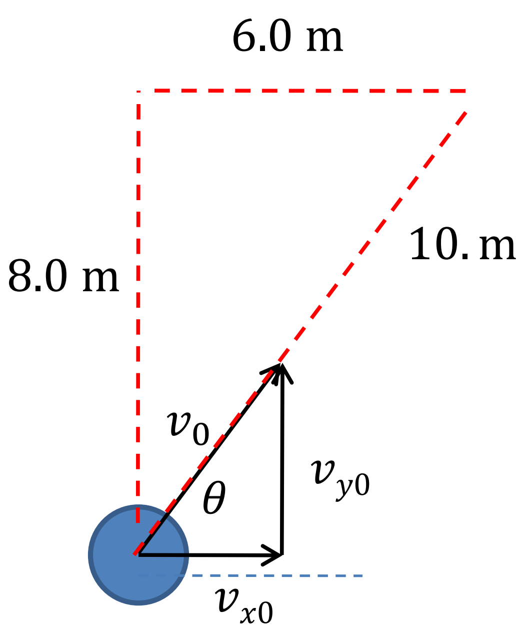
$$v_{x0} = \left(5.0\frac{\mathrm{m}}{\mathrm{s}}\right)(\cos 53°) = 3.0\ \mathrm{m/s}$$

$$v_{y0} = \left(5.0\frac{\mathrm{m}}{\mathrm{s}}\right)(\sin 53°) = 4.0\ \mathrm{m/s}$$

$$\Delta x = v_{x0}(\Delta t) = \left(3.0\frac{\mathrm{m}}{\mathrm{s}}\right)(2.0\ \mathrm{s}) = 6.0\ \mathrm{m}$$

$$\Delta y = v_{y0}(\Delta t) = \left(4.0\frac{\mathrm{m}}{\mathrm{s}}\right)(2.0\ \mathrm{s}) = 8.0\ \mathrm{m}$$

- Total distance $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(6.0\mathrm{m})^2 + (8.0\mathrm{m})^2} = 10\mathrm{m}$
- **Component method gave same result as direct method**



Quickuiz

An object starts at the origin $(x_0 = 0, y_0 = 0)$ with initial x-velocity $2.00\ \mathrm{m/s}$ and initial y-velocity $-4.00\ \mathrm{m/s}$. At a later time, its y-position is $-12.0\ \mathrm{m}$. What is its x-position at this later time?

A. $-12.0\ \mathrm{m}$

B. $8.00\ \mathrm{m}$

C. $6.00\ \mathrm{m}$

D. $12.0\ \mathrm{m}$

Redefining Vectors

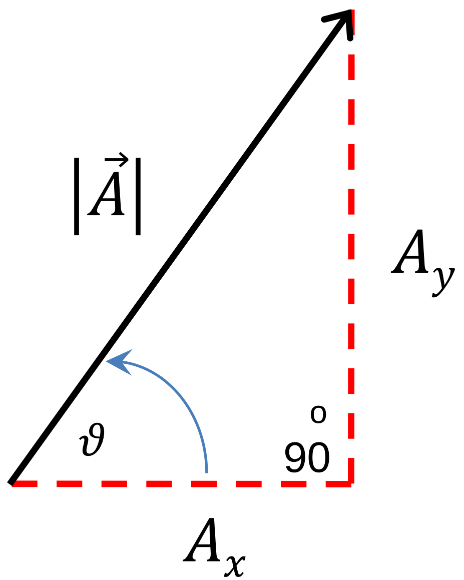
- Vectors need magnitude and direction to be specified; two numbers *(version 1)*
- In 2D we can replace magnitude and direction with x- and y-components *(version 2)*

$$\cos\vartheta = \frac{A_x}{|\vec{A}|} \Rightarrow A_x = |\vec{A}|\cos\vartheta$$

$$\sin\vartheta = \frac{A_y}{|\vec{A}|} \Rightarrow A_y = |\vec{A}|\sin\vartheta$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\vartheta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



Caution: if $\vec{A}$ is in 2[nd] or 3[rd] quadrant, you need to add 180° to ϑ output by calculator!

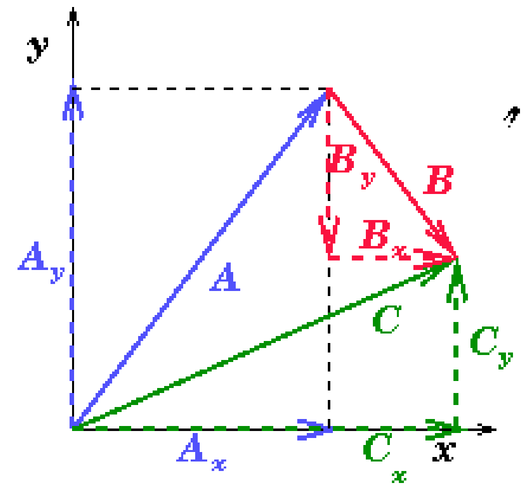
Addition of Vectors Using Components

- To add two or more vectors, we can add their components. For example:

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

where $\mathbf{A} = (A_x, A_y)$, $\mathbf{B} = (B_x, B_y)$, gives us:

$$(C_x, C_y) = (A_x, A_y) + (B_x, B_y)$$
$$(C_x, C_y) = (A_x + B_x, A_y + B_y)$$
$$C = \sqrt{(A_x + B_x)^2 + \left(A_y + B_y\right)^2}$$
$$\tan\theta = \frac{A_y + B_y}{A_x + B_x}$$



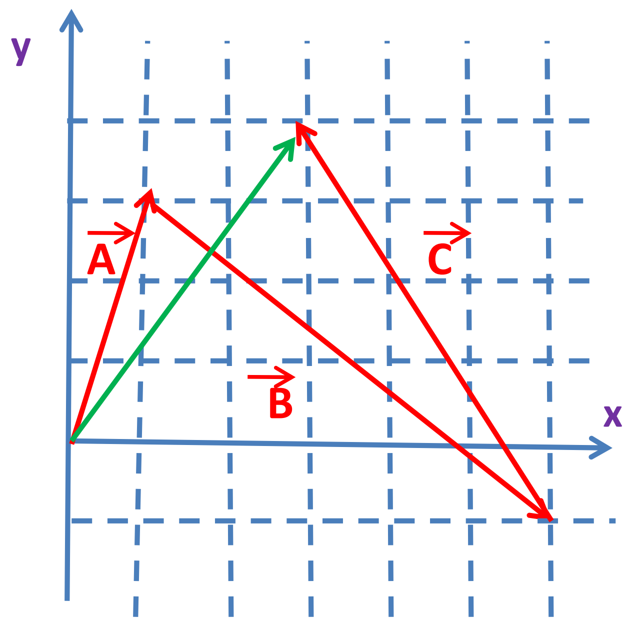
Quickuiz

Add the following three vectors:

$\mathbf{A} = (1,3)\ \mathbf{B} = (5,-4)\ \mathbf{C} = (-3,5)$

What is the length of that sum vector?

1. 2
2. $\boxed{5}$
3. 3
4. 6



Simulation: Vectors in 2D

http://phet.colorado.edu/en/simulation/vector-addition

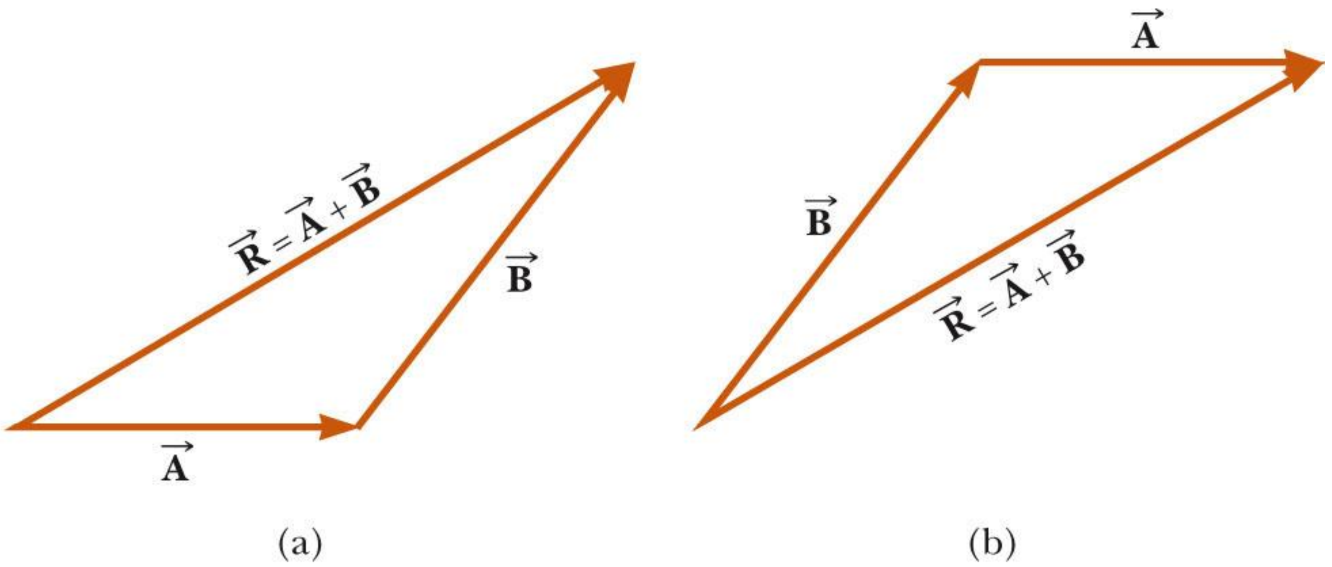
More Features of Vectors

- Multiplying by a scalar
- Subtracting vectors
- Unit of vectors (e.g velocity has m/s)
- Unit vectors

Vector Properties

- Vectors obey the commutative rule of addition:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$



Vector Properties

- Vectors can be subtracted

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

i.e. the vector is subtracted by flipping its direction