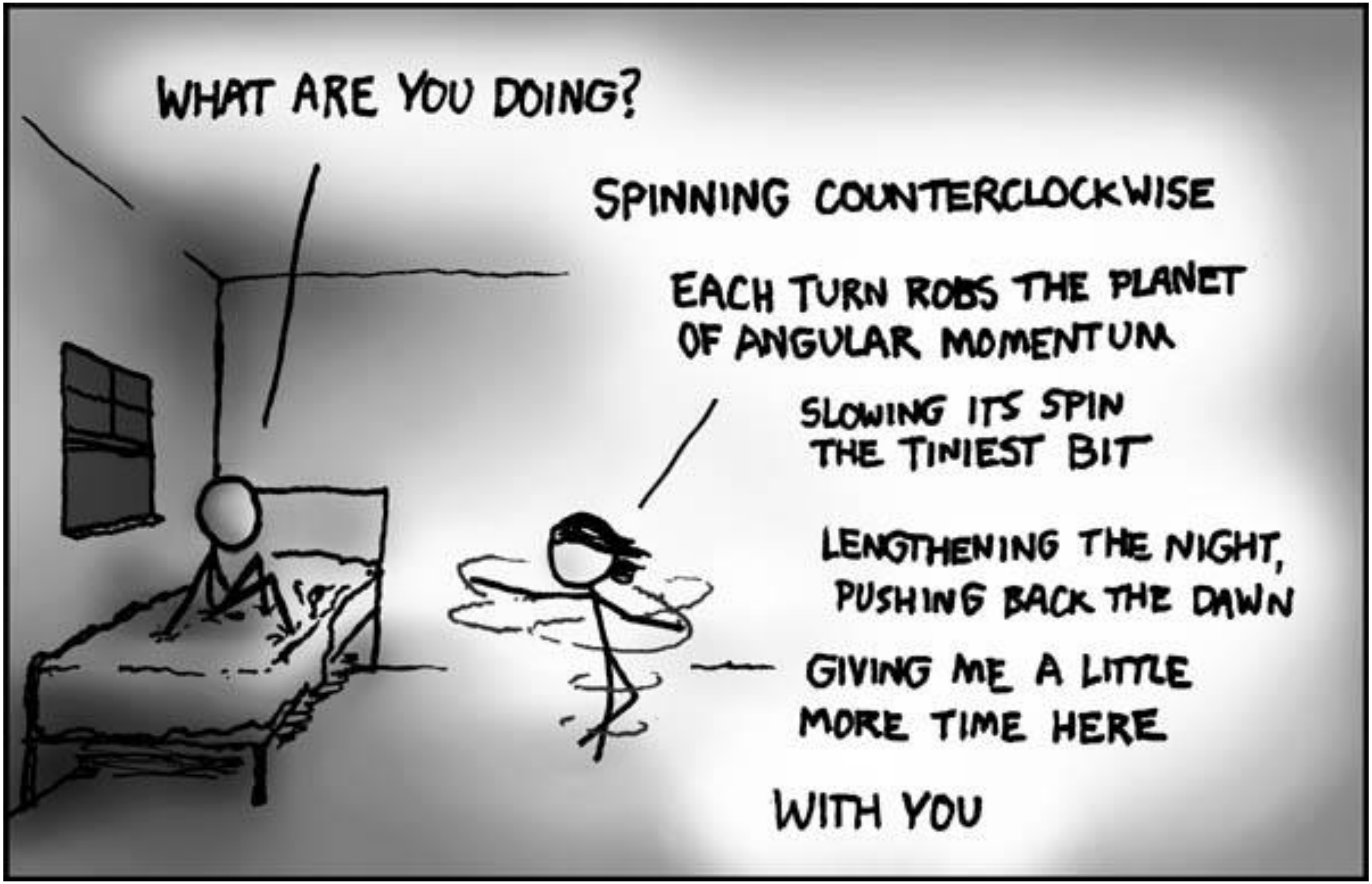
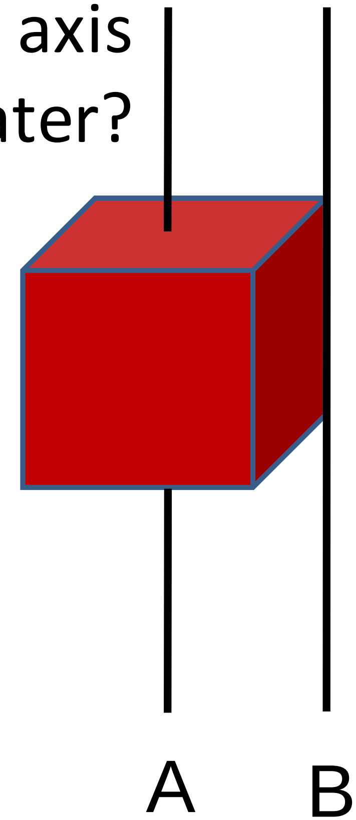
Classes 25: Rotational Kinetic Energy and Angular Momentum



Quickuiz

A cube can be rotated either around an axis A through its center or an axis B which just touches one of the edges of the cube. For which axis is the moment of inertia greater?



A. Axis A

B. Axis B

C. The moments of inertia are equal

D. Cannot be determined

Rotational Kinetic Energy

- We previously studied kinetic energy:

$$KE = \frac{1}{2}mv^2$$

- Assume this is describing an object moving on a circle at constant velocity v_t, then:

$$v_t = r\omega$$

$$KE = \frac{1}{2}mr^2\omega^2$$

- Now, if that object is a point particle, then $I = mr^2$, which means that for a point particle moving on a circle:

$$\boxed{KE = \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2}$$

- But it turns out **this is a general result for rigid objects**.

Rotation Work and Rotational Kinetic Energy

- Translational work
$$W_{trans} = F\Delta x$$
- Translational KE
$$KE_{trans} = \frac{1}{2}mv^2$$
- Work-energy theorem (translational)
$$W_{trans} = \Delta(KE_{trans})$$
- Rotational work:
$$W_{rot} = \tau\Delta\theta$$
- Rotational KE:
$$KE_{rot} = \frac{1}{2}I\omega^2$$
- Work-energy theorem (rotational):
$$W_{rot} = \Delta(KE_{rot})$$

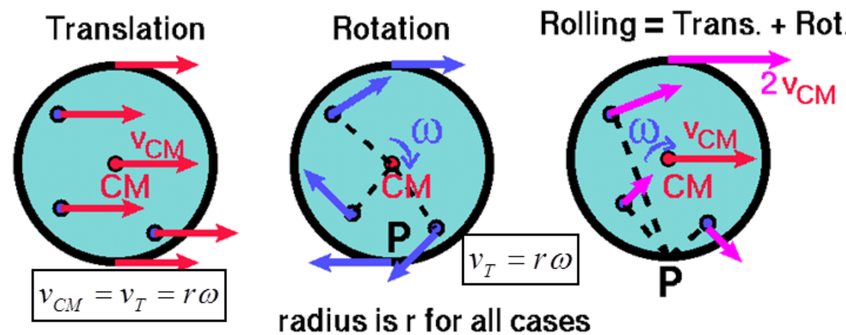
Rolling without slipping

- When a wheel rolls, a point on the edge has tangential speed $v_t = r\omega$ with respect to the center of the wheel.
- When it rolls without slipping, the translational speed of the wheel with respect to the floor has this same value (the condition for rolling without slipping):

$$v_{CM} = v_t = r\omega$$

- This implies:

$$\Delta x_{CM} = r\Delta\theta, \qquad a_{CM} = r\alpha$$



Relative to the rolling surface, a point on the top of the wheel has speed $2r\omega$, on the center has speed $r\omega$, and on the bottom has speed 0

Quickuiz

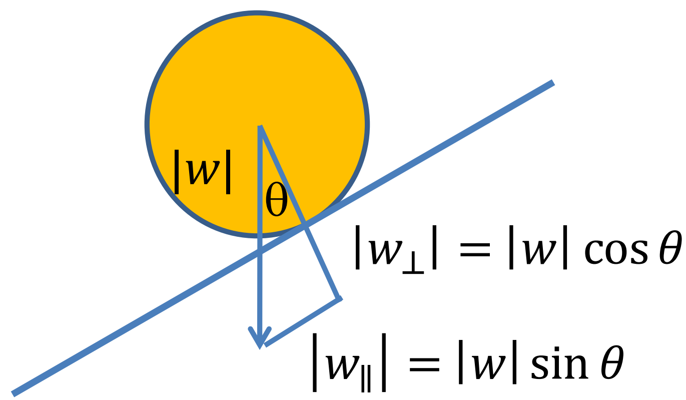
There is no relative motion between the bottom point on a wheel rolling without slipping and the surface on which it is rolling. This means that the force which creates the torque on the wheel around its center is being exerted by __________ , and the force must have a magnitude _________ the parallel component of gravity $w_{\parallel}$.

A. static friction; equal to

B. kinetic friction; smaller than

C. static friction; smaller than

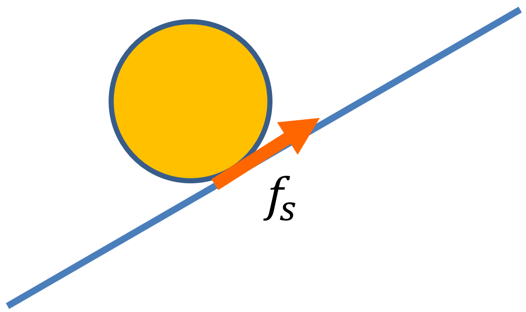
D. gravity; greater than



Rotational Kinetic Energy, and Total Mechanical Energy Conservation

- The condition of rolling without slipping leads to a very useful relationship between the translational kinetic energy, the rotational kinetic energy, and the total kinetic energy:

$$v_{CM} = v_t = r\omega \Rightarrow KE_{rot} = \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}I\left(\frac{v_{CM}}{r}\right)^2$$
$$KE_{tot} = KE_{trans} + KE_{rot}$$
$$= \frac{1}{2}m(v_{CM})^2 + \frac{1}{2}I\left(\frac{v_{CM}}{r}\right)^2$$
$$= \frac{1}{2}m(v_{CM})^2\left[1 + \left(\frac{I}{mr^2}\right)\right]$$



In rolling without slipping, the force of station friction does negative translational work, but an equal amount of positive rotational work. Thus:

$$E_{tot} = KE_{tot} + PE$$
$$= KE_{trans}(1 + C) + PE$$

Is now also conserved!

With $I = C\,mr^2$

Example

A 2.00-kg ball of radius 0.100 m rolls without slipping a distance of 5.00 m down a $30°$ incline. Treating it as a solid sphere ($C = \frac{2}{5}$), use conservation of energy to find its translational energy at the bottom of the incline B. Recall $KE_{tot} = (1 + C)KE_{trans}$, and assume the top is point A.

$$I = \frac{2}{5}mr^2 = \frac{2}{5}(2.00\text{ kg})(0.100\text{ m})^2 = 8.00 \times 10^{-3}\text{kg m}^2$$

$$TME_A = TME_B \Rightarrow GPE_A = KE_B$$

$$GPE_A = mgy_A = (2.00\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(5.00\text{ m})\sin 30° = 49.\text{J}$$

$$KE_B = \frac{1}{2}mv_B^2(1 + C) = \frac{1}{2}(2.00\text{ kg})v_B^2\left(\frac{7}{5}\right) = 1.4\text{ kg }v_B^2$$

$$\therefore 49.\text{J} = 1.4\text{ kg }v_B^2 \Rightarrow v_B = 5.92\text{ m/s}$$

Angular Momentum

- Remember how momentum was defined as
$$p = mv$$
- In a similar fashion, it is possible to define *angular momentum* L:
$$L \equiv I\omega$$
- Angular momentum is measured in $\mathrm{kg} \cdot \mathrm{m}^2/s$
- It can be *positive* (if ω is positive, i.e., for c.c.w. rotation) or *negative* (if ω is negative, i.e., for c.w. rotation)

Angular Momentum Conservation

- It can be easily shown that:

$$\sum \tau = \frac{\Delta L}{\Delta t}$$

<u>Angular Momentum Conservation</u>: When the net torque is zero, angular momentum is conserved:

$$\boxed{\sum \tau = 0 \Rightarrow L_i = L_f}$$

- Or:

$$I_i \omega_i = I_f \omega_f$$

Demonstrating Conservation of Angular Momentum

http://youtu.be/UZlW1a63KZs

Quickuiz

If the ice from the polar caps melts more water will accumulate closer to the equator of earth. If this happens, the days become:

1. Longer
2. Shorter
3. Remain the same

Ice contributes a small moment of inertia when at the poles (small radius), at the equator the moment of inertia increases, ω decreases, day increases