Math 183 | Homework 2.2 | Bowers

2.2.2 $S=\{(R,B,G): R,B,G\in\{1,\dots,6\}\}$

$R = \#$ on red
$B = \#$ on blue
$G = \#$ on green

$E = \{(R,B,G): R+B+G=5\}$

$= \{(1,1,3),(1,2,2),(1,3,1),(2,1,2),(2,2,1),(3,1,1)\}$

2.2.3 We denote an outcome by (a,b,c), where a, b, and c are positive integers, and $a<b<c\le 6$.

$E =$ "Second smallest chip is 3"

$= \{(1,3,4),(1,3,5),(1,3,6),(2,3,4),(2,3,5),(2,3,6)\}$

2.2.6 Note: N is the set of straight flushes in hearts.

$\underbrace{A\ 2\ 3\ 4\ 5}_{\text{"smallest" possible straight flush}}\ 6\ 7\ 8\ 9\ \underbrace{10\ J\ Q\ K\ A}_{\text{"largest" possible straight flush}}$

we can have an element in N start with A or 2 or 3 or ... or 10. Thus, there are 10 outcomes in N.

2.2.11 Name suspects as A, B, C, D, E.

$S=\{(X,Y): X,Y\in\{A,B,C,D,E\}\}$, where (X,Y) represents the outcome "Woman selects X and Y."

Note that (X,Y) is the same outcome as (Y,X).

We identify the guilty parties as A and B.

$A =$ "Woman makes at least one incorrect identification"

$= \{(A,B)\}^C = S\setminus\{(A,B)\}$

2.2.13
$E =$ "Shooter wins with a point of 9"

Let $A_i =$ "Rolls 9 on i^{th} roll"

$B_i =$ "Rolls 7 on i^{th} roll"

$N_i = A_i^C \cap B_i^C = (A_i\cup B_i)^C$

$=$ "Rolls neither 7 nor 9 on i^{th} roll"

$E = \{(A_1,A_2),\ (A_1,N_2,A_3),\ (A_1,N_2,N_3,A_4),\dots,(A_1,N_2,\dots,N_{k-1},A_k),\dots\}$

2.2.14 Let $X = \#$ of white chips in Urn 1 and $Y = \#$ of black chips in Urn 1.

Note: This means there are $10-X$ white chips in Urn 2 and $10-Y$ black chips in Urn 2.

$S = \{(X,Y): X,Y$ are positive integers and $1\le X+Y\le 19\}$

Note: Urn 1 must contain at least 1 chip and at most 19, because Urn 2 must also contain at least 1.

2.2.15 Let t_1 signify 1 minute until midnight, t_2 is $\frac{1}{2}$ minutes until midnight, t_3 is $\frac{1}{4}$ minutes until midnight, etc. So that, for all $k\in\mathbb{N}$:

$t_k = \frac{1}{2^{k-1}}$ minutes until midnight.

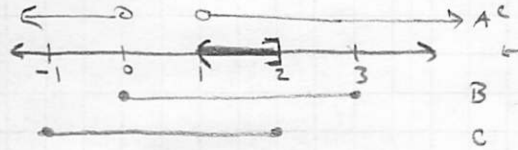
At t_1, we remove chip 1, at t_2 we remove chip 2, etc. Thus, at t_k, we remove chip k, for all $k\in\mathbb{N}$.

Therefore, given any chip, it will at some point be removed. Consequently, after midnight the urn will be empty.

Math 183 | Homework 2.2 | Bowers

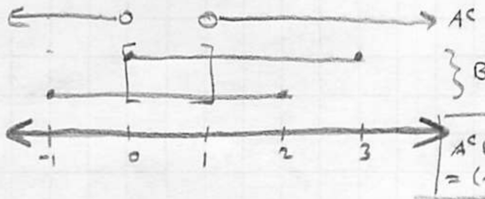
2.2.20 $A=\{x: 0\le x\le 1\}$, $B=\{x: 0\le x\le 3\}$, $C=\{x: -1\le x\le 2\}$

(a)



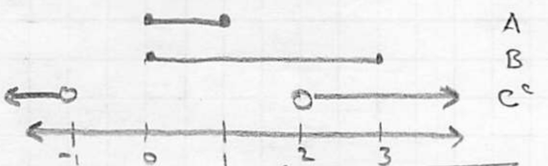
$\boxed{A^c\cap B\cap C = (1,2]}$

(b)



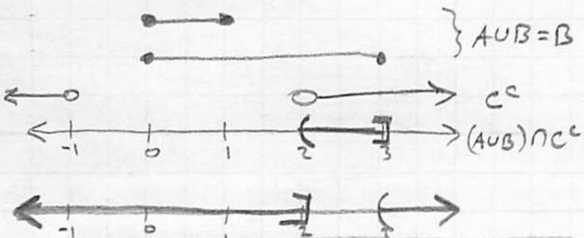
$\boxed{A^c\cup(B\cap C) = (-\infty,\infty)}$

(c)



$\boxed{A\cap B\cap C^c = \emptyset}$

(d)



$\boxed{[(A\cup B)\cap C^c]^c = (-\infty,2]\cup(3,\infty)}$

2.2.22 All letters should be considered distinct, so number letters of the same kind:

$T_1\ E_1\ S_1\ S_2\ E_2\ L_1\ L_2\ A\ T_2\ I\ O\ N$

We do this because each letter is on its own chip and the question ask which CHIPS make up the given events (not which letters).

(a) $F\cap R\cap V = \{E_1, E_2\}$

(b) $F^c\cap R\cap V^c = \{S_1, S_2, T_1, T_2\}$

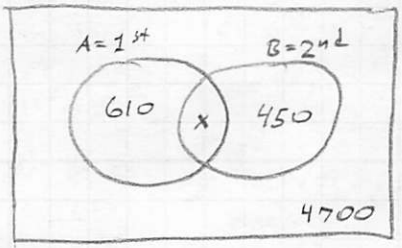
(c) $F\cap R^c\cap V = \{A, I\}$

Note:
$\underbrace{\text{ABCDEFGHIJKLM}}_{1^{st}\text{ half}}/\underbrace{\text{NOPQRSTUVWXYZ}}_{2^{nd}\text{ half}}$

2.2.29
$A=\{HHTT, THHT, TTHH, HTHT, HTTH, THTH\}$
$B=\{HTHT, THTH\}$
$C=\{HHHH, HHHT, HHTH, HHTT\}$

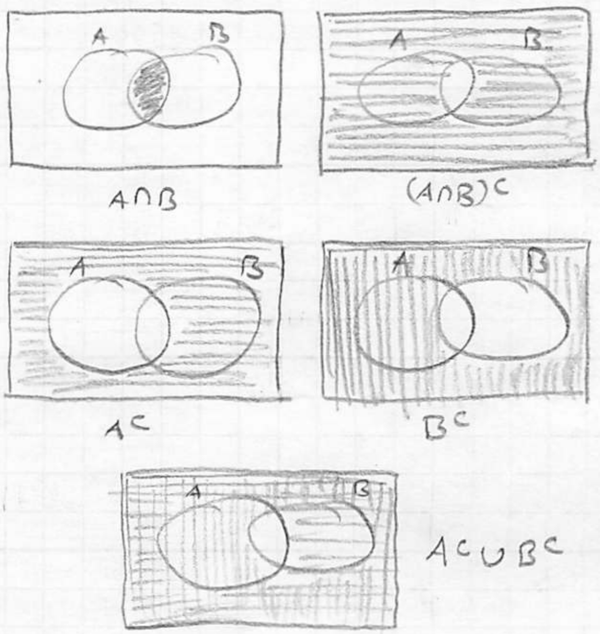
(a) $B\cap C=\emptyset$
(b) $B\subseteq A$

2.2.31

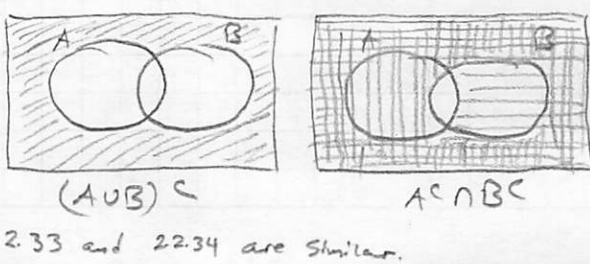


$n(A\cup B) = 6000-4700 = 1300$
$n(A\cup B) = n(A)+n(B) - n(A\cap B)$
$1300 = 850 + 690 - x$
$\boxed{x = 240}$

2.2.32 (a) Show $(A\cap B)^c = A^c\cup B^c$



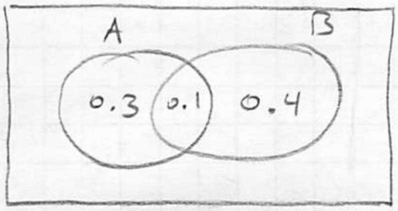
(b) Show $(A\cup B)^c = A^c\cap B^c$



2.2.33 and 2.2.34 are similar.

Math 183 | Homework 2.3 | Bowers

2.3.2 $P(A)=0.4,\quad P(B)=0.5$
$P(A\cap B)=0.1$



$P((A\cap B^c)\cup(A^c\cap B))$
$= 0.3+0.4 = 0.7$

2.3.5 $P(A_1\cup A_2\cup A_3)\neq 1/2$

$P(A_1\cup A_2\cup A_3) = P(\text{"At least one 6"})$

$= 1 - P(\text{"No six"}) = 1-\frac{5^3}{6^3}\neq \frac{1}{2}$

The events A_1, A_2, A_3 are not mutually exclusive, so
$P(A_1\cup A_2\cup A_3)\neq P(A_1)+P(A_2)+P(A_3)$

2.3.10 $\mathbb{P}(A\cup B)=\mathbb{P}(A)+\mathbb{P}(B)-\mathbb{P}(A\cap B)$

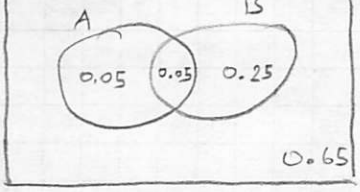
$\mathbb{P}(A)=\frac{12}{24}=\frac{1}{2}$, $\mathbb{P}(B)=\frac{8}{24}=\frac{1}{3}$

$\mathbb{P}(A\cap B) = \mathbb{P}(\{6,12,18,24\})=\frac{4}{24}=\frac{1}{6}$

$\therefore \mathbb{P}(A\cup B) = \frac{1}{2}+\frac{1}{3}-\frac{1}{6} = \frac{5}{6}-\frac{1}{6}=\frac{4}{6} = \boxed{\frac{2}{3}}$

2.3.11 A = "win Saturday"
B = "win in two weeks"

$\mathbb{P}(A)=.10,\ \mathbb{P}(B)=.30$
$\mathbb{P}(A^c\cap B^c)=.65 = \mathbb{P}((A\cup B)^c)$



$P(A\cup B) = P(A)+P(B)-P(A\cap B)$
$0.35 = 0.1+0.3-X$
$X = 0.05$
$P((A\cap B^c)\cup(A^c\cap B)) = 0.05+0.25 = \boxed{0.30}$

2.3.15 $X=\{H{*}{*}T,\ T{*}{*}H\}\quad n(X)=8$

$Y=\{HHTT, HTHT, HTTH, THHT, THTH, TTHH\}\quad n(Y)=6$

(a) $\mathbb{P}(X^c\cap Y) = \mathbb{P}(\{HTTH, THHT\})$

1st, last are same

$= \frac{2}{16} = \boxed{\frac{1}{8}}$

(b) $\mathbb{P}(X\cap Y^c) = \mathbb{P}((X^c\cup Y)^c)$

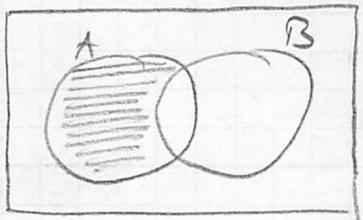
$= 1-\mathbb{P}(X^c\cup Y)$

$= 1-[\mathbb{P}(X^c)+\mathbb{P}(Y)-\mathbb{P}(X^c\cap Y)]$

$= 1-\left[\left(1-\frac{8}{16}\right)+\frac{6}{16}-\frac{1}{8}\right]$

$= \cancel{1}-\cancel{1}+\frac{1}{2}-\frac{3}{8}+\frac{1}{8} = \frac{1}{2}-\frac{2}{8} = \boxed{\frac{1}{4}}$

2.3.16
$A=\{(1,5),(2,4),(3,3),(4,2),(5,1)\}$
$B=\{(1,2),(2,4),(3,6),(6,3),(4,2),(2,1)\}$



$A\cap B^c$

$\mathbb{P}(A\cap B^c)+\mathbb{P}(A\cap B)=\mathbb{P}(A)$

$X + \frac{2}{36} = \frac{5}{36}$

$X = \boxed{3/36}$