CS 170	Algorithms	
Spring 2014	E. Mossel	HW 10

Due April 25, 6:00pm

1. (20 pts.) Bounded CLIQUE and Fake Reductions

Consider the CLIQUE problem restricted to graphs in which every vertex has degree at most 3. Call this problem CLIQUE-3.

(a) Prove that CLIQUE-3 is in **NP**.

(b) What is wrong with the following proof of **NP**-completeness for CLIQUE-3?

We know that the CLIQUE problem in general graphs is **NP**-complete, so it is enough to present a reduction from CLIQUE-3 to CLIQUE. Given a graph G with vertices of degree ≤ 3, and a parameter g, the reduction leaves the graph and the parameter unchanged: clearly the output of the reduction is a possible input for the CLIQUE problem. Furthermore, the answer to both problems is identical. This proves the correctness of the reduction and, therefore, the **NP**-completeness of CLIQUE-3.

(c) It is true that the VERTEX COVER problem remains **NP**-complete even when restricted to graphs in which every vertex has degree at most 3. Call this problem VC-3. What is wrong with the following proof of **NP**-completeness for CLIQUE-3?

We present a reduction from VC-3 to CLIQUE-3. Given a graph $G = (V,E)$ with node degrees bounded by 3, and a parameter b, we create an instance of CLIQUE-3 by leaving the graph unchanged and switching the parameter to $|V| - b$. Now, a subset $C \subseteq V$ is a vertex cover in G if and only if the complementary set $V - C$ is a clique in G. Therefore G has a vertex cover of size $\leq b$ if and only if it has a clique of size $\geq |V| - b$. This proves the correctness of the reduction and, consequently, the **NP**-completeness of CLIQUE-3.

(d) Describe an $O(|V|^4)$ algorithm for CLIQUE-3.

2. (20 pts.) Restricted Input SAT and Independent Set

Recall that 3SAT remains **NP**-complete even when restricted to formulas in which each literal (remember x and $\bar{x}$ are different literals) appears at most twice.

(a) Show that if each literal appears at most *once*, then the problem is solvable in polynomial time.

(b) Show that INDEPENDENT SET remains **NP**-complete even in the special case when all the nodes in the graph have degree at most 4.

3. (30 pts.) Proving NP-Completeness by Generalization

For each of the problems below, prove that it is **NP**-complete by showing that it is a *generalization* of an **NP**-complete problem.

(a) Subgraph Isomorphism: Given as input two undirected graphs G and H, determine whether G is a subgraph of H (that is, whether by deleting certain vertices and edges of H we obtain a graph that is, up to renaming of vertices, identical to G), and if so, return the corresponding mapping of $V(G)$ into $V(H)$.

(b) Longest Path: Given a graph G and an integer g, find in G a simple path of length g.

(c) Max SAT: Given a CNF formula and an integer g, find a truth assignment that satisfies at least g clauses.

(d) Sparse Subgraph: Given a graph and two integers a and b, find a set of a vertices of G such that there are at most b edges between them.

4. (30 pts.) Another NP-complete Graph Problem

In an undirected graph $G = (V, E)$, we say $D \subseteq V$ is a *dominating set* if every $v \in V$ is either in D or adjacent to at least one member of D. In the DOMINATING SET problem, the input is a graph and a budget b, and the aim is to find a dominating set in the graph size at most b, if one exists. Prove that this problem is **NP**-complete. *(Hint: Reduce from* VERTEX COVER*)*