MATH 203: HOMEWORK 1
DUE BY 5PM ON FRIDAY, JANUARY 24

1) Donald Trump throws a party at his home at which you are invited. To jazz up things, he decides to give out some presents to his guests. The three presents are a brand new Mercedes, an iPad, and a rubber duck. He tells you that you are to make a statement. If your statement is true, he will give you either the Mercedes or the iPad. If your statement is false, however, he will give you the rubber duck. If you'd like to be certain that you get the Mercedes, what sentence should you tell Donald Trump?

2) In the town of Arelate, no man has a beard. The barber shaves only those men who do not shave themselves. Who shaves the barber? What can you deduce about the barber?

3) In the 18th century, the Grand Lodge of Freemasons in London was a highly secretive society following some rather bizarre rules. Each of the freemasons in the lodge had shaved one other member. No freemason in the lodge had ever shaved himself. Furthermore, no freemason was ever shaved by more than one member of the lodge. There was one freemason who had never been shaved at all. The number and identity of the freemasons in the lodge was kept secret. One rumor circulating in London at that time was that there were less than a hundred freemasons in the Grand Lodge of London. Another rumor put the number at over a hundred. Which one of the two rumors is true?

4) Construct the truth tables of the following statements and determine whether each is a tautology, a contradiction, or neither:
  (a) $\neg(P \wedge Q) \to (\neg P \wedge \neg Q)$;
  (b) $(P \wedge Q) \vee (\neg P \wedge R)$;
  (c) $[(P \vee Q) \wedge R] \leftrightarrow [P \vee (Q \wedge R)]$;
  (d) $(P \leftrightarrow Q) \leftrightarrow [(P \wedge Q) \vee \neg(P \vee Q)]$;

5) For each of the following, replace the symbol $*$ with a connective so that the resulting symbolic statement is a tautology. If you can, figure these out without using truth tables, and justify your answer:
  (a) $[(P \to (Q * R)] \leftrightarrow [(P \to Q) \wedge (P \to R)]$;
  (b) $[(P * Q) \to R] \leftrightarrow [(P \to R) \wedge (Q \to R)]$;

(c) $[(P \wedge Q) \leftrightarrow P)] \leftrightarrow [P * Q]$;

(d) $[(P * Q) \rightarrow R)] \leftrightarrow [P \rightarrow (Q \rightarrow R)]$;

6) In class we introduced the "inclusive or" $\vee$ as one of the five standard connectives. In colloquial English, however, there is another "or" called the "exclusive or." An example of the latter is "I will study tonight or I will go to a party." By this it is understood that only one of the two can happen. Let us denote this "exclusive or" by the symbol $\underline{\vee}$.

(a) Construct the truth table for $P \underline{\vee} Q$.

(b) Write a statement using the five standard connectives we introduced that is equivalent to $P \underline{\vee} Q$.

(c) Write a statement using only the connectives $\neg$, $\wedge$, and $\underline{\vee}$ that is equivalent to $P \vee Q$.