MATH 203: HOMEWORK 2
DUE BY 5PM ON FRIDAY, JANUARY 31

1) An Aristotelian syllogism is a very famous classical inference rule. Look it up online, and then write a paragraph or two describing it. Be sure to provide some examples as well.

2) Replace each of the following statements by an equivalent statement that is as short as possible in number of symbols. In some cases, the answer may be the given statement itself.

(a) $\neg(P \to \neg Q)$
(b) $P \to \neg P$
(c) $(P \wedge Q) \vee (P \wedge R)$
(d) $P \vee Q \vee R$
(e) $(P \to Q) \leftrightarrow (Q \to P)$
(f) $P \to (Q \to \neg P)$

3) For each of the following statements, express its **NEGATION** in as short and simple a way as possible. Be sure to mention which tautologies regarding negations you have employed.

(a) Pigs are not blue or dogs are not green.
(b) If x^2 is positive, then x is positive.
(c) Pigs are blue if and only if dogs are not green.
(d) If set A is finite, then set B is finite and not empty.

4) Show that each of the following arguments is valid without resorting to a truth table. Quote tautologies you use by the numbers we gave them in lecture.

(a) Hypotheses: $Q \to R$, $R \vee S \to P$, $Q \vee S$. Conclusion: P.
(b) Hypotheses: $P \to (Q \leftrightarrow \neg R)$, $P \vee \neg S$, $R \to S$, $\neg Q \to \neg R$. Conclusion: $\neg R$.
(c) Babies are illogical. A person who can manage a crocodile is not despised. Illogical persons are despised. Therefore, babies cannot manage crocodiles. (This argument is due to Lewis Carroll, the author of *Alice in Wonderland*).
(d) If I oversleep, I will miss the bus. If I miss the bus, I'll be late for work unless Sue gives me a ride. If Sue's car is not working, she won't give me a ride. If I'm late for work, I'll lose my job

unless the boss is away. Sue's car is not working. The boss is not away. Therefore, if I oversleep, I will lose my job.

5) Show that $\vee$ and $\neg$ together form a complete set of connectives. Recall that we introduced five standard connectives in class. This problem is asking you to show that any statement in propositional logic can be expressed as a statement involving at most the connectives $\vee$ and $\neg$.

6) Starting with any positive number, it is possible to generate a sequence of numbers by the following rules: If the current number is even, the next number is half the current number. If the current number is odd, the next number is 1 more than 3 times the current number. For example, one such sequence begins 26, 13, 40, 20, 10, 5, 16, ...

(a) Choose three or four starting numbers, and for each of them generate the sequence just described. Keep going until the sequence stabilizes in a clear-cut way. (A good range for most of your starting numbers would be between 20 and 50).
(b) On the basis of your results in part (a), make a conjecture about what happens to these sequences, for any starting number.

DO NOT try to prove your conjecture! This is an open problem in number theory.