MATH 203: HOMEWORK 3
DUE BY 5PM ON FRIDAY, FEBRUARY 7

1) Determine whether each of the following statements is a grammatically correct symbolic statement. As usual, P, Q, and R are propositional variables, and x, y and z are mathematical variables. For each statement that is **NOT** grammatically correct, please explain why. For each statement that is grammatically correct, please list its free and bound mathematical variables.

    (a) $\forall x \, P(x, z) \leftrightarrow \exists z \, Q(y, z)$
    (b) $\exists \, (x \wedge y) \, (x > 0 \wedge y > 0)$
    (c) $\forall x \, P(x) \rightarrow \exists x$
    (d) $\neg \forall x \, \neg \forall y \, \neg \forall z \, (2 + 2 = u)$
    (e) $\forall x \, [P(x) \rightarrow \exists z \, (Q(x) \rightarrow \forall y \, R(x, y))]$

2) Represent each of the following statements symbolically, starting with only the following atomic statements: $P(x, y)$ for "x is a parent of y," $W(x)$ for "x is female," and $x = y$ meaning "x and y are the same person." All of your variables should have the set of all people as their domain. You may introduce new propositional variables for statements that you have already written symbolically in a previous part of this question.

    (a) x is y's father.
    (b) x is y's grandmother.
    (c) x is y's sibling (meaning x and y have the same parents, but they are not the same person).
    (d) x is an only child (i.e. x has no siblings).
    (e) x is y's first cousin.
    (f) x has no uncles.

3) Translate the following into symbolic notation. Temporal statements are translating by introducing time t as an additional mathematical variable. As an example, one can define a function $L(x, y, t)$ to stand for "x loves y at time t."

    (a) Not everyone likes spinach, and no one likes asparagus.
    (b) All crows are black, but not all black things are crows.
    (c) It is possible to fool all of the people some of the time and some of the people all the time, but not all people all the time.

  (d) Everybody loves somebody sometimes.
  (e) It is not true in all cases that if one person likes another, the second person likes the first.

4) Translate the following into symbolic notation, first using no existential quantifiers, and second using no universal quantifiers:

  (a) Not all cars have three wheels.
  (b) Some people are either intelligent or hard-working.
  (c) No mouse is heavier than any elephant.
  (d) Every number either is negative or has a square root.

5) Determine whether each of the following statements is true or false if all variables have the set of real numbers as their domain. Justify your answer.

  (a) $\forall x \, \exists y \, (x^2 = y)$
  (b) $\forall y \, \exists x \, (x^2 = y)$
  (c) $\exists x \, \forall y \, (x + 5 = y)$
  (d) $\forall x \, \forall y \, \exists z \, \forall u \, (x + z = y + u)$
  (e) $\forall x \, \forall y \, \exists z \, (x^2 + y^2 = z^2)$
  (f) $\exists x \, [\forall y \, (yx^2 = y) \wedge \neg \forall y \, (yx = y)]$