# MATH 203: HOMEWORK 4
## DUE BY 5PM ON FRIDAY, FEBRUARY 14

1) Translate the following into symbolic notation:

(a) There are at least two objects such that $P(x)$.
(b) There are at least three objects such that $P(x)$.
(c) There are at least $n$ objects such that $P(x)$.

For part (c), the number $n$ is any unspecified positive integer. You are allowed to use $\ldots$ as we did in lecture to denote $n$ statements of the same kind, i.e. as in $P_1 \wedge P_2 \wedge \cdots \wedge P_n$.

2) There is a first order language for plane (Euclidean) geometry. It consists of three sorts of objects: points, lines, and nonnegative real numbers (denoting magnitudes). Operator symbols $+$, $-$, $\times$, and $/$ are included for use on nonnegative real numbers only. Predicate symbols $=$ (referring to all three sorts), $<$, and $>$ (referring to nonnegative real numbers) are also included. In addition, the predicate symbol $On(A, L)$ means point $A$ is on line $L$. The operator symbol $\overline{AB}$ denotes the line segment between points $A$ and $B$, whereas the operator symbol $|\overline{AB}|$ denotes the length of that line segment. Let $A$, $B$, and $C$ denote points. Let $L$ and $M$ denote lines. Translate the following into symbolic notation making use of the quantifier $\exists!$ (wherever appropriate) in addition to the universal and existential quantifiers:

(a) Lines $L$ and $M$ are parallel, i.e. they have no point in common.
(b) Any two *distinct* lines meet in *at most* one point.
(c) Given any two distinct points, there is one and only one line passing through both of them.
(d) Given any line and any point not on that line, there exists one and only one line through that point that is parallel to the given line. (This is one version of Euclid's Postulate/The Parallel Postulate)
(e) $C$ is the midpoint of the line segment $\overline{AB}$. (Do not forget to specify that $C$ is on the line determined by $A$ and $B$.)

3) Part III of the axiomatic system on the handout Axiom.pdf posted in folder General Resources on Piazza concerns axioms fulfilled by the equal sign $=$ .

(a) Which of these equality axioms remain true if the symbol $=$ is replaced throughout by the symbol $\leq$ and all variables are assumed to represent real numbers?
(b) Repeat part (a) using the symbol $<$ .

4) Give an argument/proof for the following statement: If $n$ is any integer, then $n^2 - 3n$ must be even. (Hint: Cases come in handy here. See tautology #26 for the basis of proofs by cases.)

5) Simplify each of the following statements by moving negation signs inward as much as possible:
(a) $\neg \forall\, x, y \,\exists\, z\, (P \vee \neg \forall\, u\, Q)$
(b) $\neg \forall\, x \neg \exists\, y \neg \forall\, z\, (P \wedge \neg Q)$