MATH 203: HOMEWORK 5
DUE BY 5 PM ON WEDNESDAY, FEBRUARY 26

1) Read the first five chapters of Denis Guedj's *The Parrot's Theorem.* Write at least a page (hand-written is OK) but no more than two pages with your reaction to those five chapters and any questions or observations you have about the material contained therein. If you wish, you may look up the work of one of the mathematicians mentioned and write that page on some result/theorem of his/hers that you did not know before.

2) Suppose that we have proved steps $P \leftrightarrow Q$ and $(P \wedge R) \vee (\neg P \wedge S)$ in a proof. State all the different conclusions that can be reached from these two steps using substitution.

3)
    (a) Give an informal/semiformal proof of the following statement: For any positive real number x, the inequality
$$x + \frac{1}{x} \geq 2$$
    holds.
    (b) What happens if you drop the hypothesis that x should be positive?

4) First write the following statement symbolically and then give a formal proof for it: The number 0 does not have a multiplicative inverse.

5) Let x and y be two real numbers.
    (a) Give a formal proof for the following statement: If $0 < x$ and $x < y$, then $x^2 < y^2$.
    (b) Give a formal proof for the following statement: If $x < y$, then $x^3 < y^3$.

6) Let x and y be two real numbers that are not equal to each other.
    (a) Give an informal/semiformal proof that there exists a real number z such that $|z - x| = |z - y|$.
    (b) Give a formal proof that there exists a real number z such that $|z - x| = |z - y|$.

(c) (Extra credit) Give an informal/semiformal proof that the real
number z such that $|z - x| = |z - y|$ is unique.

It might help to think geometrically what the real number z should
be with respect to x and y, i.e. mapping these numbers on the real
number axis might be helpful.