MATH 203: HOMEWORK 8
DUE BY 5PM ON WEDNESDAY, APRIL 2

1) Read chapters 16 to 20 of Denis Guedj's *The Parrot's Theorem*. Write at least a page (hand-written is OK) but no more than two pages with your reaction to those five chapters and any questions or observations you have about the material contained therein. If you wish, you may look up the work of one of the mathematicians mentioned and write that page on some result/theorem of his/hers that you did not know before.

2) Explain where and why the reasoning in the following entertaining but spurious argument is incorrect:

Theorem: All squirrels in the world are the same color.

Proof: We proceed by induction on the number n of squirrels. For $n = 1$ the result is trivially true. We now assume that in any set of n squirrels all have the same color and seek to prove that in any set of $n+1$ squirrels they all have the same color. Consider a set A of $n+1$ squirrels. Let a be a squirrel in the set A. By the induction hypothesis, all squirrels in the set $A - \{a\}$ are the same color. Let b be another squirrel (different from a) in the set A. By the induction hypothesis, all squirrels in the set $A - \{b\}$ have the same color. Therefore, a and b must have the same color as the rest of the squirrels in the set A. It follows all squirrels in the set A of $n+1$ squirrels have the same color. $\square$

3) For each of the following statements, either prove that it is true for all sets or find a counterexample to show it fails. For parts (a) and (b), if the statement is not always true, try to prove that one side must be a subset of the other side.

(a) $A \cup (B - C) = (A \cup B) - (A \cup C)$
(b) $(A - B) \cup (A - C) = A - (B \cup C)$
(c) $A \subseteq (B \cup C)$ iff $A \subseteq B$ or $A \subseteq C$

4) For any sets A and B, define $A \,\Delta\, B$, the **symmetric difference** of A and B to be the set $(A - B) \cup (B - A)$. Prove:

(a) commutativity of $\Delta : A \,\Delta\, B = B \,\Delta\, A$
(b) the empty set is the identity for $\Delta : A \,\Delta\, \emptyset = A$

(c) each set is its own inverse for Δ : $A \,\Delta\, A = \emptyset$
(d) $\cap$ distributes over Δ : $A \cap (B \,\Delta\, C) = (A \cap B) \,\Delta\, (A \cap C)$