MATH 203: HOMEWORK 9
DUE BY 5PM ON FRIDAY, APRIL 4

1) Finish Denis Guedj's *The Parrot's Theorem* (chapters 20 to 26). Write at least a page (hand-written is OK) but no more than two pages with your reaction to those five chapters and any questions or observations you have about the material contained therein. If you wish, you may look up the work of one of the mathematicians mentioned and write that page on some result/theorem of his/hers that you did not know before.

2) Let $B = \{(x, y) \mid y = x\}$, where $x, y \in \mathbb{R}$. For each real number r, let $A_r = \{(x, y) \mid x^2 + y^2 = r^2\}$. Then $\{A_r \mid r \in \mathbb{R}\}$ is a collection of sets (circles) indexed by $\mathbb{R}$.

(a) Using $\mathbb{R}$ as the index set makes most of the circles in this collection be repeated. Name two smaller index sets that can be used to define this collection of circles without any repetition.
(b) What is the union of all the $A_r' s$?
(c) Describe $B \cap A_r$.
(d) Verify that $B \cap \left(\bigcup_{r\in\mathbb{R}} A_r\right) = \bigcup_{r\in\mathbb{R}} (B \cap A_r)$

3) Consider the Cantor set defined inductively as follows. Let $A_0 = [0, 1]$. Remove the middle third of this interval to obtain $A_1 = [0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$. Remove the middle third of each of those two intervals to obtain A_2. From A_n remove the middle third of each of the 2^n intervals in it to obtain A_{n+1}. Let $C = \bigcap_{n=1}^{\infty} A_n$.

(a) Let the length of an interval $[a, b]$ be by definition $b - a$. The length of (a, b) is likewise $b - a$. The length of a collection of disjoint intervals is the sum of the lengths of the intervals in the collection. Compute the length of each A_n in the definition of the Cantor set in terms of n. Show the length of A_n goes to 0 as n goes to ∞.
(b) Let a positive real number $\epsilon > 0$ be given. Show that for n large enough, there exists a collection of **open** intervals whose union has length at most ϵ that properly contains A_n.

4) For each of the following statements, determine whether it is either true or false and give a brief justification for your answer:

(a) $3 \in \mathcal{P}(\mathbb{N})$
(b) $\{3\} \in \mathcal{P}(\mathbb{N})$
(c) $\{3\} \subseteq \mathcal{P}(\mathbb{N})$
(d) $\{\emptyset\} \in \mathcal{P}(\{\{\emptyset\}\})$
(e) $\mathcal{P}(\mathbb{Z} \cap (2,4)) = \{\emptyset, \{3\}\}$, where $(2,4)$ means the interval with endpoints 2 and 4 on the real line.

5) In the country of Tannu Tuva, a valid license plate consists of any digit except 0, followed by any two letters of the alphabet, followed by any two digits.

(a) Let D be the set of all digits and L the set of all letters. With this notation, write the set of all possible license plates as a Cartesian product.
(b) How many possible license plates are there?

6) Let A be the set of all people who have ever lived. For each of the following, prove whether or not it is an equivalence relation on A. If it is an equivalence relation, describe its equivalence classes and give an example of an equivalence class.

(a) x and y were born in the same year.
(b) x and y were born less than a week apart.
(c) x and y have the same maternal grandfather.
(d) x and y are first cousins or $x = y$.