MATH 203: HOMEWORK 10
DUE BY 5PM ON WEDNESDAY, APRIL 16

1) Determine which of the following are groups and justify your answer.
   - (a) All complex numbers of absolute value 1 under multiplication;
   - (b) All complex numbers z of absolute value 1 under the operation $z \circ z' = |z| \cdot z'$.
   - (c) All irrational numbers under multiplication;
   - (d) All integers under the operation of subtraction.

2) In the set of equivalence classes of integers modulo 11 under multiplication denoted by $\mathbb{Z}_{11}$, which of the following form groups? Justify your answers. (Hint: It might help to draw multiplication tables as we did in class.)
   - (a) $(1, 3, 4, 5, 9)$
   - (b) $(1, 3, 5, 7, 8)$
   - (c) $(1, 8)$

3) Let a, b, and c be fixed elements of a group. Prove the equation $xaxba = xbc$ has one and only one solution.

4) In a group of $2n$ elements, prove there is another element besides the identity which is its own inverse.

5) Show that any group in which every element a satisfies $a^2 = e$ has to be abelian.

6) Prove that any group with at most 4 elements (i.e. with four elements or fewer) has to be Abelian. (Hint: In the case of a group with exactly four elements e, a, b, and ab, show that ba has to be one of these.)